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## ABSTRACT

### International Financial Integration and Crisis Contagion\*

International financial integration helps to diversify risk but also may increase the transmission of crises across countries. We provide a quantitative analysis of this trade-off in a two-country general equilibrium model with endogenous portfolio choice and collateral constraints. Collateral constraints bind occasionally, depending upon the state of the economy and levels of inherited debt. The analysis allows for different degrees of financial integration, moving from financial autarky to bond market integration and equity market integration. Financial integration leads to a significant increase in global leverage, doubles the probability of balance sheet crises for any one country, and dramatically increases the degree of 'contagion' across countries. Outside of crises, the impact of financial integration on macro aggregates is relatively small. But the impact of a crisis with integrated international financial markets is much less severe than that under financial market autarky. Thus, a trade-off emerges between the probability of crises and the severity of crises. Financial integration can raise or lower welfare, depending on the scale of macroeconomic risk. In particular, in a low risk environment, the increased leverage resulting from financial integration can reduce welfare of investors.

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Keywords: financial contagion, international financial integration, leverage and occasionally binding contracts

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# 1 Introduction

Since the global financial crisis, there has been a significant reevaluation of the effects of financial market integration. It is widely acknowledged that financial linkages between countries were critical to the rapid transmission of the crisis across national borders. A large empirical and theoretical literature has explored questions related to the nature of this transmission (see for instance, [Reinhart and Rogoff, 2009](#); [Mishkin, 2011](#); [Campello, Graham and Harvey, 2010](#)).<sup>1</sup> Many of these papers present detailed accounts of the origin and nature of the crisis.

The paper is motivated by the events of the global crisis but takes a more general perspective on the nature of international financial markets in the presence of market failures within the domestic financial system. We present an analysis of the transmission of crises in a two-country general equilibrium model with endogenous portfolio choice and collateral constraints arising from the absence of full legal enforcement of contracts. In our model, collateral constraints bind occasionally, depending on inherited debt burdens and shocks to productivity. We study three stages of financial integration: financial autarky, bond market integration and equity market integration. In each type of financial regime, an investor must raise external funds from lenders to invest in a project, but faces a collateral constraint because of default risk. In financial autarky, an investor borrows only from domestic lenders. In bond market integration, an investor obtains funding from a global bank that accepts deposits from savers in all countries. In equity market integration, investors borrow from a global bank but can make investments in domestic or foreign projects.

The aim of the paper is to explore how different levels of international financial market integration effect the level of risk-taking that investor-borrowers are willing to engage in, to explore how financial markets affect the probability of financial crises and the international transmission of crises, and what financial markets imply for the nature and severity of crises. Given answers to these questions, we can investigate the welfare effects of financial market

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<sup>1</sup>Others include [Almunia, Benetrix, Eichengreen, O'Rourke and Rua \(2010\)](#), [Puri, Rocholl and Steffen \(2011\)](#), [Chudik and Fratzscher \(2011\)](#), [Perri and Quadrini \(2011\)](#), [Eaton, Kortum, Neiman and Romalis \(2011\)](#), [Kalemli-Ozcan, Papaioannou and Perri \(2013\)](#) and [Fratzscher \(2012\)](#).

integration within a framework of endogenous financial crises. A key novelty of the analysis is that we explore these questions within a full multi-country general equilibrium model, where world interest rates, asset prices, and capital flows are all endogenously determined. In this model, crises can be specific to one country, or can occur in all countries simultaneously.

The model embodies a central trade-off inherent in much recent discussion of the nature of financial markets and international financial crises. Integrated financial markets facilitate inter-temporal capital flows and portfolio diversification, and by doing so, help to defray country-specific risk. But at the same time, more open capital markets may increase the probability of financial crises and the contagion of crises across countries. By constructing a model with endogenous portfolio choice, endogenous leverage, and endogenous financial crises which are affected by these decisions, as well as the opportunities offered by international financial markets, we can explore the nature and characteristics of this trade-off in detail.

Our results closely reflect this two-fold nature of financial market integration. First, we find that financial integration tends to increase investor leverage and risk-taking, over and above the opportunities that it affords for portfolio diversification and inter-temporal borrowing and lending. Two channels are critical for this linkage between financial opening and increased risk-taking. First, by increasing the value of existing asset holdings, financial integration increases the collateral value of investors' portfolios and facilitates an increase in borrowing capacity. Secondly, and critically, by reducing overall consumption risk, financial integration reduces precautionary saving and leads to an increase in investor's desire to borrow.

As a result of the increase in global leverage, we find that financial market integration increases the unconditional probability of financial crises. In addition, due to the linkage of borrowing costs and asset prices through international financial markets, the contagion of crises across countries is substantially higher after financial market liberalization. Because investors do not take account of how their borrowing and investment decisions impact the probability of financial crises, this represents a negative externality which reduces the social welfare benefits of financial liberalization.

While we find that financial crises are more likely in an integrated world financial market, crises are much less severe in terms of lost output and consumption than those in financial autarky. During ‘normal times’ (or in the absence of crises), the impact of financial integration is rather small - financial market openness improves allocative efficiency modestly and has a benefit in terms of slightly lower output and consumption volatility. But in a financial crisis, the output and consumption losses are much greater in an environment of financial autarky. Hence, while we see more crises unconditionally in an environment of international financial integration, they are milder events, and the costs are more evenly spread amongst countries.

In welfare terms, we can ask whether, given the existence of a trade-off between the probability of crises and the severity of crises, there is always a net gain from financial market integration. Our results indicate that this depends on the overall level of global risk. In an environment of high risk, the benefits of diversification exceed the costs of increased crisis occurrence, and both investors and savers are always better off with open capital markets. But with a lower risk environment, induced effects of financial integration on leverage and crisis probability can be more important, and we find that investors can be worse off with open financial markets than in financial market autarky.

This paper contributes to several branches of literature. First of all, the paper provides a direct link between financial integration and transmission of crisis across countries. The question of the international transmission of crises has attracted much attention in recent literature.<sup>2</sup> Recently, several authors have developed models of crisis transmission in a two-country framework with financial frictions. A paper closely related to our work is [Perri and Quadrini \(2011\)](#). They assume that investors can perfectly share their income risk across borders and consequently investors in both countries simultaneously face either slack collateral constraints or tight collateral constraints. In another words, the conditional probability of one country being in a crisis given that the other is in a crisis is one. There are two main differences between their work and ours. First, we investigate endogenous portfolio decisions made by investors, and risk sharing is imperfect between investors across borders, while they

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<sup>2</sup>See recent work on the global financial crisis by [Cetorelli and Goldberg \(2010\)](#), [de Haas and van Lelyveld \(2010\)](#), [Aizenman \(2011\)](#), [Aizenman and Hutchison \(2012\)](#) and others; on 1998 Russian default by [Schnabl \(2012\)](#).

focus mainly on perfect risk sharing for investors. Second, the mechanism is quite different. We study a channel of fire sales, in which both asset prices and quantities of assets adjust endogenously to exogenous shocks, while they focus only on the quantity adjustment of assets. Another related paper is [Kalemli-Ozcan, Papaioannou and Perri \(2013\)](#). They study a global banker who lends to firms in two countries and focus on a bank lending channel. Firms in both countries need to finance their working capital via borrowing from a banker in advance. Variation in the interest payment for working capital charged by the global banker in both countries delivers a transmission of crises across borders. Our model is quite different theirs and emphasizes the balance sheet channel of firms (investors). Moreover, we provide a model of endogenous portfolio choice by investors (firms) and bankers, and study explicitly the transmission of crises through the fire sale of assets.

Second, this paper makes a contribution to studying slow recovery in economies with occasionally binding borrowing constraints. We explore an international dimension of slow recovery. Based on a closed economy with lenders, borrowers and occasionally binding borrowing constraints, [Brunnermeier and Sannikov \(2013\)](#) show that there are two modes in the distribution of constrained borrowers' net worth. We confirm their result in our model, using the distribution of consumption for investors. Moreover, we find *multiple modes* in the consumption distribution in financial integration. There is one major mode, in which investors are financially unconstrained, and there exist at least two minor modes, in which investors in either of countries are financially constrained.

Third, this paper is complementary to a burgeoning literature of international portfolio choice (see [Devereux and Sutherland, 2011a](#); [Devereux and Sutherland, 2010](#); [Tille and van Wincoop, 2010](#) and others). Compared to these work with approximation around a deterministic steady state, we develop a model with occasionally binding collateral constraints and solve this model using a global solution method based on [Dumas and Lyasoff \(2012\)](#) and [Judd, Kubler and Schmedders \(2002\)](#). In the model, we obtain a stochastic steady state of portfolios. In terms of model setups, this paper is a variation of [Devereux and Yetman \(2010\)](#) and [Devereux and Sutherland \(2011b\)](#). They focus on a case with collateral constraints being

always binding, while we consider a model with occasionally binding collateral constraints and address quite different issues.

In a related sense, our paper makes a modest computational step forward by computing equilibria of a infinite horizon stochastic general equilibrium model with incomplete markets and multiple endogenous state variables. We make use of a recently developed method method of backward induction for stochastic general equilibrium models with incomplete markets that is outlined in [Dumas and Lyasoff \(2012\)](#).

The paper is organized as follows. Section 2 analyzes a two-country financial integration model with equity market integration, bond market integration and financial autarky. The algorithm for solving the model, some computational issues, and calibration assumptions are discussed in 3. A perfect foresight special case of the model is presented in 4. Section 5 provides the main results. The last section concludes. All detailed issues related to the solution of the model are contained in a Technical Appendix at the end of the paper.

## 2 A Two-country General Equilibrium Model of Investment and Leverage

Here we set out a basic model where there are two countries, each of which contains borrowers and lenders, and lenders make risky levered investments subject to constraints on their total borrowing. The baseline is similar to [Devereux and Yetman \(2010\)](#) and [Devereux and Sutherland \(2011b\)](#), which is essentially an international version of [Kiyotaki and Moore \(1997\)](#), except extended to allow for uncertainty in investment returns. There are two countries, country 1 (home) and 2 (foreign) in the world economy. In each country, a number of firm-investors with a measure of population  $n$  make consumption decisions, borrow from bankers and purchase capital to invest in equity markets. Investors also supply labor and earn labor income from competitive labor markets. A remaining population of  $1 - n$  workers (savers) operate capital in the informal backyard production sector, supply labor, and save in the form of risk-free debt. There is a competitive banking sector that operates in both

countries. Bankers raise funds from workers and lend to investors. We look at varying degrees of financial market integration between the two countries. In financial autarky, savers lend to domestic banks, who in turn lend to home investors, and investors can only make investments in the domestic technology (or domestic equity). In bond market integration, there is a global bank that raises funds from informal savers in both countries, and extends loans to investors. But investors are still restricted to investing in the domestic technology. Finally, in equity market integration, investors borrow from the global bankers but may make investments in the equity of either country. In all environments, there is a fixed stock of capital which may be allocated to the informal backyard sector or the domestic investment technology. Capital cannot be physically transferred across countries.

## 2.1 Equity market integration

It is convenient to first set out the model in the case of full equity market integration, and then later describe how this model is restricted in the case of bond market integration, or financial autarky. When investors trade equity assets across borders, there are three types of assets in their portfolios including domestic equities, foreign equities and borrowing from bankers.

### 2.1.1 Firm-investors

The budget constraint for a representative firm-investor in country  $l = 1, 2$ , reads as

$$-\frac{b_{l,t+1}}{R_{t+1}} + c_{l,t} + q_{1,t}k_{1,t+1}^l + q_{2,t}k_{2,t+1}^l = d_{l,t} + W_{l,t}h_{l,t} + k_{1,t}^l(q_{1,t} + R_{1k,t}) + k_{2,t}^l(q_{2,t} + R_{2k,t}) - b_{l,t} \quad (1)$$

The right hand side of the budget constraint states income sources including labor income  $W_{l,t}h_{l,t}$ , profit from the ownership of domestic firms,  $d_{l,t}$ , gross return on equities issued by country 1 and held by investor  $l$ ,  $k_{1,t}^l(q_{1,t} + R_{1k,t})$ , gross return on equity 2,  $k_{2,t}^l(q_{2,t} + R_{2k,t})$ , less debt owed to the bank  $b_{l,t}$ . The left hand side denotes the investor's consumption  $c_{l,t}$ , and portfolio decisions,  $(k_{1,t+1}^l, k_{2,t+1}^l, b_{l,t+1})$ . Asset prices for country 1 and 2 equities and the international bond are  $q_{1,t}$ ,  $q_{2,t}$ ,  $\frac{1}{R_{t+1}}$ , respectively. Dividends for equities come from the

marginal product of capital,  $R_{1k,t}$  and  $R_{2k,t}$ .

Profit  $d_{l,t}$  is then defined as

$$d_{l,t} \equiv \frac{1}{n} [F(A_{l,t}, H_{l,t}, K_{l,t}) - W_{l,t}H_{l,t} - K_{l,t}R_{lk,t}]$$

where the total cost of labor services then reads  $W_{l,t}H_{l,t}$ .

Investors need to finance their inter-period borrowing to smooth their consumption over time. They face a collateral (or leverage) constraint as in Kiyotaki and Moore (1997) when borrowing from a bank

$$b_{l,t+1} \leq \kappa E_t \{q_{1,t+1}k_{1,t+1}^l + q_{2,t+1}k_{2,t+1}^l\} \quad (2)$$

where  $\kappa$  characterizes the upper bound for loan-to-value.

We assume that investors can collateralize all of their equity assets at hand to finance borrowing from bankers. Domestic and foreign equity assets are perfect substitutes when they are used to obtaining external funds for investors in either of the countries.<sup>3</sup> Preferences of investors are given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta_l^t U(c_{l,t}, h_{l,t}) \right\}$$

where  $0 < \beta_l < 1$  is investors' subjective discount factor and  $U(c_{l,t}, h_{l,t})$  is their utility function.  $E_0$  stands for mathematical expectations conditional on information up to period 0.

The optimality condition for labor supply states that marginal rate of substitution between consumption and leisure equals the wage rate,

$$-\frac{U_h(c_{l,t}, h_{l,t})}{U_c(c_{l,t}, h_{l,t})} = W_{l,t}, \quad l = 1, 2 \quad (3)$$

where  $U_c(c_{l,t}, h_{l,t})$  and  $U_h(c_{l,t}, h_{l,t})$  denote an investor's marginal utility of consumption and labor supply in country  $l$ , respectively.

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<sup>3</sup>Several recent studies explore asymmetric efficiency of channeling funds through financial markets (Mendoza, Quadrini and Rios-Rull, 2009) or through financial intermediations (Maggiore, 2012) across countries.

Let the Lagrange multiplier for the collateral constraint (2) be  $\mu_{l,t}$ . The optimal holdings of equity for investors must satisfy the following conditions:

$$q_{i,t} = \frac{\beta_l E_t U_c(c_{l,t+1}, h_{l,t+1})(q_{i,t+1} + R_{ik,t+1}) + \mu_{l,t} \kappa E_t q_{i,t+1}}{U_c(c_{l,t}, h_{l,t})}, \quad i = 1, 2 \quad (4)$$

The left hand side is the cost of one unit of equity at time  $t$ . The right hand side indicates that the benefit of an additional unit of equity is twofold. First, there is a direct increase in wealth in the next period from the direct return on capital plus the value of equity. Secondly, an additional unit of equity relaxes the collateral constraint (2). If  $\mu_t > 0$ , then this increases the borrowing limit at time  $t$ .

The optimal choice of bond holdings must satisfy

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_l E_t U_c(c_{l,t+1}, h_{l,t+1}) + \mu_{l,t}}{U_c(c_{l,t}, h_{l,t})} \quad (5)$$

When the collateral constraint (2) is binding, reducing one unit of borrowing has an extra benefit  $\mu_{l,t}$ , by relaxing the constraint. Rearranging the equation above yields,

$$1 = \frac{\beta_l E_t U_c(c_{l,t+1}, h_{l,t+1}) R_{t+1}}{U_c(c_{l,t}, h_{l,t})} + EFP_{l,t+1}$$

where  $EFP_{l,t+1} \equiv \frac{\mu_{l,t} R_{t+1}}{U_c(c_{l,t}, h_{l,t})}$  measures the external finance premium in terms of consumption in period  $t$  faced by investors in country  $l$ .

The complementary slackness condition for the collateral constraint (2) can be written as

$$(\kappa E_t \{q_{1,t+1} k_{1,t+1}^l + q_{2,t+1} k_{2,t+1}^l\} - b_{l,t+1}) \mu_{l,t} = 0 \quad (6)$$

with  $\mu_{l,t} \geq 0$ .

The critical focus of the computation will be to investigate the extent to which constraint (2) binds in an equilibrium that is represented by a stationary distribution of decision rules made by savers and investors, and to see how that depends on the realizations of productivity, and on the degree of financial market integration across countries. The fact that productivity

is stochastic, inducing riskiness in the return on equities to investors, is critical. In a deterministic environment, the constraint will always bind in a steady state equilibrium (with any degree of financial market integration). This is because following [Kiyotaki and Moore \(1997\)](#), we assume that investors are more impatient than savers. Thus, in an infinite horizon budgeting plan, investors will always wish to front-load their consumption stream so much that (2) will always bind. But this is not true generally, in a stochastic economy, since then investors will have a precautionary savings motive that leads them to defer consumption as a way to self-insure against low productivity (and binding collateral constraints) states in the future.

### 2.1.2 Global bankers

In both countries, there are worker/savers who supply labor, earn income, employ capital to use in an informal sector, and save. They save by making deposits in a ‘bank’, which in turn makes loans to investors. Workers are subject to country specific risk coming from fluctuations in wage rates and in the price of domestic capital. Because idiosyncratic variation in workers consumption and savings decisions plays no essential role in the transmission of productivity shocks across countries, we make a simplification that aids the model solution by assuming that with financial market integration (either bond market integration or equity market integration), workers’ preferences are subsumed by a ‘global banker’ who receives their deposits and chooses investment and lending to maximize utility of the global representative worker. Hence, there is full risk-sharing across countries among worker/savers. This assumption substantially simplifies the computation of equilibrium without changing the nature of the results in any essential way.

Hence, there is a representative banker in the world financial market. The banker runs two branches, one branch in each country with which the representative worker/saver conducts business. As we noted, the worker receives a competitive wage rate in the local labor market and operates a safe but informal production function. Given perfect risk sharing among workers within the bank, the objective of the banker is to maximize a representative worker’s

lifetime utility. Let subscript 3 indicate variables for the banker. The budget constraint for the global banker can be written as

$$\begin{aligned}
-\frac{b_{3,t+1}}{R_{t+1}} + \frac{c_{1,t}^3 + c_{2,t}^3}{2} + \frac{q_{1,t}k_{1,t+1}^3 + q_{2,t}k_{2,t+1}^3}{2} &= \frac{W_{1,t}h_{1,t}^3 + W_{2,t}h_{2,t}^3}{2} + \frac{q_{1,t}k_{1,t}^3 + q_{2,t}k_{2,t}^3}{2} \\
&- b_{3,t} + \frac{G(k_{1,t}^3) + G(k_{2,t}^3)}{2}
\end{aligned} \tag{7}$$

The left hand side of the equation above states expenditures for a representative worker in the bank, including borrowing  $\frac{b_{3,t+1}}{R_{t+1}}$ , consumption  $c_{l,t}^3$  and physical capital  $\frac{q_{1,t}k_{1,t+1}^3 + q_{2,t}k_{2,t+1}^3}{2}$  employed in informal production sectors. The right hand side describes labor income per worker  $\frac{W_{1,t}h_{1,t}^3 + W_{2,t}h_{2,t}^3}{2}$ , the value of existing capital holdings  $\frac{q_{1,t}k_{1,t}^3 + q_{2,t}k_{2,t}^3}{2}$ , debt repayment  $b_{3,t}$  and production in the informal sectors  $\frac{G(k_{1,t}^3) + G(k_{2,t}^3)}{2}$ .  $G(k_{1,t}^3)$  and  $G(k_{2,t}^3)$  denote the production technology in the informal sector of country 1 and 2 with physical capital  $k_{1,t}^3$  and  $k_{2,t}^3$  as inputs, respectively. We assume that  $G(\cdot)$  is increasing and concave.

The global banker internalizes the preferences of worker savers, maximizing an objective function given by

$$E_0 \left\{ \left( \frac{1}{2} \right) \sum_{t=0}^{\infty} \beta_3^t \{ U(c_{1,t}^3, h_{1,t}^3) + U(c_{2,t}^3, h_{2,t}^3) \} \right\}$$

where  $\beta_3$  stands for the subjective discount factor for a worker. As noted above, we assume that workers are more patient than investors, so that  $\beta_l < \beta_3 < 1$ . This ensures that investors are borrowers in the stationary equilibrium.

The optimality condition for labor supply in each country is

$$-\frac{U_h(c_{l,t}^3, h_{l,t}^3)}{U_c(c_{l,t}^3, h_{l,t}^3)} = W_{l,t}, \quad l = 1, 2 \tag{8}$$

In addition, through the global banker, consumption risk-sharing among worker/savers across borders is attained, so that

$$U_c(c_{1,t}^3, h_{1,t}^3) = U_c(c_{2,t}^3, h_{2,t}^3) \quad (9)$$

The optimal choices of capital holdings for the banker are represented as:

$$q_{i,t} = \frac{\beta_3 E_t U_c(c_{i,t+1}^3, h_{i,t+1}^3)(q_{i,t+1} + G'(k_{i,t+1}^3))}{U_c(c_{i,t}^3, h_{i,t}^3)}, \quad i = 1, 2 \quad (10)$$

Finally the optimality condition for the global banker bond holdings is as follows

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_3 E_t U_c(c_{i,t+1}^3, h_{i,t+1}^3)}{U_c(c_{i,t}^3, h_{i,t}^3)} \quad (11)$$

We note that the global banker faces no separate borrowing constraints such as (2).

### 2.1.3 Production and market clearing conditions

In the formal sector of country  $i$  with  $i = 1, 2$ , goods are produced by competitive goods producers, who hire domestic labor services and physical capital in competitive factor markets. Taking the formal sector production function as  $Y_{i,t} = F(A_{i,t}, H_{i,t}, K_{i,t})$ , where  $A_{i,t}$  represents an exogenous productivity coefficient, we have in equilibrium, the wage rate equaling the marginal product of labor and the return on capital being equal to the marginal product of capital

$$W_{i,t} = F_h(A_{i,t}, H_{i,t}, K_{i,t}), \quad i = 1, 2 \quad (12)$$

$$R_{ik,t} = F_k(A_{i,t}, H_{i,t}, K_{i,t}), \quad i = 1, 2 \quad (13)$$

Labor market and rental market clearing conditions are written as

$$H_{i,t} = nh_{i,t} + (1 - n)h_{i,t}^3, \quad i = 1, 2. \quad (14)$$

Total labor employed is the sum of employment of investors and savers.

$$K_{i,t} = n(k_{i,t}^1 + k_{i,t}^2), \quad i = 1, 2. \quad (15)$$

Capital employed in the formal sector is the sum of equity holdings of the domestic capital stock by both domestic and foreign investors.

Asset market clearing conditions read

$$nk_{1,t+1}^1 + nk_{1,t+1}^2 + (1-n)k_{1,t+1}^3 = 1, \quad nk_{2,t+1}^1 + nk_{2,t+1}^2 + (1-n)k_{2,t+1}^3 = 1 \quad (16)$$

$$nb_{1,t+1} + nb_{2,t+1} + 2(1-n)b_{3,t+1} = 0 \quad (17)$$

The top line says that equity markets in each country must clear, while the bottom line says that bond market clearing ensures that the positive bond position of the global bank equals the sum of the bond liabilities of investors in both countries.

Finally, there is only one world good, so the global resource constraint can be written as:

$$n(c_{1,t} + c_{2,t}) + (1-n)(c_{1,t}^3 + c_{2,t}^3) = F(A_{1,t}, H_{1,t}, K_{1,t}) + F(A_{2,t}, H_{2,t}, K_{2,t}) + (1-n)(G(k_{1,t}^3) + G(k_{2,t}^3)) \quad (18)$$

#### 2.1.4 A competitive recursive stationary equilibrium

We define a competitive equilibrium which consists of a sequence of allocations  $\{c_{l,t}\}_{t=0,1,2,\dots}$ ,  $\{c_{l,t}^3\}_{t=0,1,2,\dots}$ ,  $\{k_{l,t}^i\}_{t=0,1,2,\dots}$ ,  $\{b_{i,t}\}_{t=0,1,2,\dots}$ ,  $\{h_{l,t}\}_{t=0,1,2,\dots}$ ,  $\{h_{l,t}^3\}_{t=0,1,2,\dots}$ ,  $\{H_{l,t}\}_{t=0,1,2,\dots}$ ,  $\{K_{l,t}\}_{t=0,1,2,\dots}$ , a sequence of prices  $\{q_{i,t}\}_{t=0,1,2,\dots}$ ,  $\{W_{l,t}\}_{t=0,1,2,\dots}$  and  $\{R_{lk,t}\}_{t=0,1,2,\dots}$ , and a sequence of Lagrange multipliers  $\{\mu_{l,t}\}_{t=0,1,2,\dots}$ , with  $l = 1, 2$ ,  $i = 1, 2, 3$ , such that (a) consumption  $\{c_{l,t}\}_{t=0,1,2,\dots}$ ,  $\{c_{l,t}^3\}_{t=0,1,2,\dots}$ , labor supply,  $\{h_{l,t}\}_{t=0,1,2,\dots}$ ,  $\{h_{l,t}^3\}_{t=0,1,2,\dots}$ , with  $l = 1, 2$ , and portfolios  $\{k_{l,t}^i\}_{t=0,1,2,\dots}$ ,  $\{b_{i,t}\}_{t=0,1,2,\dots}$ , with  $l = 1, 2$ ,  $i = 1, 2, 3$ , solve the investors' and bankers' problem; (b) labor demand  $\{H_{l,t}\}_{t=0,1,2,\dots}$  and physical capital demand  $\{K_{l,t}\}_{t=0,1,2,\dots}$ , with  $l = 1, 2$ , solve for firms' problem; (c) wages  $\{W_{l,t}\}_{t=0,1,2,\dots}$ , with  $l = 1, 2$ , clear labor markets and  $\{R_{lk,t}\}_{t=0,1,2,\dots}$ , with  $l = 1, 2$ , clear physical capital markets; (d) asset prices  $\{q_{i,t}\}_{t=0,1,2,\dots}$ , with  $i = 1, 2, 3$ , clear the corresponding equity markets and bond markets; (e) the associated Lagrange multipliers  $\{\mu_{l,t}\}_{t=0,1,2,\dots}$ , with  $l = 1, 2$ , satisfy the complementary slackness conditions.

Our interest is in developing a global solution to the model, where the collateral constraint

may alternate between binding and non-binding states. A description of the solution approach is contained in Section 3 below, and fully exposited in the Technical Appendix.

## 2.2 Bond market integration

We wish to compare the equilibrium with fully integrated global equity markets with one where there is restricted financial market integration. Take the case where there is a global bond market, but equity holdings are restricted to domestic agents. All returns on capital in the formal sector must accrue to domestic firm-investors, although they can finance investment by borrowing from the Global Bank. To save space, we outline only the equations that differ from the case of equity market integration.

A representative firm-investor's budget constraint in the bond market integration case is given by

$$-\frac{b_{l,t+1}}{R_{t+1}} + c_{l,t} + q_{l,t}k_{l,t+1}^l = d_{l,t} + W_{l,t}h_{l,t} + k_{l,t}^l(q_{l,t} + R_{lk,t}) - b_{l,t}. \quad (19)$$

The collateral constraint now depends only on domestic equity values

$$b_{l,t+1} \leq \kappa E_t q_{l,t+1} k_{l,t+1}^l \quad (20)$$

A firm-investor maximizes his life-time utility

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta_t^l U(c_{l,t}, h_{l,t}) \right\}$$

Consumption Euler equations for portfolio holdings imply

$$q_{l,t} = \frac{\beta_l E_t \{ U_c(c_{l,t+1}, h_{l,t+1}) (q_{l,t+1} + R_{lk,t+1}) \} + \mu_{l,t} \kappa E_t \{ q_{l,t+1} \}}{U_c(c_{l,t}, h_{l,t})}, \quad l = 1, 2 \quad (21)$$

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_l E_t \{ U_c(c_{l,t+1}, h_{l,t+1}) \} + \mu_{l,t}}{U_c(c_{l,t}, h_{l,t})} \quad (22)$$

The complementary slackness condition implied by the collateral constraint (20) can be

written as

$$(\kappa E_t \{q_{l,t+1} k_{l,t+1}^l\} - b_{l,t+1}) \mu_{l,t} = 0 \quad (23)$$

with  $\mu_{l,t} \geq 0$ .

The global banker's problem is identical to the condition under equity market integration, and so is omitted here.

Market clearing conditions for rental and equity assets are as follows

$$K_{l,t} = nk_{l,t}^l, \quad l = 1, 2 \quad (24)$$

$$nk_{1,t+1}^1 + (1-n)k_{1,t+1}^3 = 1, \quad nk_{2,t+1}^2 + (1-n)k_{2,t+1}^3 = 1 \quad (25)$$

A competitive recursive stationary equilibrium in bond market integration is similar to equity market integration in section 2.1.4.

## 2.3 Financial autarky

Finally, we define a market structure without any financial interaction between countries at all, referred to as financial autarky. Since there is only a single good in the world economy, in financial autarky there is no trade between countries at all, so the two countries are essentially closed economies. Investors obtain external funds only from local bankers and hold only local equity assets. Therefore, their budget constraints and collateral constraints are the same as in bond market integration (equation (19)-(20)). Now, local bankers receive deposits only from local savers.

A representative local banker's problem in country  $l = 1, 2$  reads

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta_3^t U(c_{l,t}^3, h_{l,t}^3) \right\}$$

subject to

$$-\frac{b_{3,t+1}^l}{R_{t+1}} + c_{l,t}^3 + q_{l,t} k_{l,t+1}^3 = W_{l,t} h_{l,t}^3 + q_{l,t} k_{l,t}^3 - b_{3,t}^l + G(k_{l,t}^3) \quad (26)$$

The optimality conditions yield

$$-\frac{U_h(c_{l,t}^3, h_{l,t}^3)}{U_c(c_{l,t}^3, h_{l,t}^3)} = W_{l,t}, \quad l = 1, 2 \quad (27)$$

$$q_{l,t} = \frac{\beta_3 E_t U_c(c_{l,t+1}^3, h_{l,t+1}^3)(q_{l,t+1} + G'(k_{l,t+1}^3))}{U_c(c_{l,t}^3, h_{l,t}^3)} \quad (28)$$

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_3 E_t U_c(c_{l,t+1}^3, h_{l,t+1}^3)}{U_c(c_{l,t}^3, h_{l,t}^3)} \quad (29)$$

The market clearing condition for the domestic bond market now becomes

$$nb_{l,t+1} + (1-n)b_{3,t+1}^l = 0. \quad (30)$$

The resource constraint in financial autarky is written as

$$nc_{l,t} + (1-n)c_{l,t}^3 = F(A_{l,t}, H_{l,t}, K_{l,t}) + (1-n)G(k_{l,t}^3) \quad (31)$$

A competitive recursive stationary equilibrium in financial autarky is similar to equity market integration in section 2.1.4.

## 3 Calibration and Model Solution

### 3.1 Specific functional forms

We make the following set of assumptions regarding functional forms for preferences and technology. First, all agents are assumed to have GHH preferences so that

$$U(c, h) = \frac{[c - v(h)]^{1-\sigma} - 1}{1-\sigma}$$

with

$$v(h) = \chi \frac{h^{1+\nu}}{1+\nu}$$

In addition, the formal good production function is Cobb-Douglas with the form

$$F(A_{i,t}, H_{i,t}, K_{i,t}) = A_{i,t} H_{i,t}^\alpha K_{i,t}^{1-\alpha}, \quad i = 1, 2 \quad (32)$$

Informal backyard production has a technology of  $G(k_{i,t}^3) = Z(k_{i,t}^3)^\gamma$ . Parameter  $Z$  denotes a constant productivity in the informal sector.

## 3.2 Solution Method

The solution of the model with stochastic productivity shocks, occasionally binding collateral constraints, multiple state variable for capital holdings and debt, and endogenous asset prices and world interest rates, represents a serious computational challenge. The solution approach is described at length in the Technical Appendix. The key facilitating feature of the solution is that the model structure allows us to follow the approach of [Dumas and Lyasoff \(2012\)](#). Their method involves a process of backward induction on an event tree. Current period consumption shares in total world GDP are treated as endogenous state variables. The construction of equilibrium is done by a change of variable, so that the equilibrium conditions determine future consumption and end of period portfolios as functions of current exogenous and endogenous state variables. From these functions, using consumption-Euler equations we can recursively compute asset prices and the end of period financial wealth. We then iterate on this process using backward induction until we obtain time-invariant policy functions. Once we have these policy functions, we can make use of the initial conditions, including initial exogenous shocks and initial portfolios, and of equilibrium conditions in the first period to pin down consumption, end of period portfolios, output and asset prices in the initial period.

## 3.3 The role of asset constraints

Although the basic economics of the model is described in the previous section, an additional set of constraints has to be incorporated as part of the implementation, in order to

ensure the existence of a well-behaved solution to the general equilibrium model. What we have described is a general equilibrium model with incomplete markets (GEI). In order to ensure an equilibrium solution, it is necessary to rule out paths in which agents accumulate unbounded debts. Without imposing some additional constraints, an equilibrium may not exist (e.g. Krebs, 2004). There are several ways to restrict the model so as to ensure existence. One approach is to directly put short-sale constraints on asset holdings. For our purposes, it is more useful to follow Judd et al. (2002) and impose a utility penalty on holdings of assets. The benefit of imposing a penalty rather than a short-sale constraint is that adding a penalty function to the problem of the investors or the global banker does not increase the numbers of equations to be solved in the solution procedure.

Following Judd et al. (2002), the penalty function for investor  $l$  has a form of

$$\rho^l(k_{1,t}^l, k_{2,t}^l) = \kappa_1 \min(0, k_{1,t}^l - \underline{k}_1^l)^4 + \kappa_2 \min(0, k_{2,t}^l - \underline{k}_2^l)^4 \quad (33)$$

where  $\underline{k}_i^l$ , with  $i = 1, 2$ , denotes the lower bound for holding equity  $i$  by investor  $l$ , and  $\kappa_i > 0$  is a penalty parameter. Whenever equity holdings are lower than their lower bounds, investors will receive a penalty.<sup>4</sup> As described later, the calibration for the penalty functions in the model ensures that there is only very tiny probability of hitting the lower bound constraints on asset holdings along the equilibrium path of the model. Also, we do not need to impose a constraint on bond holdings since the collateral constraint (2) already limits the highest borrowing an investor can incur.

The portfolio penalty is added to the utility function of investors in each country and leads to an amendment of investor's necessary conditions in a straightforward way. The amended set of first-order conditions is described in the Technical Appendix.

For the global banker, we also need a penalty function constraining the equity and debt positions. We use the following penalty function

$$\rho^3(k_{1,t}^3, k_{2,t}^3, b_{3,t}) = \kappa_1 \min(0, k_{1,t}^3 - \underline{k}_1^3)^4 + \kappa_2 \min(0, k_{2,t}^3 - \underline{k}_2^3)^4 + 2\kappa_b \min(0, \bar{b}_3 - b_{3,t})^4 \quad (34)$$

---

<sup>4</sup>We use a power of 4 here to make sure that the first-order partial derivative of  $\rho^l$  with respect to any argument is twice continuously differentiable.

where  $\kappa_b > 0$ ,  $\underline{k}_i^3$  with  $i = 1, 2$  is the lower bound for physical capital held by the banker, and  $\bar{b}_3$  is the upper bound for borrowing by the banker. It will be a property of equilibrium that physical capital held by the banker is never below zero, since the marginal product of capital in backyard production tends to infinity when capital approaches zero. Here, the lower bound for capital in backyard production in each country captures a minimum requirement of capital. Again, this penalty function is added to the preferences of the global banker and leads to an amended set of first order conditions in a straightforward way, as documented in the Technical Appendix.

The set of penalty functions differs under the bond market equilibrium. For the investors, in the bond market equilibrium it is not necessary to impose any penalties, since bond positions are constrained by (2), and given strict decreasing returns in formal and informal production, along with the penalty imposed on global bankers, there is no need for a penalty on investors. For the global banker, the penalty function is the same in the bond market integration equilibrium as in the equity market integration, since the global banker is assumed not to directly access equity markets in any case.

Finally, under financial autarky, the investors problem is the same as under the bond market integration equilibrium (no penalty function), while the domestic banker's problem is a straightforward restriction on (34), where the first order conditions are described in the Technical Appendix.

### 3.4 Calibration

The model has relatively few parameters. The period of measurement is one year. Parameter values in the baseline model are mostly taken from the literature and are listed in table 1.<sup>5</sup> The population of investors in each country is  $n = 0.5$ . The coefficient of relative risk aversion  $\sigma$  is set equal to 2.

The key features of the calibration involve the productivity shock processes. This is done in a two-fold manner. First, we specify a conventional AR process for the shock. But we

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<sup>5</sup>In the deterministic steady state, both financial integration regimes have the same values for aggregate variables.

Table 1: Parameter values

Parameter		Value
<i>Preferences</i>		
$n$	Population size	0.5
$\sigma$	Relative risk aversion	2
$\beta_l, l = 1, 2$	Subjective discount factor for investors	0.954
$\beta_3$	Subjective discount factor for workers	0.96
$\nu$	Inverse of Frisch labor supply elasticity	0.5
$\chi$	Parameter in labor supply (H=1 at steady state)	0.58
$\kappa$	Loan-to-value ratio for inter-period borrowing	0.5
<i>Production</i>		
$\alpha$	Labor share in formal production	0.64
$\gamma$	Informal production	0.3
$Z_l, l=1,2$	TFP level in informal production (capital share in the formal production=0.8)	0.7
$\rho_z$	Persistence of TFP shocks	0.65
$\sigma_\epsilon$	Standard deviation of TFP shocks	0.02
$D$	Disaster risk	-0.1054
$\pi$	Probability of disaster risk	0.025
$K$	Normalized physical capital stock	1
<i>Penalty for portfolios</i>		
$\underline{k}_i^l$	Lower bound for equity holdings	-0.35
$\bar{b}_3$	Upper bound for a banker's borrowing	2
$\kappa_1, \kappa_2, \kappa_b$	Penalty parameter	1,000

append to this process a small probability of a large negative shock (a ‘rare disaster’). The  $AR(1)$  component of the shock can be represented as:

$$\ln(A_{l,t+1}) = (1 - \rho_z) \ln(A_l) + \rho_z \ln(A_{l,t}) + D\phi_{l,t+1} + \epsilon_{l,t+1}, \quad l = 1, 2 \quad (35)$$

where  $A_l$  is the unconditional mean of  $A_{l,t}$ ,  $\rho_z$  characterizes the persistence of the shock, and  $\epsilon_{l,t+1}$  denotes an innovation in period  $t + 1$ , which is assumed to follow a normal distribution with zero mean and standard deviation  $\sigma_\epsilon$ .

The disaster component of the shock is captured by the  $\phi_{l,t+1}$  term. This follows a Bernoulli distribution and takes either value of  $\{-\pi, (1 - \pi)\}$ , with probability of  $\{1 - \pi, \pi\}$  respectively,  $0 < \pi < 1$ . The scale parameter  $D < 0$  measures a disaster risk in productivity.<sup>6</sup>

<sup>6</sup>The mean of the disaster risk is zero  $E(\phi_{l,t}) = 0$  and its standard deviation equals  $-D\sqrt{\pi(1 - \pi)}$ .

We take  $\rho_z = 0.65$  and  $\sigma_\epsilon = 0.02$  as in [Bianchi \(2011\)](#). We assume that the cross-country correlation of productivity shocks is zero. The mean of each country's productivity shock is normalized to be one. The distribution of  $\phi_{l,t+1}$  is taken from the rare disaster literature (see for instance, [Barro and Ursua, 2012](#)) so that  $\pi = 0.025$ , which implies that the probability of an economy entering a disaster state is 2.5% per year. Once an economy enters a disaster state, productivity will experience a drop by 10%, that is,  $D = \ln(0.9)$ . The number is chosen such that investors' consumption will drop by around 30% in disasters relative to that in normal times.

With this calibration, the unconditional standard deviation of TFP shocks is 4 percent per annum. In some of the analysis below, we look at a low-risk case, where the unconditional standard deviation of the TFP shocks is 2 percent per annum. This has some important implications for welfare comparisons. Aside from welfare implications though, all the results discussed in the paper are robust to a change from the high-risk to a low-risk economy.

In solving the model, we need to discretize the continuous state  $AR(1)$  process above into a finite number of exogenous states. The Technical Appendix describes in detail how we accomplish this task. In the baseline model, we choose three grid points for technological levels  $A_{l,t}$ . The grid points and the associated transition matrix in each country are as follows

$$A_l \equiv \begin{bmatrix} A_L \\ A_M \\ A_H \end{bmatrix} = \begin{bmatrix} 0.9271 \\ 0.9925 \\ 1.0269 \end{bmatrix} \quad \Pi_l = \begin{bmatrix} 0.6311 & 0.2723 & 0.0967 \\ 0.1312 & 0.5739 & 0.2949 \\ 0.0078 & 0.2321 & 0.7601 \end{bmatrix}$$

Given this specification, productivity in each country will visit its lowest state 0.9271 with a probability of 15%. The lowest state is associated with the disaster state but because the continuous distribution is projected into a three state Markov Chain, this is not identical to the disaster itself. The exogenous state of the world economy in financial integration is characterized by a pair  $(A_{1,t}, A_{2,t})$ , which takes nine possible values. Its associated transition matrix is simply the Kronecker product of transition matrices in both countries,  $\Pi \equiv \Pi_1 \otimes \Pi_2$ ,

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Compared to the standard  $AR(1)$  process without disaster risk (say  $D = 0$ ), the standard deviation of innovations increases by  $-D\sqrt{\pi(1-\pi)}$ .

since the shocks are independent across countries.

Recall that we require portfolio penalties to restrict equity holdings by investors in equity market integration. In the baseline model, we set  $\underline{k}_i^l = -0.35$ . The results show that the economy never hits these equity boundaries in simulations. Equity short-sale constraints are redundant in bond market integration because of the concavity of production in capital. As for the upper bound for the banker's borrowing  $\bar{b}_3$ , we choose a positive number  $\bar{b}_3 = 2$ . This upper bound is never reached in our case since we assume investors are less patient than bankers. The penalty for exceeding the preset bounds is set as  $\kappa_1 = \kappa_2 = \kappa_b = 1,000$ .

The loan-to-value ratio parameter is set to be  $\kappa = 0.5$ , which states that the maximal leverage is 2 in the stationary distribution of the economy. This leverage ratio is consistent with evidence from non-financial corporations in the United States ([Graham, Leary and Roberts, 2013](#)). The subjective discount factor for bankers is  $\beta_3 = 0.96$ , which implies an annualized risk free rate of 4%. Investors are less patient, and their subjective discount factor is chosen to be  $\beta_l = 0.954$ . Together with the productivity shock process and the loan-to-value ratio of  $\kappa = 0.5$ , this implies that in financial autarky, the economy visits the state where the collateral constraint is binding and productivity is at its lowest level with a probability of around 3.5%, or approximately every 30 years.

In the preference specification, the inverse of the Frisch labor supply elasticity is set to be 0.5, which is consistent with business cycle observations ([Cooley, ed, 1995](#)). We normalize the steady state labor supply to be  $H = 1$ , which implies the parameter  $\chi = 0.58$ .

The share of labor in formal production is set to be  $\alpha = 0.64$ . In the informal backyard production, the marginal product of capital is characterized by parameter  $\gamma = 0.3$ , which is lower than that in the formal production sector. The level of productivity in the informal backyard production is set to be  $Z_l = 0.7$ . This implies that around 80% of physical capital is employed in the formal production in the stationary distribution of the economy.

## 4 A perfect foresight special case

Before we present the main results of the paper, it is worthwhile to explore the workings of the model in a simpler environment. Here we look at the impact of productivity shocks in a deterministic version of the model, under financial autarky. This can give some insight into the states of the world in which the collateral constraint on investor's borrowing does or does not bind. As we have noted, in a deterministic steady state, the constraint will always bind, since investors are more impatient than savers, and wish to front-load their consumption stream. But in response to a transitory productivity shock, the endogenous movement in borrowing, asset prices and investment may lead to either a relaxation or a further tightening of the collateral constraint.

Figure 1 illustrates the impact of an unanticipated fall in productivity in the final goods sector that starts in period 2 and is known to last for 3 periods, following which productivity returns to the steady state level and remains at that level thereafter.<sup>7</sup> The 6 panels in the figure show the responses of investor borrowing, the Lagrange multiplier on the investor's collateral constraint, the price of capital, productivity in the formal sector, capital employed in the formal sector, and output in the formal sector. At the initial level of investor debt, the collateral constraint is binding. The unanticipated fall in productivity leads to an immediate and large drop in the price of capital. This causes a tightening of the collateral constraint, illustrated by a jump in the Lagrange multiplier  $\mu_t$ . Investors are forced to de-lever, and there is a large reallocation of capital out of the final goods sector and into the backyard production sector. In the transitional period, following the return of productivity to its steady state level, investor's borrowing gradually returns to its initial steady state, mirrored by an increased allocation of capital to the formal sector. Although the shock expires after period 3, there is a prolonged period of adjustment as the collateral constraint gradually eases over time. Note that the constraint continues to bind before and after the shock in this experiment.

Figure 2 looks at the same size shock, but now assuming a *positive* rather than a negative

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<sup>7</sup>The calibration of this version of the model follows the calibrated values as described above, except for the absence of stochastic shocks.

shock to productivity in the final goods sector. Again, the shock is unanticipated, and known to last for 3 periods, before returning to the steady state. The immediate effect is to cause a rise in the price of capital. But this now relaxes the collateral constraint such that the constraint ceases to bind. The Lagrange multiplier on the collateral constraint goes to zero. Investor borrowing and investment in the final goods sector increases for the duration of the shock. Although the size of the productivity shock is exactly the reverse of Figure 1, the magnitude of responses is much less. The forced de-leveraging in response to a temporary negative productivity shock is over 50% greater than the peak increase in debt accumulation in the case of a temporary positive productivity shock. The fall in capital use in the formal sector after the negative shock is more than twice the rise in capital use following the positive shock. The follow-on adjustment to the positive productivity shock is also significantly different than that for the negative shock. The collateral constraint remains non-binding for a significant time after the expiry of the shock. Even though the price of capital returns very quickly to its steady state level, the higher level of capital in the hands of investors enhances the value of collateral so as to exceed the borrowing limit for a prolonged period after the shock. We note also that in this case, there is much less persistence in investor borrowing, capital dynamics, and output. Both in the magnitude of response and in the persistence of responses, we see a clear asymmetry in the dynamics of the economy following a positive shock relative to a negative shock.

This comparison suggests that, although the borrowing constraint on investors will always bind in a deterministic steady, this is not likely to be true in a stochastic economy, where productivity shocks generate unanticipated movements in asset prices and the value of collateral. Moreover, in a stochastic economy, given CRRA preferences, investors will engage in precautionary saving; their desired consumption profile for any given path of interest rates will be less front-loaded than in the deterministic economy.

Figure 3 illustrates the impact of a negative productivity shock, but now assuming that the shock is anticipated. We assume that the shock again lasts for three periods, but the shock is known 3 periods in advance. The immediate effect is an increase in borrowing,

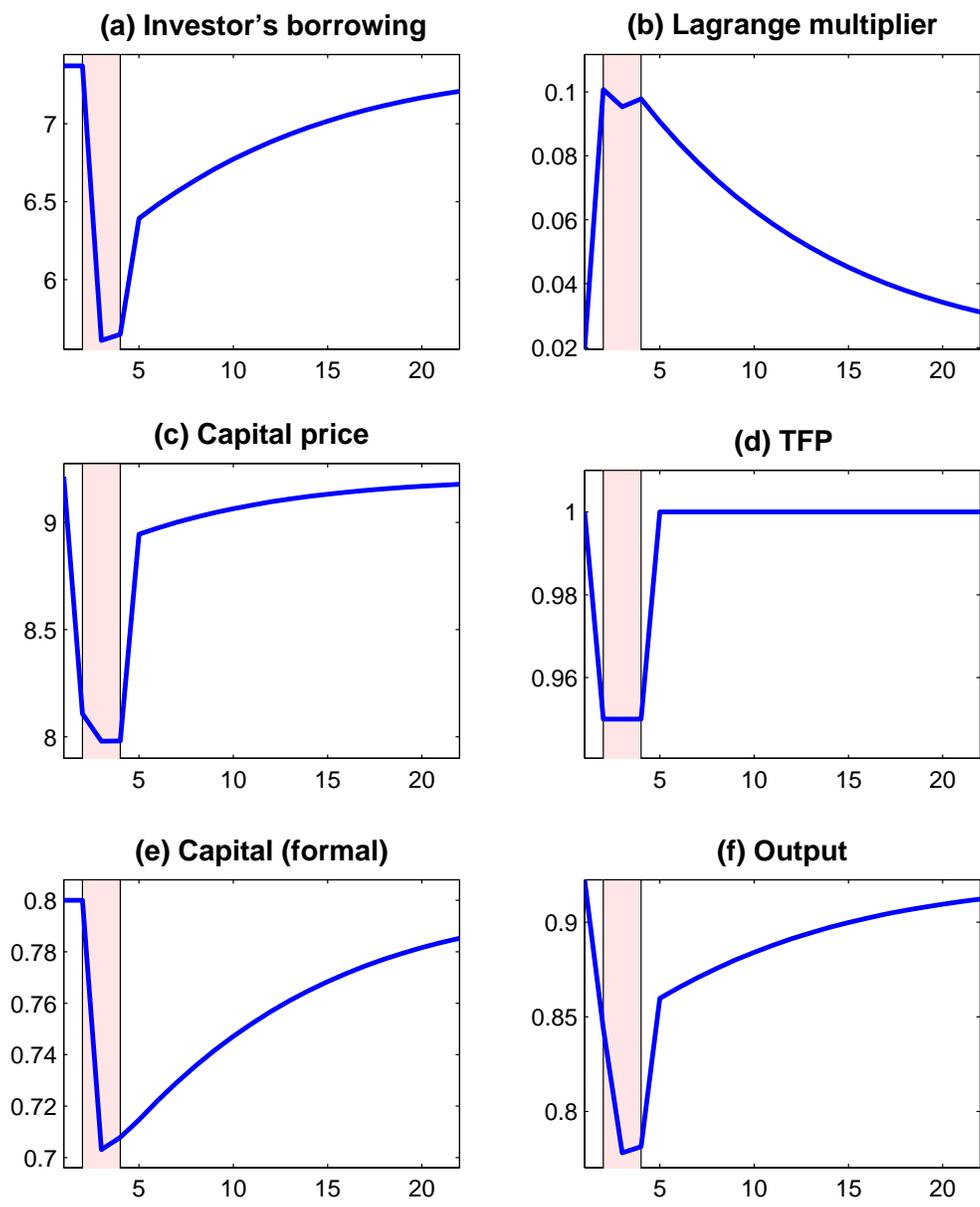


Figure 1: This figure reports responses of investors' borrowing, Lagrange multiplier, capital price, capital and output in the formal production sector to an *unanticipated negative* productivity (TFP) shock. The economy stays at its steady state in period 1. TFP shock occurs unexpectedly in period 2 and lasts for 3 periods (the shaded region), and it returns to its steady state from period 5 onwards.

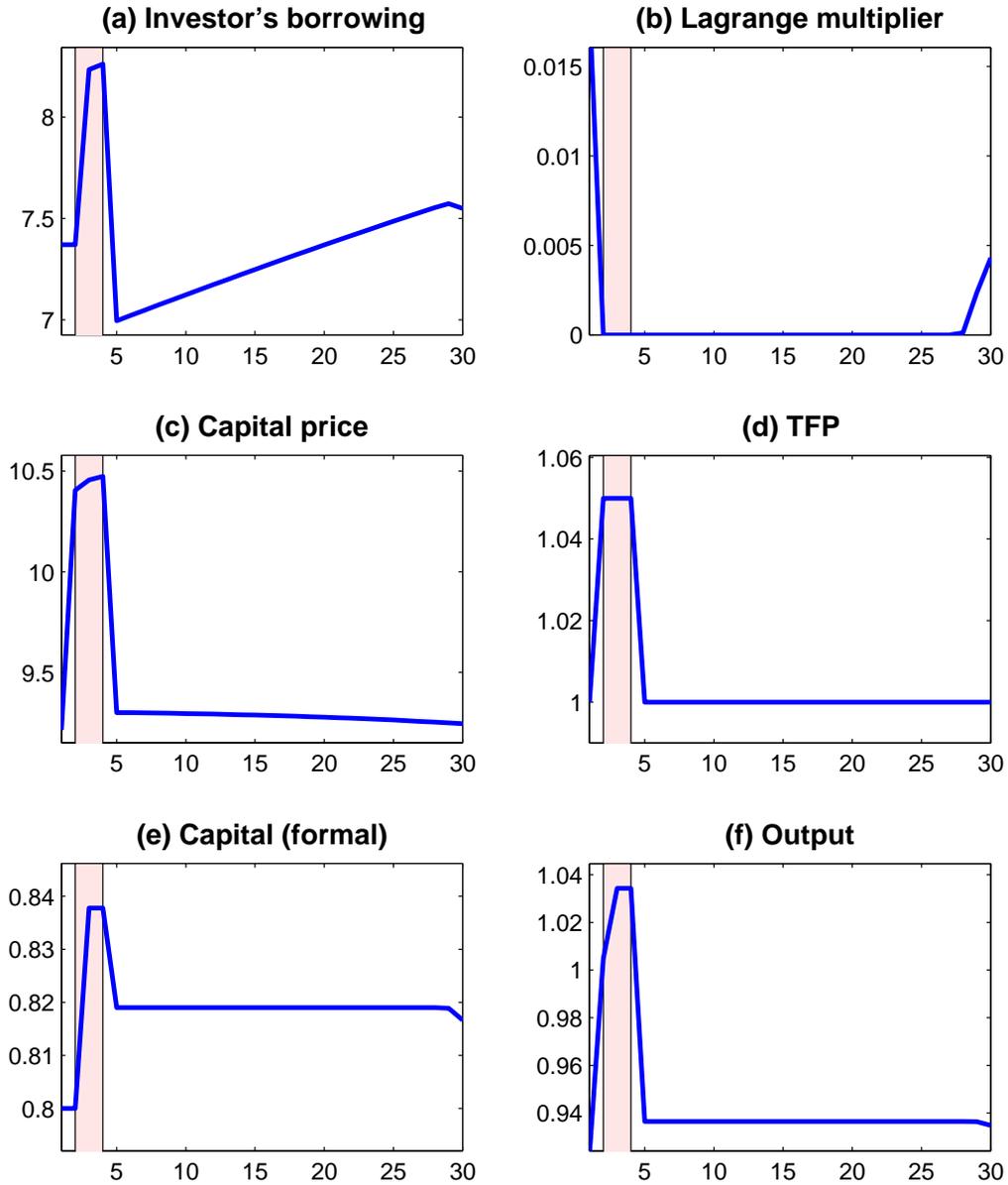


Figure 2: This figure reports responses of investors' borrowing, Lagrange multiplier, capital price, capital and output in the formal production sector to an *unanticipated positive* productivity (TFP) shock. The economy stays at its steady state in period 1. TFP shock occurs unexpectedly in period 2 and lasts for 3 periods (the shaded region), and it returns to its steady state from period 5 onwards.

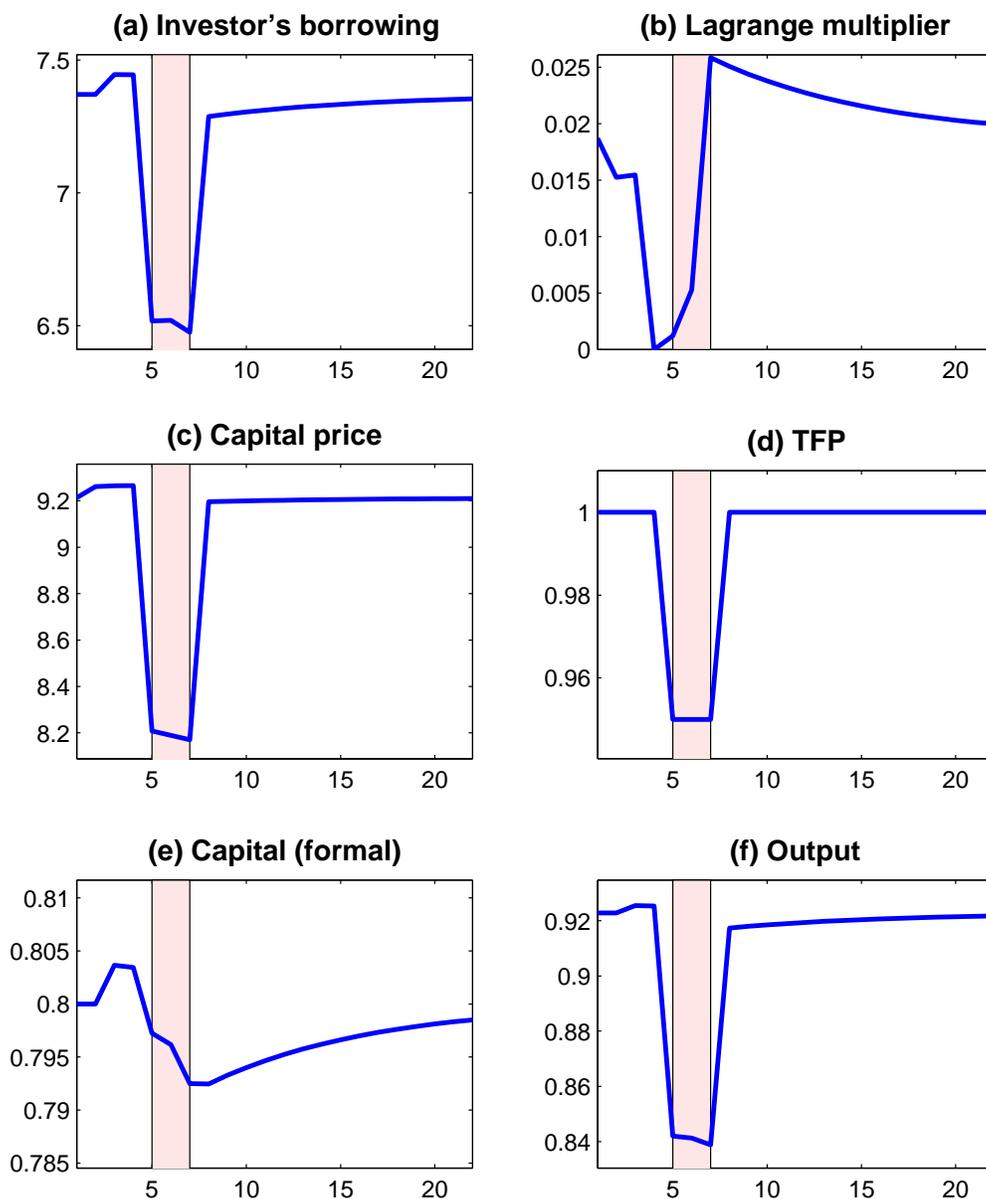


Figure 3: This figure reports responses of investors' borrowing, Lagrange multiplier, capital price, capital and output in the formal production sector to an *anticipated negative* productivity (TFP) shock. The economy stays at its steady state in period 1. TFP shock occurs expectedly in period 5 and lasts for 3 periods (the shaded region), and it returns to its steady state from period 8 onwards.

a rise in the price of capital, and a weakening of the collateral constraint. The constraint becomes non-binding in advance of the shock, as savers build up capital to partially absorb the negative effects of the shock on consumption. This example illustrates that the impact of productivity shocks on the borrowing constraint depends sensitively not just on the sign of the shock, but also on the timing of the shock.

Finally, these examples pertain only to the closed economy setting. When we open up financial markets, either with bond trade, or full equity market integration, these shocks can be absorbed by international borrowing and lending, or diversified away by holding a portfolio of domestic and foreign equities. But by altering the stochastic environment in which investors operate, financial integration will also affect the frequency with which borrowing constraints are binding, and the intensity to which they bind. We now turn to an analysis of the effects of financial market integration in the full model.

## **5 Results: simulations over alternative financial regimes**

### **5.1 The effect of financial integration on balance sheet constraints**

We simulate the stationary policy rules obtained from the model, using the stochastic processes for productivity described in the previous section, for the three different financial regimes described above. The simulations are done for  $T=210,000$  periods, with the first 10,000 periods dropped from the sample. The first issue of interest is the degree to which the debt collateral constraint binds, and how this differs across the different financial regimes. Figure 4 provides an illustration and contrast between the different regimes. The figure presents illustrations of the fraction of time spent at the leverage constraints, and the degree to which the leverage constraints bind simultaneously in the two countries. Starting with the financial autarky case, we find that under the calibration and the distribution of productivity shocks in the baseline model, the leverage constraint binds only 10 percent of the time. So in the simulations, in financial autarky, investors in each country are strictly below the borrowing limit for 90 percent of the time. Considering that in a deterministic steady state,

the constraint will always bind, this dramatically illustrates the strength of the precautionary savings motive in the economy with technological uncertainty. Investors will consistently limit their borrowing so as to avoid instances where they are constrained in the future.

Because productivity shocks are independent across countries, there is a very small probability that the constraint binds simultaneously in both countries - there is no 'contagion' in leverage crises in the absence of financial linkages.

How does opening up to international financial markets affect the likelihood of leverage crises? In the bond market case, there is a big increase in the likelihood that the leverage constraint binds, for any one country. In a global bond market equilibrium the probability that any one country is constrained effectively doubles, to 19 percent. Moreover, to a large extent this increase in the probability of leverage crises is associated with correlated crises across countries. The probability that the constraint binds in both countries simultaneously is now 13 percent (relative to 1 percent in financial autarky). Hence there is a jump in financial contagion associated with the opening up of international bond markets.

What drives the high correlation in leverage crises across countries with bond market integration? In this form of financial linkages, equities are not tradable across countries, so it is only the linkages in debt markets that lead to a correlation in financing of investment behaviour across countries. As we will see below in more detail, a tightening of the collateral constraint in one country drives up the cost of borrowing for investors, and this will spill over to the cost of financing investment in the other country, increasing the likelihood that the leverage constraint binds in that country also.

But there is a second critical element at work in the comparison of the financial autarky regime with the bond market integration. This is the scale of overall borrowing. In financial autarky in one country, investors limit their debt accumulation due to precautionary motives. Hence, as we have seen, the leverage constraint binds on average only 10 percent of the time. When bond markets become integrated, the overall scale of consumption risk falls significantly, as shown below. This leads to a reduction in precautionary saving motives, increasing the willingness to borrow, raising leverage closer to the limit implied by equation

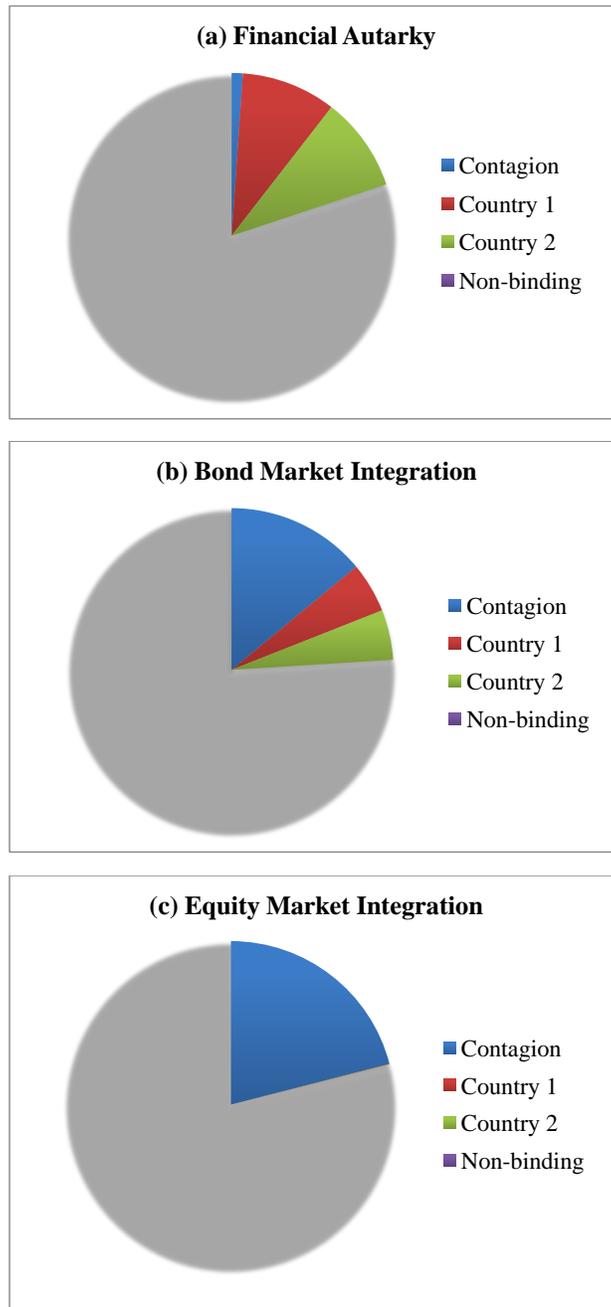


Figure 4: This chart shows the joint distribution of collateral constraints being binding or not in financial autarky (panel a), bond market integration (panel b) and equity market integration (panel c). Contagion is defined as  $\text{prob}(\mu_1 > 0, \mu_2 > 0)$ , Country  $i$  in the chart denotes  $\text{prob}(\mu_i > 0, \mu_j = 0)$  with  $i \neq j$ , and Non-binding is  $\text{prob}(\mu_1 = 0, \mu_2 = 0)$ .

(2). As a result, the leverage constraint binds almost 20 percent of the time.

Hence the two key features of financial integration are a substantial increase in the correlation of crises across countries, and an increased willingness to assume a higher level of leverage due to a fall in overall investment risk. Figure 4 illustrates now the impact of moving from integration in bond markets alone to a full integration of equity markets and bond markets<sup>8</sup>. With integrated equity markets, the correlation of leverage crises across countries becomes complete. That is, conditional on one country being leverage constrained, the probability that the second country is also constrained is effectively 100 percent. Hence, crisis contagion is complete when equity markets are integrated. Compared with the bond market integration, the unconditional probability of any one country being constrained is only slightly higher (21 percent relative to 20 percent). But the key difference is that there is essentially zero probability that a country will be subject to a balance sheet constraint on its own.

As we show in more detail below, the move from bond market integration to full bond and equity market integration has only a marginal increase in the degree of risk-sharing across countries. But the key feature that effects the linkage of financial contagion is the direct interdependence of balance sheets. As illustrated by equation (2), with equity market integration, the collateral value of investors portfolios are directly interdependent via the prices of domestic and foreign equity. Investors in one country hold a world portfolio, and shocks which affect foreign equity prices directly impact on the value of domestic collateral, independent of the world cost of capital. This leads to a dramatic increase in the degree of financial contagion in the equity market equilibrium relative to the bond market equilibrium.

Since the leverage constraint depends on the collateral value of capital, it is natural to conjecture that the constraint is more likely to be binding in low productivity states. Figure 5 illustrates the probability of binding constraints conditional on the state of productivity in any one country, contrasting this across all three financial regimes. In financial autarky, the probability of being constrained is much higher in the low productivity state - conditional

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<sup>8</sup>We note that although equity markets enhance the possibilities for cross-country risk-sharing relative to bond market integration alone, this still falls short of a full set of arrow Debreu markets for risk-sharing.

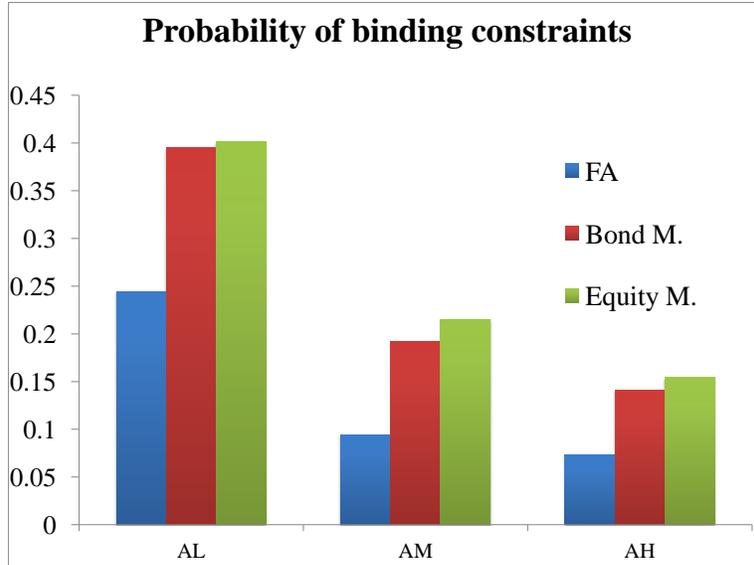


Figure 5: This chart shows the conditional distribution of collateral constraints being binding or not in financial autarky (FA), bond market integration (Bond M.) and equity market integration (Equity M.) under different shock states. *AL*, *AM* and *AH* denote the low, medium and high state of productivity *A*, respectively.

on a low state, the probability of the constraint binding is about 0.25. The corresponding probabilities under the medium and high states are .09 and .07 respectively. But when we open up international bond markets and international debt markets, it is much more likely for a country to be leverage constrained in medium or high productivity states, as well as states of low productivity.

## 5.2 Moment analysis

The impact of alternative degrees of financial integration can be seen more directly in the moments reported in Tables 2 and 3. Table 2 shows the simulated mean levels of investor consumption, formal output, capital in production, investor borrowing and leverage, employment, the price of equity, the interest rate (borrowing rate), and the external finance premium. Table 3 shows standard deviations, in percentage terms, as well as the cross-country correlations, under each financial regime. In each case, we report first the moments over the whole sample, then the moments restricted to states where the leverage constraint

binds in both countries simultaneously (or in the case of financial autarky, just cases where the leverage constraint is binding).

In the discussion above, we noted that financial integration leads to an increase in the overall probability of leverage crises, as well as a dramatic increase in the cross-country correlation of crises. The first factor comes about because financial integration leads to a substantial increase in investor debt accumulation. Table 2 shows that the mean level of investor borrowing increases by about 25 percent when we move from financial autarky to bond or equity integration. This translates into a shift from an average leverage rate of 1.46 under autarky to a leverage of 1.68 with financial integration.

What accounts for the rise in investor borrowing in a global financial system? There are two main channels. The first is a fall in the desire to engage in precautionary saving. Table 3 shows moving from autarky to bond or equity integration reduces the volatility of consumption. This leads directly to a rise in willingness to accumulate debt. The second factor is that the average equity price tends to rise, increasing the value of collateral, thus increasing the ability to service debt without violating the leverage constraint. But since financial integration does not change the limit on investor leverage, the first factor is the key reason for a rise in average leverage, and as a consequence, a rise in the unconditional probability of leverage crises when financial markets become integrated.

We now focus more carefully on a comparison of the first and second moments in Tables 2 and 3 for the different degrees of financial market integration. The first point of note is that the mean consumption level of investors is lower under bond market and equity market integration than in financial autarky. This is not surprising, since we have already seen that opening financial markets leads to an increase in leverage, and investors attempt to engage in more front-loading of consumption. As a result there is a rise in anticipated debt service payments and fall in expected consumption in a stationary equilibrium.

When we compare over the whole sample path, aside from the rise in leverage and the fall in mean consumption, there is little difference between the different financial regimes in terms of first moments. Mean output is slightly higher with full equity market integration,

as is capital used in the formal sector. Mean employment is unchanged, while the mean price of equity rises very slightly when bond or equity markets are opened up.

Opening up financial markets has a greater impact on volatilities, compared over the whole sample path. We have noted already that the volatility of consumption of investors drops by a full percentage point. The volatility of output and employment also fall, but by a relatively small magnitude.

These results are, on the whole, consistent with existing business cycle literature, suggesting that financial market integration enhances consumption risk-sharing but does not have major effects on other macro aggregates in terms of first or second moments. But this observation pertains only to the comparison across the whole sample path. The middle panels of Table 2 and 3 focus on moments taken from a sample path where the leverage constraint binds in both countries simultaneously. Here we see a very sharp difference between outcomes in financial autarky and those under financial market integration.

First, note from Table 2, that while mean consumption is on average 5 percent higher in financial autarky than in financial integration, over the whole sample path, this comparison is completely reversed during a global leverage crisis, where both collateral constraints bind. Average consumption within a leverage crisis is 6 to 7 percent lower in financial autarky than under bond or equity market integration. Likewise, output, employment, and capital in production falls by much more during a leverage crisis in the absence of international financial markets.

A similar picture emerges in the comparison of second moments during a leverage crisis. Table 3 shows that the rise in consumption, output, employment and capital stock volatility during a leverage crisis is dramatically greater under financial autarky than when financial markets are integrated through bond trade or equity trade.

What accounts for the major difference between ‘normal times’ and ‘crisis times’ as regards the effects of financial market openness? We know from previous literature that a binding collateral constraint introduces a ‘financial accelerator’, so that a negative shock to productivity leads to a greater fall in equity prices, borrowing, and formal sector output

through the process of forced deleveraging. The same process is taking place in this model. In the comparison of the performance during leverage crisis, the financial accelerator is in operation under all degrees of financial market integration. But because financial markets allow for diversification, when the underlying fundamental shocks are not perfectly correlated across countries, they also allow the multiplier effects of these shocks to be cushioned through a smaller volatility in world interest rates and asset prices. Thus, while volatility is magnified during leverage crises under all regimes, the impact is much greater in the absence of this international diversification. This accounts for the much smaller volatility of macro aggregates during a leverage crisis in the presence of international financial markets. We see this much more clearly in the ‘event analysis’ described below.

Why is it that first moments are also lower during a leverage crisis, under financial autarky? This is due to the asymmetry between positive and negative shocks, as we pointed out in Figures 1-2 above. Since a negative productivity shock is more likely to lead to binding collateral constraints, and the response to a negative shock will be greater under financial autarky than with international financial integration, it follows that international financial markets facilitate higher average levels of consumption, output and employment, even during episodes of leverage crises. Tables 2 indicates that the rise in interest rates, affecting the borrowing costs facing investors, is significantly larger in leverage crises in the financial autarky environment than when capital markets are open.

Hence, while on average, international capital markets lead to a rise in the probability of binding leverage constraints, and an increase in financial contagion, they have the benefit that crises are much less severe with financial market integration than under financial autarky. This points to a clear trade-off between the benefits of integration and the increased preponderance of balance sheet crises under integration. In section 5.6 below, we explore the welfare implications of this trade-off.

Table 2: Simulated means

	Autarky	Bond	Equity
<i>Panel A: Full sample</i>			
Consumption	1.06	1.02	1.02
Formal output	0.92	0.92	0.93
Capital	0.78	0.78	0.78
Debt (borrowing)	4.50	5.80	5.90
Leverage	1.46	1.67	1.69
Labor	1.01	1.01	1.01
Equity Price	9.19	9.28	9.29
Interest Rate	1.0417	1.0417	1.0417
External Finance Premium	0.0022	0.0033	0.0034
<i>Panel B: A subsample with <math>\mu_1 &gt; 0</math> and <math>\mu_2 &gt; 0</math></i>			
Consumption	0.83	0.86	0.88
Formal output	0.81	0.84	0.86
Capital	0.67	0.70	0.72
Debt (borrowing)	5.84	6.22	6.46
Leverage	2.00	2.00	2.00
Labor	0.92	0.95	0.96
Equity Price	8.60	8.74	8.86
Interest Rate	1.0497	1.0486	1.0474
External Finance Premium	0.0207	0.0196	0.0161
<i>Panel C: A subsample with <math>\mu_1 &gt; 0</math>, <math>\mu_2 &gt; 0</math> and <math>A_1 = AL</math></i>			
Consumption	0.72	0.77	0.80
Formal output	0.69	0.72	0.74
Capital	0.60	0.64	0.67
Debt (borrowing)	4.70	5.30	5.88
Leverage	2.00	2.00	2.00
Labor	0.85	0.87	0.88
Equity Price	7.49	8.06	8.27
Interest Rate	1.0760	1.0610	1.0617
External Finance Premium	0.0262	0.0247	0.0206

Notes: This table reports simulated means for variables of interest in the model economies. Column Autarky, Bond and Equity denote financial autarky, bond market integration and equity market integration respectively. Corr.(bond) and Corr.(Equity) are for cross correlations in bond and equity market integration. Model parameters are the same as the baseline model. The results are obtained through simulating the model economy 210,000 periods and the first 10,000 periods are discarded to get rid of the impact of initial conditions. *AL* denotes the low state of productivity. All model economies use the same realized shock sequences.

Table 3: Simulated standard deviations

	Autarky	Bond	Equity	Corr.(bond)	Corr.(equity)
<i>Panel A: Full sample</i>					
Consumption	4.61	3.76	3.63	0.64	0.78
Formal output	4.62	4.42	4.45	0.11	0.11
Capital	1.99	1.65	1.67	0.57	0.67
Debt (borrowing)	35.00	28.11	26.18	0.57	1.00
Labor	3.16	3.04	3.07	0.07	0.07
Equity Price	47.18	34.73	34.62	0.96	1.00
External Finance Premium	0.18	0.21	0.21	0.42	1.00
<i>Panel B: A subsample with <math>\mu_1 &gt; 0</math> and <math>\mu_2 &gt; 0</math></i>					
Consumption	6.36	4.79	4.43	0.49	0.78
Formal output	6.86	5.78	5.76	0.17	0.19
Capital	4.16	3.12	2.99	0.60	0.75
Debt (borrowing)	68.92	50.27	47.39	0.72	1.00
Labor	4.61	3.96	3.93	0.09	0.11
Equity Price	63.41	43.03	42.52	0.89	1.00
External Finance Premium	0.55	0.50	0.43	0.38	1.00
<i>Panel C: A subsample with <math>\mu_1 &gt; 0</math>, <math>\mu_2 &gt; 0</math> and <math>A_1 = AL</math></i>					
Consumption	3.85	3.53	3.46	0.56	0.92
Formal output	3.25	2.74	2.57	0.40	0.54
Capital	4.19	3.67	3.50	0.57	0.91
Debt (borrowing)	42.82	43.59	46.91	0.61	1.00
Labor	3.08	2.54	2.39	0.36	0.48
Equity Price	15.36	27.31	29.62	0.82	1.00
External Finance Premium	0.96	0.77	0.73	0.52	1.00

Notes: This table reports simulated standard deviations for variables of interest in the model economies. Column Autarky, Bond and Equity denote financial autarky, bond market integration and equity market integration respectively. Corr.(bond) and Corr.(Equity) are for cross correlations in bond and equity market integration. Model parameters are the same as the baseline model. The results are obtained through simulating the model economy 210,000 periods and the first 10,000 periods are discarded to get rid of the impact of initial conditions.  $AL$  denotes the low state of productivity. All model economies use the same realized shock sequences. The model period is one year and variables are HP-filtered with parameter  $\lambda = 100$ .

### 5.3 Comparison of financial autarky and financial integration under low states of productivity.

In the previous section, we noted that leverage crises are much less severe under financial market integration, whether with bond trade or equity market integration, than with financial autarky. But we had previously noted that leverage crises are more likely to occur in the first place when financial markets are integrated, and as pointed out in Figure 5, crises are more likely to occur in low productivity states. So a natural question to ask is whether this beneficial effect of international financial markets in responding to crises is robust to a comparison carried out solely during states when productivity is at its lowest level. The bottom panels in Table 2-3 show that this is the case. These tables illustrate the comparison of first and second moments for the three different financial regimes when leverage constraints bind, and productivity in the home country is at its lowest level (i.e.  $A = A_L$ ).<sup>9</sup> We see that average levels of consumption, output and employment are still significantly higher under either bond market integration or equity integration than with financial autarky. Likewise, consumption, output and employment volatility are much lower in the presence of international financial market integration. Hence, even when the comparison is restricted to the low productivity state, we find nonetheless that there remains a major cushioning effect of financial markets in times of crises.

### 5.4 The distribution of investors consumption under alternative financial market outcomes.

Figures 6-8 illustrate the empirical distributions of investor's consumption from the model simulations. In financial autarky, Figure 6 shows that investor's consumption has a bi-modal characteristic with a fat left tail. The blue shades indicate states where the leverage constraint is non-binding, while the red shaded areas indicate states with the binding constraint. Clearly consumption is lower in the latter case. But the presence of the fat left tail of the distribution

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<sup>9</sup> These moments now pertain to the home country alone, since this comparison involves an asymmetric outcome across countries.

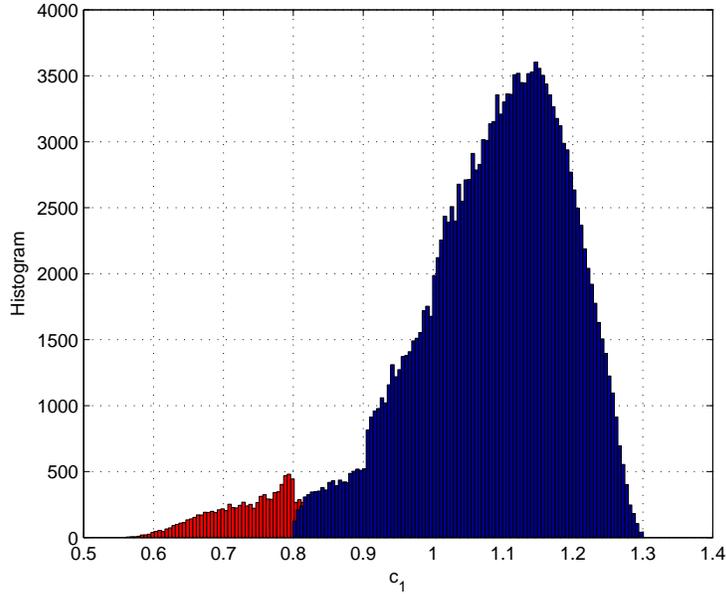


Figure 6: Investors' consumption distribution along a simulated path of 200,000 periods in financial autarky. The blue region denotes the distribution of consumption when collateral constraints don't bind and the red region is for binding collateral constraints. There are also overlapping areas in the middle. Model parameters are the same as the baseline model.

indicates that constrained states tend to be more persistent than unconstrained states. This is consistent with previous analysis (e.g. [Brunnermeier and Sannikov, 2013](#)). Once the economy is in a debt constrained equilibrium, the convergence to a steady state slows down significantly.

Figures 7 and 8 illustrate the joint consumption distribution for country 1 and country 2 in four separate panels, depending upon whether they are both unconstrained, both constrained, or just one country constrained. Figure 7 illustrates the distribution under bond market integration. Again, when both countries are unconstrained, consumption is substantially higher on average, and the distribution of consumption is more tightly ordered. When both countries are constrained, consumption rates are lower, and the distribution is more spread out in both directions. More generally, we see a multi-modal characteristic of the consumption distribution in this case, where depending on constraints that bind, there are multiple local maxima in the distribution of consumption. Figure 7 also indicates two features of the nature of leverage crises under both market equilibrium. First, as in the financial autarky case, there

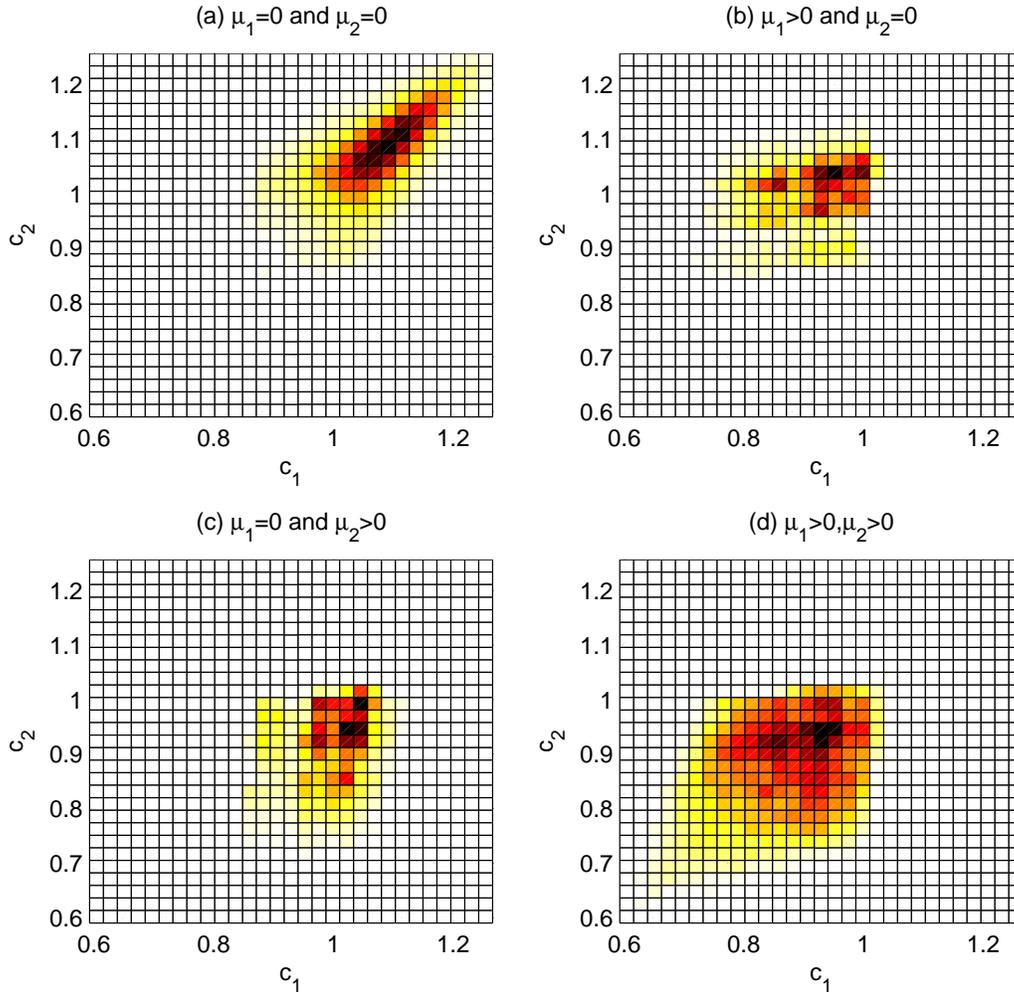


Figure 7: Investors' consumption distribution along a simulated path of 200,000 periods in bond market integration. The darker an area is, the higher the frequency becomes. Panel (a) displays the distribution of consumption when investors in both countries are financially unconstrained, panel (b) for only investors in country 1 constrained, panel (c) for only investors in country 2 constrained, and , panel (d) for investors in both countries constrained. Model parameters are the same as the baseline model.

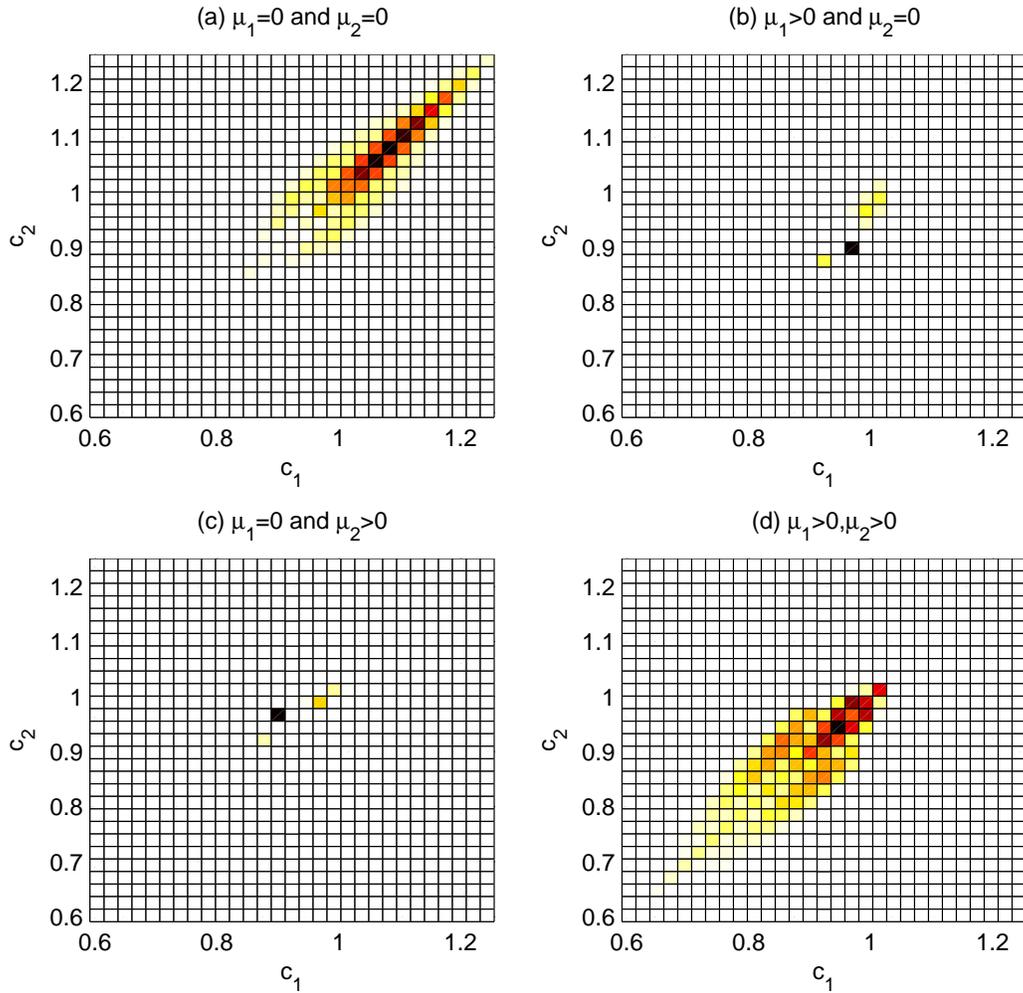


Figure 8: Investors' consumption distribution along a simulated path of 200,000 periods in equity market integration. The darker an area is, the higher the frequency becomes. Panel (a) displays the distribution of consumption when investors in both countries are financially unconstrained, panel (b) for only investors in country 1 constrained, panel (c) for only investors in country 2 constrained, and panel (d) for investors in both countries constrained. Model parameters are the same as the baseline model.

is substantially more persistence in the economy constrained by leverage limits. But also, given that the consumption distribution is more laterally spread, the impact of leverage constraints dramatically reduces the degree to which bond market integration facilitates international consumption risk-sharing. This point can be seen equivalently in going back to Table 3, where we see that the cross country consumption correlation drops significantly when the sample is restricted to episodes of binding leverage constraints.

Finally, Figure 8 shows the joint consumption distribution but now under full equity integration. The sparseness of the distributions with just one constraint binding confirms our previous results that financial contagion is almost complete in the equity integration case. Again we find that the distribution is shifted down substantially when the leverage constraints are binding, and tends to be more spread out. But unlike the bond market case, the distribution is not noticeably more laterally spread out when leverage constraints are binding. The implication of this is that unlike the bond market equilibrium, the presence of binding leverage constraints does not clearly interfere with cross country consumption risk sharing when equity markets are integrated. While leverage crises reduce average consumption rates, they do not limit the degree to which equity markets can share risk across countries. This is confirmed also by noticing that in Table 3, the cross country correlation of consumption is not reduced in simulations that are restricted to the binding leverage states relative to those from the overall sample.

## 5.5 Event Analysis

Figure 9-10 organizes the simulation results in terms of an ‘Event Analysis’. Because the responses to shocks to productivity in the model depend upon the existing states, it is not possible to conduct conventional impulse response calculations as in models analyzed with linear approximations. Instead, we define a particular ‘event’, characterized by a particular set of criteria, and group together all simulation runs in the model which satisfy this criteria, and then taking average of these runs over the whole sample. In this instance, we define the event as a situation where the home country (country 1) experiences a binding leverage

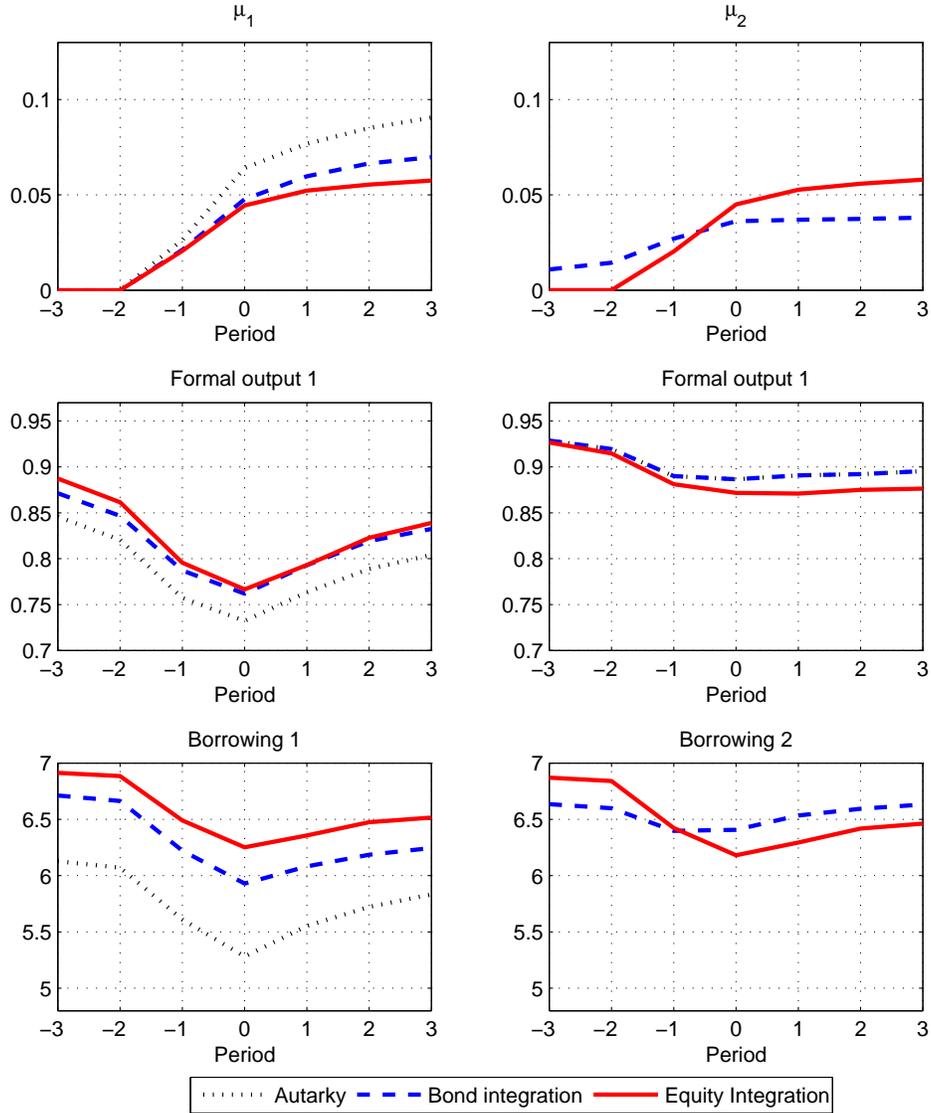


Figure 9: Event analysis in financial autarky (black dots), bond market integration (blue dashed lines) and equity market integration (solid red lines). The figure shows an average of events with a seven-year window along a simulated path with 200,000 periods. The selection of a seven-year window satisfies that collateral constraints don't bind in the first two periods and bind in the following three period (period -1 to 1) in country 1 and country 1 experiences the lowest productivity in the middle period 0.

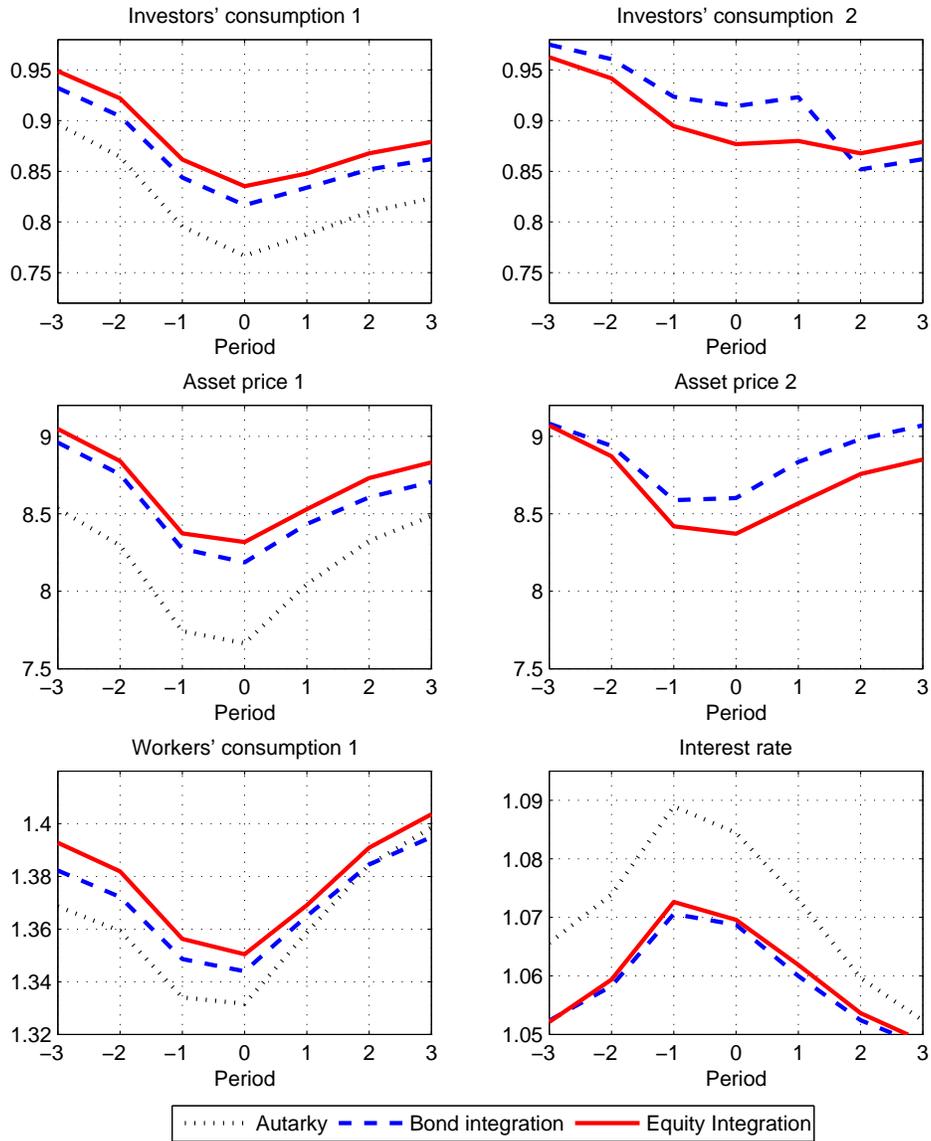


Figure 10: Event analysis in financial autarky (black dots), bond market integration (blue dashed lines) and equity market integration (solid red lines). The figure shows an average of events with a seven-year window along a simulated path with 200,000 periods. The selection of a seven-year window satisfies that collateral constraints don't bind in the first two periods and bind in the following three period (period  $-1$  to  $1$ ) in country 1 and country 1 experiences the lowest productivity in the middle period 0.

constraint for three successive periods, and in addition, the home country experiences the lowest productivity outcome in the *middle* period.<sup>10</sup> All other variables are left unconstrained.

Using this definition for each of the three financial market regimes, we then compare the outcomes in Figure 9-10. For the case of financial autarky, we show only the home country responses, because under financial autarky, the two country's responses are independent of one another. The Figure shows the average time path of the home country and foreign country Lagrange multiplier on the leverage constraint under the three separate regimes.

The results from these pictures are in accord with the previous analysis from the first and second moments in Tables 2-3). In particular, the response of all macro variable in financial autarky is more 'severe' than that under bond or equity market integration. The Lagrange multiplier on the home leverage constraint is uniformly higher in financial autarky, having the interpretation that the constraint is more binding in financial autarky (equivalently, the shadow price of an additional unit of collateral is greater in financial autarky than with integrated bond or equity markets). Output, borrowing, investor's consumption, and the equity price falls substantially more in financial autarky than in either of the integrated regimes. The domestic interest rate also increases much more in financial autarky, increasing the cost of borrowing for domestic investors.

With bond and equity market integration, there is a higher correlation of macro aggregates across the two countries, but the degree of response is less. In general, with bond market integration, the correlation of responses is less than with equity market integration. With bond market integration, output falls by more in the home country, which is the source of the 'event', and also borrowing, investor's consumption, and asset prices fall by more. Bond integration helps insulate the country from the shock, but by less than can be achieved by full equity market integration. With full equity market integration, we see that the responses of the Lagrange multipliers are essentially identical. This translates into a much greater, but still less than perfect, co-movement in macro aggregates. Asset prices, consumption and borrowing move closely together in the two countries, even though the home country is the

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<sup>10</sup>We experimented with other definitions of an event, for instance assuming only that the leverage constraint binds without specifying the productivity state. The results were similar to those in Figure 9-10.

source of the ‘shock’.

## 5.6 Welfare analysis

What are the welfare implications of financial market integration in this model? The answer to this is not immediately obvious, because the presence of leverage constraints in each domestic economy prior to financial opening implies a pre-existing financial distortion, and hence expanding existing financial trading opportunities is not necessarily welfare enhancing. More precisely, a number of authors in recent literature have pointed out (e.g. [Bianchi, 2011](#); [Bianchi and Mendoza, 2010](#); [Korinek, 2011](#); [Korinek and Simsek, 2014](#); [Jeanne and Korinek, 2010](#)) that the existence of balance sheet constraints in financial markets introduces a pecuniary externality associated with the failure of individual financial market participants to take account of their trading activity on the constraints faced by other traders. Hence, in general, investment and portfolio choices will fail to be socially optimal, and as a result it is not guaranteed that opening up financial markets will raise welfare. More generally, the presence of balance sheet constraints opens up the possibility for macro-prudential policy instruments applied to bond or equity trading that may improve upon the unrestricted free market outcomes.

Given that this is a heterogeneous agent model within each country, the evaluation of welfare must involve weighting preferences in some way. This introduces some complications with respect to the nature of the social welfare function, given that investors have a higher rate of time preference than savers. In particular, it would not be a valid comparison to focus only on a stationary or unconditional measure of welfare, since, as shown in [Table 2](#), in a stationary equilibrium with open international financial markets, investors will have lower mean consumption levels, given that they reduce precautionary saving and tilt consumption more toward the present and away from the future. An unconditional measure of welfare solely based on the stationary distribution will neglect the additional welfare benefits that investors obtain from being able to consume earlier, as a result of the lower-volatility environment induced by international financial integration.

To take account of this we compute welfare from a conditional distribution. Specifically, we compute welfare under all three different financial environments using the equilibrium policy functions for each environment, conditional on the same initial conditions for investor's portfolios and debt, where these initial portfolio and debt liabilities are determined at their mean levels under financial autarky. By using this conditional measure of welfare, we incorporate the full potential benefit that investor's receive by reducing their precautionary saving in the lower risk environment, thereby tilting their consumption profile more towards the present.

The conditional utility for a representative agent  $l$  ( $= 1, 2$  for an investor in either country or  $= 3$  for a banker) is defined as

$$Wel_l \equiv E_0 \left\{ \sum_{t=1}^{\infty} \beta_l^{t-1} U(c_{l,t}, h_{l,t}) \right\} \quad (36)$$

We focus on the certainty equivalence of effective consumption  $\tilde{c}_l$ , which is given by<sup>11</sup>

$$Wel_l = E_0 \left\{ \sum_{t=1}^{\infty} \beta_l^{t-1} \frac{[c_{l,t} - v(h_{l,t})]^{1-\sigma} - 1}{1-\sigma} \right\} = \frac{\tilde{c}_l^{1-\sigma} - 1}{1-\sigma} \frac{1}{1-\beta_l}$$

Rearranging the equation above, yields

$$\tilde{c}_l = [Wel_l(1-\sigma)(1-\beta_l) + 1]^{\frac{1}{1-\sigma}} \quad (37)$$

Suppose that economy-wide social welfare is defined as the equally weighted sum of utilities for all agents in an economy, and then we have a measure of economy-wide welfare

$$Wel = nWel_l + (1-n)Wel_3, l = 1, 2 \quad (38)$$

We compute both the conditional welfare and the effective consumption certainty equiv-

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<sup>11</sup>Certainty equivalences of effective consumption across different financial integration regimes are comparable. Let the  $\tilde{c}_l^r$  denote effective consumption certainty equivalence in regime  $r$ ,  $r = 1, 2, 3$  for financial autarky, bond market integration and equity market integration, and then  $(\tilde{c}_l^j/\tilde{c}_l^i - 1) \times 100$  measures the percentage increase of effective consumption in regime  $i$  such that welfare for agent  $l$  in regime  $i$  is the same as that in regime  $j$ .

Table 4: Conditional welfare in the baseline models

	Utility $Wel_l$	Certainty equivalence $\tilde{c}_l$	Change (%)
<i>Panel A: Financial autarky</i>			
Investor	-10.2859	0.6788	
Worker	-2.3124	0.9153	
Economy-wide	-6.2992	0.7971	
<i>Panel B: Bond market integration</i>			
Investor	-10.2725	0.6791	0.042
Worker	-2.1492	0.9208	0.601
Economy-wide	-6.2109	0.8000	
<i>Panel C: Equity market integration</i>			
Investor	-10.2667	0.6792	0.018
Worker	-2.1406	0.9211	0.032
Economy-wide	-6.2036	0.8002	

Notes: This table reports the conditional welfare and certainty equivalence of effective consumption for all agents in various financial integration regimes. The volatility of shocks are the same as in the baseline model.  $Wel_l$  denotes the life time discounted utility. Economy wide welfare is a weighted average of the life time discounted utility for investors and savers (workers). The initial portfolio and shock in period 0 read  $k_{1,0}^1 = k_{2,0}^2 = 1.6$ ,  $b_{1,0} = 4.5$  and  $A_{1,0} = A_{2,0} = A_M$  where  $A_M$  is the middle state of productivity. The last column shows the percentage change of effective consumption in regime  $j$  relative to the previous regime  $i$ ,  $(\tilde{c}_l^j / \tilde{c}_l^i - 1) \times 100$ .

Table 5: Conditional welfare in the low risk economies

	Utility $Wel_l$	Certainty equivalence $\tilde{c}_l$	Change (%)
<i>Panel A: Financial autarky</i>			
Investor	-10.1504	0.6817	
Worker	-2.2617	0.9170	
Economy-wide	-6.2060	0.7994	
<i>Panel B: Bond market integration</i>			
Investor	-10.1780	0.6811	-0.086
Worker	-2.1260	0.9216	0.500
Economy-wide	-6.1520	0.8014	
<i>Panel C: Equity market integration</i>			
Investor	-10.1775	0.6811	0.001
Worker	-2.1242	0.9217	0.007
Economy-wide	-6.1508	0.8014	

Notes: This table reports the conditional welfare and certainty equivalence of effective consumption for all agents in various financial integration regimes. The standard deviation of shocks is one-half of the baseline shocks (2% vs. 4%).  $Wel_l$  denotes the life time discounted utility. Economy wide welfare is a weighted average of the life time discounted utility for investors and savers (workers). The initial portfolio and shock in period 0 read  $k_{1,0}^1 = k_{2,0}^2 = 1.6$ ,  $b_{1,0} = 4.5$  and  $A_{1,0} = A_{2,0} = A_M$  where  $A_M$  is the middle state of productivity. The last column shows the percentage change of effective consumption in regime  $j$  relative to the previous regime  $i$ ,  $(\tilde{c}_l^j / \tilde{c}_l^i - 1) \times 100$ .

alence for each set of agents (investors and savers) in all financial integration regimes. Computation of conditional welfare is done through two steps. First, we compute the stationary value function for each agent in each regime. Second, starting at the same initial conditions mentioned above in period 0, we apply the transition matrix and the value function to calculate the conditional welfare for each agent  $Wel_t$  in each financial integration regime.

Table 4 indicates the results of the comparison of welfare across the different regimes for the baseline calibration of the model. We find that overall social welfare as measured by (38) is higher when we move from financial autarky to bond integration or equity integration. This is also true separately for investors and savers/workers, although as to be expected from previous literature, the gains are very small. Investors gain in terms of consumption certainly equivalence by 0.04% moving from autarky to bond market integration, and then by about another 0.02% moving from bond market integration to equity market integration. The gain for savers/workers is significantly larger, equal to 0.6% and 0.03% respectively. Thus, for the baseline (or high-risk) economy, although there is no theoretical guarantee that financial market integration will raise welfare, we find that this is indeed the case. Welfare rises by a small amount when financial markets are opened, and most of the gains can be accrued by opening international bond markets alone.

But these results are likely to depend on the overall scale of the risk in the economy. In the previous section, we discussed the trade-off between the benefits of diversification arising from international financial market integration and the costs of an increase in the probability of binding balance sheet constraints. A fall in overall risk may be expected to tilt the calculation towards an increase in the importance the costs of financial distortions and away from the benefits of risk sharing. To explore this, Table 5 reports the same welfare calculations in a low-risk economy. This is defined in the same way as before, but now assuming that the unconditional standard deviation of productivity shocks is 2% instead of 4%.<sup>12</sup> Now we find that again, overall social welfare increases with financial integration, as before. But this overall measure hides a conflict among groups. Savers/workers are better off

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<sup>12</sup>Aside from the welfare differences, the qualitative results of the paper do not change under this alternative low-risk analysis.

in either bond market integration or equity integration than under financial autarky. But now, investors are slightly worse off with integrated financial markets, both in bond Integration and equity Integration. As expected, the magnitude of welfare changes in terms of effective consumption equivalencies is again very small.<sup>13</sup> Nevertheless, the negative effects of financial market integration for the welfare of investors indicates that the presence of balance sheet externalities are important enough to reverse the normal presumption of welfare gains from financial market integration for this version of the model.

## 6 Conclusions

This paper constructs and solves a two-country general equilibrium model with endogenous portfolio choice, occasionally binding collateral constraints, and within and across country trade in equity and bond assets. Leverage is time varying and will depend on the nature of international financial markets. The paper finds that international financial integration introduces a trade-off between the frequency and severity of financial crises. Opening up financial markets leads to a higher degree of global leverage, increasing the frequency of financial crises for any one country and dramatically increasing the correlation (or contagion) of crises across countries. But crises in an open world capital market are less severe than in closed economies. In terms of welfare, financial market integration may be positive or negative.

The paper naturally suggests a number of extensions. One major question we have not addressed is the role for economic policy, whether it terms of macro-prudential tools that affect leverage and investment choices of agents, or other more general tools of fiscal or monetary policy. In addition, we have focused on shocks coming from real economic fundamentals - productivity. An obvious further question would be how shocks arising from the financial system itself would affect the nature and workings of international financial

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<sup>13</sup>It is important to note that these welfare calculations are not approximation errors. In calculating welfare effects, we choose a tolerance size and the number of grid points for endogenous state variables so that the approximation error is several orders of magnitude smaller than calculated differences in welfare across financial market regimes.

markets. Finally, we have not allowed a role for aggregate demand deficiencies following crises. These would naturally arise in an extended model that incorporated slow price or wage adjustment.

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# Not For Publication Technical Appendix

## A Asset constraints and model solution

The text discussed the need for imposing asset trading constraints as part of the solution of the general equilibrium model with incomplete markets. Here we report how these asset market constraints affect the first order conditions of investors and bankers in the model.

In the case of full equity market integration, investors first order condition for equity holding are given by:

$$q_{i,t} = \frac{\beta_l E_t \{U_c(c_{l,t+1}, h_{l,t+1})(q_{i,t+1} + R_{ik,t+1})\} + \mu_{l,t} \kappa E_t \{q_{i,t+1}\} - \rho_{i,t+1}^l}{U_c(c_{l,t}, h_{l,t})}, \quad i = 1, 2 \quad (\text{A.1})$$

where  $\rho_{i,t+1}^l$  is defined as

$$\rho_{i,t+1}^l = \frac{\partial \rho^l(k_{1,t+1}^l, k_{2,t+1}^l)}{\partial k_{i,t+1}^l} = 4\kappa_i \min(0, k_{i,t+1}^l - \bar{k}_i^l)^3 \leq 0$$

The numerator of equation (A.1) states three types of gains for an investor from increasing an additional unit of equity holdings: (a) increasing consumption tomorrow, (b) relaxing a borrowing constraint for inter-period loans, (c) reducing penalties of hitting lower bounds of equity holdings.

That analogous optimality condition for bond holdings reads

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_l E_t \{U_c(c_{l,t+1}, h_{l,t+1})\} + \mu_{l,t}}{U_c(c_{l,t}, h_{l,t})} \quad (\text{A.2})$$

For the global banker under equity market integration, the optimality condition for equity holdings are

$$q_{i,t} = \frac{\beta_3 E_t \{U_c(c_{l,t+1}^3, h_{l,t+1}^3)(q_{i,t+1} + G'(k_{i,t+1}^3))\} - \rho_{i,t+1}^3}{U_c(c_{l,t}^3, h_{l,t}^3)}, \quad i = 1, 2 \quad (\text{A.3})$$

where the marginal penalty for holding physical capital  $\rho_{i,t+1}^3$  is similar to  $\rho_{i,t+1}^1$ .

Again, for the banker under equity market integration, the optimality condition for bond holdings is as follows

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_3 E_t \{U_c(c_{l,t+1}^3, h_{l,t+1}^3)\} + \frac{1}{2} \rho_{3,t+1}^3}{U_c(c_{l,t}^3, h_{l,t}^3)} \quad (\text{A.4})$$

where  $\rho_{3,t+1}^3$  is defined as

$$\rho_{3,t+1}^3 = \frac{\partial \rho^3(k_{1,t+1}^3, k_{2,t+1}^3, b_{3,t+1})}{\partial b_{3,t+1}} = -8\kappa_b \min(0, \bar{b}_3 - b_{3,t+1})^3 \geq 0$$

Under bond market integration, the optimality conditions for equity holdings are

$$q_{l,t} = \frac{\beta_l E_t \{U_c(c_{l,t+1}, h_{l,t+1})(q_{l,t+1} + R_{lk,t+1})\} + \mu_{l,t} \kappa E_t \{q_{l,t+1}\}}{U_c(c_{l,t}, h_{l,t})}, \quad l = 1, 2 \quad (\text{A.5})$$

and for bond holding, the optimality condition is

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_l E_t \{U_c(c_{l,t+1}, h_{l,t+1})\} + \mu_{l,t}}{U_c(c_{l,t}, h_{l,t})} \quad (\text{A.6})$$

The global banker's problem under bond market integration is the same as under equity market integration.

Finally, in financial autarky, the investor's optimality condition is the same as that under bond market integration. The banker's problem (now a domestic banker) is written as: The optimality conditions yield

$$q_{l,t} = \frac{\beta_3 E_t \left\{ U_c(c_{l,t+1}^3, h_{l,t+1}^3)(q_{l,t+1} + G'(k_{l,t+1}^3)) \right\} - \rho_{l,t+1}^3}{U_c(c_{l,t}^3, h_{l,t}^3)} \quad (\text{A.7})$$

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_3 E_t \left\{ U_c(c_{l,t+1}^3, h_{l,t+1}^3) \right\} + \rho_{3,t+1}^3}{U_c(c_{l,t}^3, h_{l,t}^3)} \quad (\text{A.8})$$

with penalty on excessive holdings of a portfolio

$$\begin{aligned} \rho_{l,t+1}^3(k_{l,t+1}^3, b_{3,t+1}^l) &= 4\kappa_l \min(0, k_{l,t}^3 - \underline{k}_l^3)^3 \\ \rho_{3,t+1}^3 &= \frac{\partial \rho^3(k_{l,t+1}^3, b_{3,t+1}^l)}{\partial b_{3,t+1}^l} = -4\kappa_b \min(0, \bar{b}_3^l - b_{3,t+1}^l)^3 \geq 0 \end{aligned}$$

## B An event tree approach to solving the model

This section shows in detail how we solve a model with equity market integration. The other two financial integration regimes can be solved similarly and are omitted here. Basically, we need to rewrite the forward-looking recursive competitive equilibrium in terms of a backward-looking system. The event tree approach developed by [Dumas and Lyasoff \(2012\)](#) is used. We use B-splines smooth functions with degree three to interpolate and approximate policy functions on discrete state grid points. Accordingly, the transformed system of equations in equilibrium are twice continuously differentiable by construction. Therefore, a Newton-type method can be used. Next, we calculate the system based on a discrete consumption share distribution, which lies between zero and one, and exogenous shocks  $A_{l,t}$ . Let  $S$  be the number of exogenous states and  $J$  be the number of consumption distribution grid points in the economy. After simplifying the dynamic system, there are  $2S + 6$  ( $2S + 4$  or  $S + 2$ ) number of equations in equity market integration (bond market integration or financial autarky). Let's take equity market integration for example. In equity market integration, we need to solve  $2S + 6$  nonlinear equations at  $S \times J$  different nodes in each iteration. As is well known, it is not a trivial task to solve a system of nonlinear equations

using Newton method, particularly at so many different nodes, unless we provide the solver with an extremely good initial guess at each node. In practice, we adopt a method similar to that in [Judd, Kubler and Schmedders \(2002\)](#), which uses a homotopy path-following algorithm to solve a system of nonlinear equations. Open sources for FORTRAN supply necessary subroutines to use a homotopy path-following algorithm, such as HOMPACT77/90, as well as subroutines for nonlinear solvers. Of course the key challenge for using this algorithm is in constructing a homotopy function.

Nevertheless, a remaining challenge still presents here. The homotopy path-following algorithm requires certain conditions to be satisfied (see, [Schmedders, 1998](#); [Eaves and Schmedders, 1999](#); [Watson, 1990](#)). In practice, it is not easy to verify these conditions, particularly when the system is complicated, like the system here. So we take a practical perspective. That is, we will use the algorithm as long as it leads us to find optimal policy functions. First of all, we need to construct a proper homotopy function such that the homotopy path-following algorithm works effectively. In the GEI literature, several ways are proposed to use the homotopy path-following algorithm. Basically, in these exercises, researchers attempt to find current optimal consumption and exiting portfolio of assets given *current portfolio of assets* and shocks. In order to solve these models, it is normal to introduce a penalty function for asset trading, which guarantees continuity in excess demand functions. Conditional on continuous excess demand functions, the approach rephrases agents' objective functions such that financial autarky for each agent in each period is a good start for the homotopy path-following algorithm. However, the way of constructing homotopy functions in these papers seems improper in our model, because we don't have any information on the *current portfolio of assets*. The main fact about the system here is that it should have a solution under a proper set of parameter values and that the Jacobian of the system should have full rank. After trying several ways of homotopy functions, we found that the Newton homotopy works quite well in the current model.

To illustrate, let  $F(x)$  be a system of nonlinear equations with an endogenous variable vector  $x$ .  $F(x)$  contains all of the equilibrium conditions in the model and it is a square nonlinear system. A homotopy function  $H(x, \lambda)$  is defined as

$$H(x, \lambda) \equiv \lambda F(x) + (1 - \lambda)(F(x) - F(x_0)) \tag{B.1}$$

where  $x_0$  is a starting point for the homotopy path-following and  $0 \leq \lambda \leq 1$ . When  $\lambda = 0$ , the homotopy function is degenerated to a simple system  $H(x_0, 0) = F(x_0) - F(x_0) = 0$ . This simple system  $F(x) - F(x_0) = 0$  has a unique and robust solution at  $x = x_0$ . When  $\lambda = 1$ , the homotopy function becomes the original function  $H(x, 1) = F(x)$ . Observe that if the Jacobian of  $F(x)$  has a full rank, the Jacobian of the homotopy function  $H(x, \lambda)$  also has a full row rank. Based on this constructed homotopy function, we can solve the transformed system for the optimal policy functions. We next need to simulate a path for exogenous shocks to pin down other endogenous variables. Combining the optimal policy functions with the initial conditions for shocks and portfolios, we can solve for all endogenous variables along the simulation path.

## B.1 Equilibrium conditions

Following the algorithm in [Dumas and Lyasoff \(2012\)](#), we rewrite the whole equilibrium system in the period  $t + 1$  and get rid of all endogenous variables in period  $t$ .

The so-called ‘marketability conditions’ (budget constraints) of [Dumas and Lyasoff \(2012\)](#) are as follows

$$\tilde{c}_{l,t+1} + F_{l,t+1} = NW_{l,t+1} - v(H_{l,t+1}) + k_{1,t+1}^l (q_{1,t+1} + R_{1k,t+1}) + k_{2,t+1}^l (q_{2,t+1} + R_{2k,t+1}) - b_{l,t+1} \quad (\text{B.2})$$

where exiting wealth is  $F_{l,t+1} \equiv q_{1,t+1}k_{1,t+2}^l + q_{2,t+1}k_{2,t+2}^l - b_{l,t+2}/R_{t+2}$  with  $l = 1, 2$ , and effective consumption is  $\tilde{c}_{l,t+1} \equiv c_{l,t+1} - v(H_{l,t+1})$ . The effective non-financial income becomes  $NW_{l,t+1} - v(H_{l,t+1})$ , where  $NW_{l,t+1}$  is defined as

$$\begin{aligned} NW_{l,t+1} &\equiv \frac{1}{n} [F(A_{l,t+1}, H_{l,t+1}, K_{l,t+1}) - W_{l,t+1}H_{l,t+1} - K_{l,t+1}R_{lk,t+1}] + W_{l,t+1}H_{l,t+1} \\ &= W_{l,t+1}H_{l,t+1} \end{aligned} \quad (\text{B.3})$$

where we have used the fact that the formal production is CRS.

Kernel conditions read

$$\begin{aligned} q_{i,t} &= \frac{\beta_l E_t \{U_c(c_{l,t+1}, h_{l,t+1})(q_{i,t+1} + R_{ik,t+1})\} + \mu_{l,t} \kappa E_t \{q_{i,t+1}\} - \rho_{i,t+1}^l}{U_c(c_{l,t}, h_{l,t})} \\ &= \frac{\beta_3 E_t \{U_c(c_{l,t+1}^3, h_{l,t+1}^3)(q_{i,t+1} + G'(k_{i,t+1}^3))\} - \rho_{i,t+1}^3}{U_c(c_{l,t}^3, h_{l,t}^3)} \end{aligned} \quad (\text{B.4})$$

with  $l = 1, 2$  and  $i = 1, 2$ . And

$$q_{3,t} = \frac{\beta_l E_t \{U_c(c_{l,t+1}, h_{l,t+1})\} + \mu_{l,t}}{U_c(c_{l,t}, h_{l,t})} = \frac{\beta_3 E_t \{U_c(c_{l,t+1}^3, h_{l,t+1}^3)\} + \frac{1}{2}\rho_{3,t+1}^3}{U_c(c_{l,t}^3, h_{l,t}^3)}, \quad l = 1, 2 \quad (\text{B.5})$$

The collateral constraint reads

$$\left( \kappa E_t \{q_{1,t+1}k_{1,t+1}^l + q_{2,t+1}k_{2,t+1}^l\} - b_{l,t+1} \right) \mu_{l,t} = 0 \quad (\text{B.6})$$

Equilibrium labor yields

$$H_{l,t+1} = \left[ \frac{\alpha A_{l,t+1} K_{l,t+1}^{1-\alpha}}{\chi} \right]^{\frac{1}{1+\nu-\alpha}} \quad (\text{B.7})$$

Let the global output be

$$Y_t \equiv A_{1,t} H_{1,t}^\alpha K_{1,t}^{1-\alpha} + A_{2,t} H_{2,t}^\alpha K_{2,t}^{1-\alpha} + (1-n) (Z(k_{1,t}^3)^\gamma + Z(k_{2,t}^3)^\gamma)$$

and the effective global output is given by

$$\tilde{Y}_t \equiv Y_t - v(H_{1,t}) - v(H_{2,t})$$

## B.2 Scaled equilibrium conditions

The share of investors' effective consumption in country  $l$  is defined as

$$\omega_{l,t+1} \equiv \frac{n\tilde{c}_{l,t+1}}{\tilde{Y}_{t+1}}, \quad l = 1, 2 \quad (\text{B.8})$$

Then bankers' effective consumption share becomes  $\omega_{3,t+1} = \frac{2(1-n)\tilde{c}_{1,t+1}^3}{\tilde{Y}_{t+1}} = 1 - \omega_{1,t+1} - \omega_{2,t+1}$ . Effective consumption for each agent thus reads

$$\tilde{c}_{1,t+1} = \frac{\omega_{1,t+1}\tilde{Y}_{t+1}}{n}, \quad \tilde{c}_{2,t+1} = \frac{\omega_{2,t+1}\tilde{Y}_{t+1}}{n}, \quad \tilde{c}_{1,t+1}^3 = \frac{(1 - \omega_{1,t+1} - \omega_{2,t+1})\tilde{Y}_{t+1}}{2(1-n)}$$

Therefore, consumption levels can be obtained as

$$\begin{aligned} c_{1,t+1} &= \tilde{c}_{1,t+1} + v(H_{1,t+1}) & c_{2,t+1} &= \tilde{c}_{2,t+1} + v(H_{2,t+1}) \\ c_{1,t+1}^3 &= \tilde{c}_{1,t+1}^3 + v(H_{1,t+1}) & c_{2,t+1}^3 &= \tilde{c}_{1,t+1}^3 + v(H_{2,t+1}) \end{aligned}$$

Notice that current output  $\tilde{Y}_t$  is an endogenous variable, we then scale asset prices and exiting wealth by  $1/\tilde{Y}_t^\sigma$

$$\tilde{q}_{i,t} = \frac{q_{i,t}}{\tilde{Y}_t^\sigma}, \quad i = 1, 2, 3 \quad \tilde{F}_{l,t} = \frac{F_{l,t}}{\tilde{Y}_t^\sigma}, \quad l = 1, 2. \quad (\text{B.9})$$

Substituting the effective consumption share  $\{\omega_1, \omega_2\}$ , scaled asset prices  $\tilde{q}_i$  and exiting financial wealth  $\tilde{F}_l$  in period  $t$  and  $t + 1$  into the system of equations (B.4-B.7) to replace the effective consumption, asset prices and exiting financial wealth, we then obtain a transformed system of equations, which are listed as follows

$$\begin{aligned} \omega_{l,t+1} + \frac{n}{\tilde{Y}_{t+1}} \tilde{Y}_{t+1}^\sigma \tilde{F}_{l,t+1} &= \frac{n}{\tilde{Y}_{t+1}} (NW_{l,t+1} - v(H_{l,t+1})) + \frac{n}{\tilde{Y}_{t+1}} k_{1,t+1}^l (\tilde{Y}_{t+1}^\sigma \tilde{q}_{1,t+1} + R_{1k,t+1}) + \\ &\frac{n}{\tilde{Y}_{t+1}} k_{2,t+1}^l (\tilde{Y}_{t+1}^\sigma \tilde{q}_{2,t+1} + R_{2k,t+1}) - \frac{n}{\tilde{Y}_{t+1}} b_{l,t+1}, \quad l = 1, 2 \end{aligned} \quad (\text{B.10})$$

$$\begin{aligned} \tilde{q}_{i,t} &= \frac{\beta_l E_t \left\{ \left( \frac{\omega_{l,t+1} \tilde{Y}_{t+1}}{n} \right)^{-\sigma} \left( \tilde{Y}_{t+1}^\sigma \tilde{q}_{i,t+1} + R_{ik,t+1} \right) \right\} + \mu_{l,t} \kappa E_t \left\{ \tilde{Y}_{t+1}^\sigma \tilde{q}_{i,t+1} \right\} - \rho_{i,t+1}^l}{\left( \frac{\omega_{l,t}}{n} \right)^{-\sigma}} \\ &= \frac{\beta_3 E_t \left\{ \left( \frac{\omega_{3,t+1} \tilde{Y}_{t+1}}{2(1-n)} \right)^{-\sigma} \left( \tilde{Y}_{t+1}^\sigma \tilde{q}_{i,t+1} + G'(k_{i,t+1}^3) \right) \right\} - \rho_{i,t+1}^3}{\left( \frac{\omega_{3,t}}{2(1-n)} \right)^{-\sigma}} \end{aligned} \quad (\text{B.11})$$

with  $l = 1, 2$  and  $i = 1, 2$ .

$$\tilde{q}_{3,t} = \frac{\beta_l E_t \left\{ \left( \frac{\omega_{l,t+1} \tilde{Y}_{t+1}}{n} \right)^{-\sigma} \right\} + \mu_{l,t}}{\left( \frac{\omega_{l,t}}{n} \right)^{-\sigma}} = \frac{\beta_3 E_t \left\{ \left( \frac{\omega_{3,t+1} Y_{t+1}}{2(1-n)} \right)^{-\sigma} \right\} + \frac{1}{2} \rho_{3,t+1}^3}{\left( \frac{\omega_{3,t}}{2(1-n)} \right)^{-\sigma}}, \quad l = 1, 2 \quad (\text{B.12})$$

$$\mu_{l,t} \left( \kappa E_t \left\{ \tilde{Y}_{t+1}^\sigma \tilde{q}_{1,t+1} k_{1,t+1}^l + \tilde{Y}_{t+1}^\sigma \tilde{q}_{2,t+1} k_{2,t+1}^l \right\} - b_{l,t+1} \right) = 0, \quad l = 1, 2 \quad (\text{B.13})$$

$$H_{l,t+1} = \left[ \frac{\alpha A_{l,t+1} K_{l,t+1}^{1-\alpha}}{\chi} \right]^{\frac{1}{1+\nu-\alpha}} \quad (\text{B.14})$$

Consumption share simplex reads

$$\Omega \equiv \left\{ (\omega_{1,t}, \omega_{2,t}, \omega_{3,t}) : \omega_{l,t} > 0, \quad l = 1, 2, 3, \quad \text{and} \quad \sum_{l=1}^3 \omega_{l,t} = 1 \quad \text{for all } t \right\} \quad (\text{B.15})$$

Let the exogenous shock vector be  $A_t = (A_{1,t}, A_{2,t})$ . Policy functions for variables of interest can be expressed as functions of investors' effective consumption distribution  $(\omega_{1,t}, \omega_{2,t})$ . Given future policy functions  $\{\tilde{q}_{i,t+1}(\omega_{1,t+1}, \omega_{2,t+1}; A_{t+1}), \tilde{F}_{l,t+1}(\omega_{1,t+1}, \omega_{2,t+1}; A_{t+1})\}$ , with  $i = 1, 2, 3$  and  $l = 1, 2$ , given current effective consumption shares  $\{\omega_{1,t}, \omega_{2,t}\}$ , we solve for future state-contingent effective consumption shares  $\{\omega_{l,t+1}(\omega_{1,t}, \omega_{2,t}; A_t; A_{t+1})\}_{l=1,2}$ , the Lagrange multiplier for the inter-period collateral constraint  $\{\mu_{l,t}(\omega_{1,t}, \omega_{2,t}; A_t)\}_{l=1,2}$ , state-contingent equilibrium labor  $\{H_{l,t+1}(\omega_{1,t}, \omega_{2,t}; A_t; A_{t+1})\}_{l=1,2}$ , end-of-period  $t$  equity portfolio  $\{k_{l,t+1}^i(\omega_{1,t}, \omega_{2,t}; A_t)\}_{i=1,2,3}$  and bond portfolio  $\{b_{i,t+1}(\omega_{1,t}, \omega_{2,t}; A_t)\}_{l=1,2}$ . Asset price and exiting financial wealth are updated via corresponding conditions  $\{\tilde{q}_i(\omega_{1,t}, \omega_{2,t}; A_t)\}_{i=1,2,3}$  and  $\{\tilde{F}_l(\omega_{1,t}, \omega_{2,t}; A_t)\}_{l=1,2}$ , which are derived in the following subsection in detail.

Let  $S$  be the number of exogenous states in the economy. there are then  $2S + 7 + 2S = 4S + 7$  equations and variables to be solved at each grid point and at each iteration.

### B.3 Dealing with inequality constraints

Following [Judd, Kubler and Schmedders \(2002\)](#), we make the following transformation

$$\mu_{l,t} = (\max\{0, \eta_{l,t}\})^L, \quad l = 1, 2 \quad (\text{B.16})$$

$$\kappa E_t \left\{ q_{1,t+1} k_{1,t+1}^l + q_{2,t+1} k_{2,t+1}^l \right\} - b_{l,t+1} = (\max\{0, -\eta_{l,t}\})^L, \quad l = 1, 2$$

where  $\eta_{l,t}$  is a real number and  $L$  is an integer, usually taking  $L = 3$ . These two equations are equivalent to the slackness conditions in the system. Notice that function  $(\max\{0, \eta_{l,t}\})^L$  is a  $(L - 1)$ th continuously differentiable function. Therefore, the transformed equilibrium system is twice continuously differentiable, and a Newton method can be applied here.

Rearranging the collateral constraint above, yields

$$b_{l,t+1} = \kappa E_t \left\{ \tilde{Y}_{t+1}^\sigma \tilde{q}_{1,t+1} k_{1,t+1}^l + \tilde{Y}_{t+1}^\sigma \tilde{q}_{2,t+1} k_{2,t+1}^l \right\} - (\max \{0, -\eta_{l,t}\})^L, \quad l = 1, 2 \quad (\text{B.17})$$

Accordingly, we use  $\eta_{l,t}$  to replace  $\mu_{l,t}$  in the computation.

## B.4 Simplifying the system

Notice that the sum of consumption shares equals unity. We can use only consumption shares for investors as independent state variables here. In addition, we can also get rid of bond holdings  $b_{l,t+1}$  by using equation (B.17). Asset market clearing conditions imply that one agent's portfolio of assets is pinned down by the portfolios of the rest of agents. Consequently, we have a sequence of independent variables  $\{\omega_{l,t+1,s}\}_{l=1,2;s=1,\dots,S}$ ,  $\{k_{i,t}^l\}_{l=1,2;i=1,2}$ ,  $\{\eta_{l,t}\}_{l=1,2}$ , totally  $2S + 6$  variables. The system of equations consists of  $2S + 6$  equations (B.10)-(B.12), with  $l = 1, 2$ ,  $i = 1, 2$ .

## B.5 Updating asset prices and exiting financial wealth

Once solving the system of equations above, we update asset prices according to equation (B.11)-(B.12). Multiplying  $\beta_l U_c(c_{l,t+1}, h_{l,t+1})$  on both sides of equation (B.2), taking expectations conditional on information up to period  $t$ , and replacing relevant terms with the ones in corresponding consumption Euler equations and complementary slackness conditions for collateral constraints, yields

$$F_{l,t} = \frac{1}{U_c(c_{l,t}, h_{l,t})} \left\{ \begin{array}{l} \beta_l E_t [U_c(c_{l,t+1}, h_{l,t+1})(F_{l,t+1} + \tilde{c}_{l,t+1} - NW_{l,t+1} + v(H_{l,t+1}))] \\ -k_{1,t+1}^l \rho_{1,t+1}^l - k_{2,t+1}^l \rho_{2,t+1}^l \end{array} \right\}$$

where net non-financial income  $NW_{l,t+1}$  can be written as

$$NW_{l,t+1} = W_{l,t+1} H_{l,t+1}$$

Normalizing this equation by  $Y_t^\sigma$ , yields

$$\tilde{F}_{l,t} = \frac{1}{\left(\frac{\omega_{l,t}}{n}\right)^{-\sigma}} \left\{ \begin{array}{l} \beta_l E_t \left[ \left(\frac{\omega_{l,t+1} \tilde{Y}_{t+1}}{n}\right)^{-\sigma} \left( \tilde{Y}_{t+1}^\sigma \tilde{F}_{l,t+1} + \frac{\omega_{l,t+1} \tilde{Y}_{t+1}}{n} - NI_{l,t+1} + v(H_{l,t+1}) \right) \right] \\ -k_{1,t+1}^l \rho_{1,t+1}^l - k_{2,t+1}^l \rho_{2,t+1}^l \end{array} \right\}, \quad l = 1, 2 \quad (\text{B.18})$$

## B.6 The initial period t=0 and simulated paths

At period  $t \geq 1$ , we can solve for endogenous variables  $\{H_{l,t+1}, K_{l,t+1}, k_{i,t+1}^l, b_{l,t+1}, \mu_{l,t}, \omega_{l,t+1}\}$ , with  $l = 1, 2$ ,  $i = 1, 2, 3$ , as functions of consumption share distribution in period  $t$  (also in period  $t+1$  for state-contingent variables). Asset prices  $\tilde{q}_{i,t}$  and exiting financial wealth  $\tilde{F}_{i,t}$  can be updated based on the corresponding consumption Euler equations. Although we can't prove the existence of an equilibrium in such a complicated model as ours, we take a more practical approach as in most

of the literature. As long as policy functions of interest converge after a long enough period of time, we assume that an equilibrium exists over an appropriate domain. Nevertheless, the economy starts with some initial conditions such as initial portfolio,  $k_{l,0}^i, b_{l,0}$ , initial interest rate  $R_0$  and shocks  $A_0$ . The path for variables of interest should be calculated given these initial conditions. We first solve for  $\omega_{l,0}, H_{l,0}$  with  $l = 1, 2$  based on the following four equations.

$$\begin{aligned} \omega_{l,0} + \frac{n}{\tilde{Y}_0} \tilde{Y}_0^\sigma \tilde{F}_{l,0} &= \frac{n}{\tilde{Y}_0} (NW_{l,0} - v(H_{l,0})) + \frac{n}{\tilde{Y}_0} k_{1,0}^l (\tilde{Y}_0^\sigma \tilde{q}_{1,0} + R_{1k,0}) + \\ &\frac{n}{\tilde{Y}_0} k_{2,0}^l (\tilde{Y}_0^\sigma \tilde{q}_{2,0} + R_{2k,0}) - \frac{n}{\tilde{Y}_0} b_{l,0}, \quad l = 1, 2, 3 \end{aligned} \quad (\text{B.19})$$

$$H_{l,0} = \left[ \frac{\alpha A_{l,0} K_{l,0}^{1-\alpha}}{\chi} \right]^{\frac{1}{1+\nu-\alpha}} \quad (\text{B.20})$$

Notice that equilibrium labor is a function of state variables and becomes known at the beginning of a period. We need essentially to solve two budget constraints for consumption share distribution  $\{\omega_{1,0}, \omega_{2,0}\}$ . Once obtaining the current consumption distribution  $\{\omega_{1,0}, \omega_{2,0}\}$ , the end-of-period portfolio  $k_{l,1}^i, b_{l,1}$  are obtained via interpolating relevant policy functions. We then move the process forwarding redoing the calculation for  $\{\omega_{1,t}, \omega_{2,t}\}$  based on four equations in period  $t$  and given portfolio  $k_{l,t}^i, b_{l,t}$  with  $t \geq 1$  along the simulation path. Other endogenous variables can be found accordingly along the simulation path.

## B.7 The algorithm for solving the model

Assume that exogenous shocks  $(A_{1,t}, A_{2,t})$  follow a Markovian process. We can use time-iteration (backward induction) to solve the system. At the last period  $T$ ,  $\tilde{q}_{i,T} = \tilde{F}_{l,T} = 0$  with  $i = 1, 2$  and  $l = 1, 2$ . The algorithm is summarized as follows

- Step 1.** Choose an appropriate function tolerance  $\epsilon$ . In the baseline model we use  $\epsilon = 10^{-5}$ . Discretize the exogenous state space  $(A_{1,t}, A_{2,t})$  into  $S$  grid points  $\{(a_{1,s}, a_{2,s})\}_{s=1, \dots, S}$  and endogenous state space  $\Omega$  into  $J = n_x n_y$  grid points  $\{(\omega_{1,i}, \omega_{2,j})\}_{j=1, \dots, n_y}^{i=1, \dots, n_x}$ . Set period  $T$  long enough.
- Step 2.** Given asset price functions  $\tilde{q}_{i,t+1}$  and exiting wealth functions  $\tilde{F}_{l,t+1}$  with  $i = 1, 2, 3$  and  $l = 1, 2$ ,  $t = T - 1, T - 2, \dots$ , for each grid point  $\{(a_{1,s}, a_{2,s}; \omega_{1,i}, \omega_{2,j})\}$ , we solve equation (B.10)-(B.12) with  $l = 1, 2$ ,  $i = 1, 2$  for state consumption share  $\{\omega_{l,t+1}\}_{l=1,2}$ , current portfolio  $\{k_{i,t+1}^l\}_{i=1,2}^{l=1,2}$  and current Lagrange multipliers  $\{\eta_{l,t}\}_{l=1,2}$ . Asset price  $\tilde{q}_{i,t+1}$  and exiting wealth  $\tilde{F}_{l,t+1}$  are obtained through interpolation at a specific point of  $\{(a_{1,t+1}, a_{2,t+1}; \omega_{1,t+1}, \omega_{2,t+1})\}$ . Current asset prices  $\tilde{q}_{i,t}$ ,  $i = 1, 2, 3$ , are updated through equation (B.11)-(B.12), and exiting wealth  $\tilde{F}_{l,t}$ ,  $l = 1, 2$ , through equation (B.18).

**Step 3.** Compare the distance between two consecutive asset prices and exiting wealths

$$dist = \max\{|k_{i,t+1}^l - k_{i,t}^l|, |\tilde{q}_{i,t+1} - \tilde{q}_{i,t}|, |\tilde{F}_{l,t+1} - \tilde{F}_{l,t}|\}_{l=1,2; i=1,2,3}$$

If  $dist \geq \epsilon$ , go to step 2; otherwise terminate the calculation and go to step 4.

**Step 4.** Once obtaining a convergent solution, we simulate the model forwardly for given initial conditions  $\{k_{i,0}^l\}_{i=1,2}^{l=1,2,3}$ ,  $\{b_{l,0}\}_{l=1,2,3}$  and shock  $A_0$  to obtain state consumption levels  $\{c_{l,t}\}_{l=1,2,3}$ , labor  $\{H_{l,t}\}_{l=1,2}$ , portfolios  $\{k_{i,t+1}^l\}_{i=1,2}^{l=1,2,3}$ ,  $\{b_{l,t+1}\}_{l=1,2,3}$ , Lagrange multipliers  $\{\mu_{l,t}\}_{l=1,2}$ , asset prices  $\{q_{i,t}\}_{i=1,2,3}$  and exiting wealth  $\{F_{l,t}\}_{l=1,2}$ , for  $t = 0, 1, 2, \dots$ .

## C Interpolation and approximation

### C.1 Consumption simplex

In the model, the domain of consumption shares is a triangle, which is not easy to cope with directly in computation. There are several methods to investigate this issue in numerical analysis, for instance, Barycentric coordinates on surfaces. However, we will avoid this computational issue by making a one-to-one mapping between a triangle and a rectangle.<sup>14</sup> Consumption share simplex in the economy is rewritten here for convenience

$$\Omega \equiv \left\{ (\omega_1, \omega_2, \omega_3) : \omega_l > 0, l = 1, 2, 3, \text{ and } \sum_{l=1}^3 \omega_l = 1 \right\}$$

Here, we treat  $(\omega_1, \omega_2)$  as a pair of free states. The consumption share simplex is equivalent to  $0 < \omega_i < 1$ ,  $i = 1, 2$ ,  $\omega_1 + \omega_2 < 1$  and  $\omega_3 = 1 - \omega_1 - \omega_2$ . We employ a trick in the following way. First, write  $(\omega_1, \omega_2)$  as functions of two other variables, say,  $z$ ,  $w$

$$\omega_1 = \frac{z}{1+z+w} \qquad \omega_2 = \frac{w}{1+z+w}$$

Here,  $\omega_1$  is increasing in  $z$  and decreasing in  $w$ .  $\omega_2$  is increasing in  $w$  and decreasing in  $z$ . These two functions map  $(0, +\infty) \cup (0, +\infty)$  onto the consumption share simplex  $\Omega$ . Second, finding a mapping between a rectangle and  $(0, +\infty) \cup (0, +\infty)$ , we use two new variables  $(\theta_1, \theta_2)$  which are defined over  $(0, 1) \cup (0, 1)$ , and two new functions here

$$z = -\log(\theta_1) \qquad w = -\log(\theta_2)$$

Consequently, we build a one-to-one mapping from a rectangle  $(0, 1) \cup (0, 1)$  onto the simplex  $\Omega$ .<sup>15</sup>

<sup>14</sup>We thank Hiroyuki Kasahara for his helpful suggestion.

<sup>15</sup>Of course, there are many other transformations. Let's discuss some of them. Example 1:  $(\theta_1, \theta_2) \in$

## C.2 B-spline interpolation and approximation

In the algorithm, we need to know the policy functions  $\tilde{q}_{h,t+1}(\theta_{1,t+1}, \theta_{2,t+1}, s_{t+1})$  and  $\tilde{F}_{l,t+1}(\theta_{1,t+1}, \theta_{2,t+1}, s_{t+1})$  with  $h = 1, 2, 3$ ,  $l = 1, 2$  and  $s_{t+1}$  is the state of Nature. Since we cannot obtain closed-form expressions for these functions, we take use of B-spline smooth functions with degree 3 to approximate policy functions. Therefore the approximated functions are twice continuously differentiable. We approximate asset prices  $\tilde{q}_h(\theta_1, \theta_2, s)$  with  $h = 1, 2, 3$  and exiting wealth  $\tilde{F}_l(\theta_1, \theta_2, s)$  with  $l = 1, 2$  parametrically by functions

$$\hat{q}_h(\theta_1, \theta_2, s) = \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} a_{ij}^{qhs} B_i(\theta_1) B_j(\theta_2)$$

$$\hat{F}_l(\theta_1, \theta_2, s) = \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} a_{ij}^{fls} B_i(\theta_1) B_j(\theta_2)$$

where  $a_{ij}^{qhs}$  and  $a_{ij}^{fls}$  are coefficients in B-spline approximations.

B-splines are piecewise polynomial. The B-splines of degree 0 are defined as

$$B_i^0(x) = \begin{cases} 0 & \text{if } x < x_i \\ 1 & \text{if } x_i \leq x < x_{i+1} \\ 0 & \text{if } x_{i+1} \leq x \end{cases}$$

where  $\{x_i\}_{i=1, \dots, n}$  are grid points on  $x$ . The B-splines of degree 1 follow as

$$B_i^1(x) = \begin{cases} 0 & \text{if } x < x_i \\ \frac{x-x_i}{x_{i+1}-x_i} & \text{if } x_i \leq x < x_{i+1} \\ \frac{x_{i+2}-x}{x_{i+2}-x_{i+1}} & \text{if } x_{i+1} \leq x < x_{i+2} \\ 0 & \text{if } x_{i+2} \leq x \end{cases}$$

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$(0, \frac{\pi}{2}) \cup (0, \frac{\pi}{2})$

$$z = \tan(\theta_1)$$

$$w = \tan(\theta_2)$$

Example 2:  $(\theta_1, \theta_2) \in (0, 1) \cup (0, 1)$

$$z = \frac{1}{\theta_1} - 1$$

$$w = \frac{1}{\theta_2} - 1$$

Example 3:  $(\theta_1, \theta_2) \in (0.5, 1) \cup (0.5, 1)$

$$z = \log(\theta_1) - \log(1 - \theta_1)$$

$$w = \log(\theta_2) - \log(1 - \theta_2)$$

Example 4:  $(\theta_1, \theta_2) \in (-1, 1) \cup (-1, 1)$

$$\omega_1 = \frac{1 + \theta_1}{2}$$

$$\omega_2 = \frac{1}{4}(1 + \theta_2)(1 - \theta_1)$$

The key differences among these transformations are the density of consumption shares on the simplex  $\Omega$  even when grid points for  $(\theta_1, \theta_2)$  are equidistant.

The B-splines of degree  $k$  have a recursive form of

$$B_i^k(x) = \frac{x - x_i}{x_{i+k} - x_i} B_i^{k-1}(x) + \frac{x_{i+k+1} - x}{x_{i+k+1} - x_{i+1}} B_{i+1}^{k-1}(x), \quad k \geq 1$$

The B-splines of degree  $k$  require  $(n + k)$  grid points. In the algorithm, we need to implement interpolation and approximation of policy functions. In the interpolation part, asset prices and exiting wealth are obtained through interpolating approximated asset price functions  $\hat{q}_h(\theta_1, \theta_2, s)$  and end-of-period wealth  $\hat{F}_h(\theta_1, \theta_2, s)$ . Once we have updated current asset prices and exiting wealth through equation (B.11) and (B.18), coefficients in the approximated functions  $\{a_{ij}^{qhs}, a_{ij}^{fls}\}$  can be obtained through interpolation.

### C.3 Interpolation on a bounded set

In interpolation, we want to find an asset pricing (exiting wealth) function  $f(state, \cdot) : \Omega \rightarrow \Omega$ , where  $\Omega$  is a bounded set. In practice, we might only use a subset of  $\Omega$  to enhance the accuracy given the number of nodes (the nodes are more dense on a subset of  $\Omega$  for a given number of grid points). The grid points we are mainly concerned with are boundary points. For instance, when they are patient and have high consumption shares today, say at the boundary points in the domain, investors will hold more assets to smooth consumption whenever they have the chance, say, exogenous constraints on asset holdings are not tight. It would be the case that their optimal consumption shares are higher than the upper bound of the bounded set. The zero-finding problem then becomes truncated and it's possible that the nonlinear system doesn't have a solution. There are at least three ways to deal with this situation.

One is increasing the range of the bounded set. The problem is how large the highest consumption share is since the domain in the original problem is an open set (say,  $\Omega$ ). For given preference parameters, there should exist, we think, an upper bound for consumption shares. But as a matter of fact, this approach will produce a policy function surface with many curvatures, which in turn requires very dense grid points to obtain an accurate solution.

A second way is to make investors more patient, in which investors prefer consuming today to tomorrow. Therefore, consumption share domain is smaller than that in the original problem. When they are impatient enough, investors would borrow a lot and they will face binding borrowing constraints in equilibrium. Thereby, given other parameter values unchanged, an economy with more impatient investors would make collateral constraints bind more frequently.

Another way to deal with unbounded asset holdings is to allow for large penalties when asset holdings exceed some bounds. For instance, if we don't allow for short positions (or small positive positions) in assets, agents can not accumulate a large position in any of assets. In this case, their consumption shares will lie in a narrow range of its natural domain.

Our approach takes a practical perspective. First, we set the penalties for over holdings of portfolios are large enough, choose a relatively large tolerance size, and try a relatively large domain for consumption shares to obtain a stationary solution to the model. Note that the achieved solution might be inaccurate. We then narrow the domain for consumption shares based on simulations,

use the obtained stationary solution as an new initial guess, and then rerun the solution procedure above to obtain an accurate solution.

## C.4 Discretizing an AR(1) process with a disaster risk

We discretize the continuous state technological shock Markov process into a discrete and finite state Markov process. Researchers usually make use of [Hussey and Tauchen \(1991\)](#)'s method to find a finite state Markov process based on a standard AR(1) Markov process. However their method doesn't apply directly here because of the additional disaster risk  $\phi_t$ . We use the following approach instead.<sup>16</sup>

Assume that the finite state space for the exogenous state variable is  $A_l$  in country  $l$ . For the sake of simplicity but without loss of generality, we choose three states to characterize the exogenous process (C.1) in each country,

$$\ln(A_{l,t+1}) = (1 - \rho_z) \ln(A_l) + \rho_z \ln(A_{l,t}) + D\phi_{l,t+1} + \epsilon_{l,t+1}, \quad l = 1, 2 \quad (\text{C.1})$$

$A_l = \{lo, mid, hi\}$ . These three states could be any numbers around the unconditional mean of  $A_{l,t}$ . We set the middle state  $mid$  equals the mean of  $A_{l,t}$ , the lowest state  $lo = mid - 2 \times std(A_{l,t})$ , and the highest state  $hi = mid + std(A_{l,t})$ , where  $std$  stands for the unconditional standard deviation of  $A_{l,t}$ . Now we need an associated transition matrix  $\Pi$  such that the discrete state process generates the same moments as the original AR(1) process with a disaster shock. Since  $\Pi$  is a 3 by 3 matrix and the sum of each row is one, there are 6 free unknowns in  $\Pi$ . Consequently, we need 6 moments (constraints) to pin down these unknowns.

In the calculation, we choose the first three unconditional moments including mean, variance and skewness, and three auto-correlations with lagged 1, 2 and 3 periods. Then we simulate the original exogenous process (C.1) with 2500 periods (discard the first 500 periods) and 5000 times to calculate the 6 unconditional moments for equation (C.1). Next we then use nonlinear solvers to find 6 unknowns in matrix  $\Pi$  such that the unconditional moments generated by matrix  $\Pi$  are very closed to the original process.

Some remarks are in order. First, the discrete state Markov process characterizes the original continuous state Markov process in terms of unconditional moments. There might be other moments one could choose, but we prefer these six moments in the calculation. Second, we want to associate the lowest technological state with an event of disaster, but it isn't necessarily the disaster state itself. One could use a much lower value for the lowest state, say,  $lo = mid - 3 \times std(A_{l,t})$  and obtain a different transition matrix. The choice of state values doesn't affect the business cycle property of the model. It only affects how we define a recession scenario.

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<sup>16</sup>We thank Victor Rios-Rull for his valuable suggestion and encouragement.

## C.5 Accuracy

Once obtaining policy functions for asset prices and exiting wealth, we can implement a simulation starting from the initial period (given the portfolio and state of Nature in the first period) to obtain all other variables. Along a simulated path, we have a sequence of consumption levels and a sequence of portfolio of assets for each agent in the world economy. The accuracy of the solution is based on relative consumption between actual consumption  $\tilde{c}_{l,t}$  along the simulated path and the consumption  $\hat{c}_{l,t}$  that is derived from Euler equations, given current portfolio choices and future state contingent consumption and asset prices,

$$\hat{c}_{l,t} = \left\{ \frac{\beta_l E_t \{U'(\tilde{c}_{l,t+1})(q_{i,t+1} + R_{ik,t+1})\} + \mu_{l,t} \kappa E_t \{q_{i,t+1}\} - \rho_{i,t+1}^l}{q_{i,t}} \right\}^{-\frac{1}{\sigma}} \quad \text{with } l = 1, 2, i = 1, 2, 3$$

The banker's Euler equations deliver similar current consumption  $\hat{c}_{3,t}$ . The absolute relative error is defined as

$$\epsilon_{l,i} = \left| \frac{\hat{c}_{l,t}}{\tilde{c}_{l,t}} - 1 \right| \quad \text{with } l, i = 1, 2, 3$$

When the mean and maximum of  $\epsilon_{l,i}$  along the simulated is small enough, the solution to the model is accurate. In the baseline model, we obtain an average error of  $\log(\epsilon_{l,i}) < -6.98$  and maximal error of  $\log(\epsilon_{l,i}) < -2.8$ .

## D Value functions and welfare

Once obtaining policy functions for variables of interest, we arrive at the policy function for consumption at period  $t$  as  $\tilde{c}_i(\omega_{1,t}, \omega_{2,t}, A_t, A_{t-1})$ . Then the value function for agent  $i$  (investors and workers in either country) is defined as,

$$\begin{aligned} V_i(\omega_{1,t}, \omega_{2,t}, A_t, A_{t-1}) &= \max_{\{\tilde{c}_{i,t}\}} \{U(\tilde{c}_{i,t}) + \beta_i E [V_i(\omega_{1,t+1}, \omega_{2,t+1}, A_{t+1}, A_t)]\} \\ &= U(\tilde{c}_i(\omega_{1,t}, \omega_{2,t}, A_t, A_{t-1})) + \beta_i E [V_i(\omega_{1,t+1}, \omega_{2,t+1}, A_{t+1}, A_t)]. \end{aligned} \quad (\text{D.1})$$

where the second equality uses the optimal effective consumption  $\tilde{c}_i(\omega_{1,t}, \omega_{2,t}, A_t, A_{t-1})$ . The time-invariant function  $V_i(\omega_{1,t}, \omega_{2,t}, A_t, A_{t-1})$  satisfying the equation above is the value function we look for. Note that the value function can also be expressed as a function of predetermined portfolios and exogenous shocks  $V_i(\{k_{j,t}^i, b_{i,t}\}_{j=1,2}^{i=1,2,3}, A_{t+1})$ .

### D.1 Unconditional welfare evaluation

Investors are less patient than workers, so they will front-load their consumption by borrowing more from workers. In the long run ergodic distribution, investors accumulate higher debts and consume less, while workers' consumption will have the opposite feature. Financial integration which enhances risk-sharing for investors is associated with higher indebtedness and lower consumption in

the long run. How does financial integration alter risk sharing in the long run? We can characterize the unconditional stationary value of welfare by calculating the unconditional mean of value function  $V_i$ . To economize on notation, we use  $V_i^r$  as such an unconditional mean in financial integration regime  $r = u, b, e$ , with  $u$  standing for financial autarky,  $b$  for bond market integration and  $e$  for equity market integration. The unconditional mean of value function  $V_i^r$  is calculated based on 100 simulation runs, each of which contains 210000 periods (the first 10000 periods are discarded). Along these simulation paths, portfolio choices never exceed their preset lower bounds.

Specifically, in order to make welfare gains from financial integration comparable to consumption changes, we apply the certainty equivalent of effective consumption  $\tilde{c}_i$ , which is defined as,

$$V_i^r = \sum_{t=0}^{\infty} \beta_i^t U(\tilde{c}_i^r). \quad (\text{D.2})$$

Given the preference specification in the main text, the certainty equivalent of consumption  $\tilde{c}_i^r$  reads,

$$\tilde{c}_i^r = [V_i^r(1 - \sigma)(1 - \beta_i) + 1]^{\frac{1}{1-\sigma}}. \quad (\text{D.3})$$

Unconditional welfare gains now can be written as effective consumption changes across different financial integration regimes. Let  $\lambda_{r1,r2,i}$  be the change of effective consumption from regime  $r1$  to regime  $r2$  for agent  $i$ ,

$$\lambda_{r1,r2,i} = \frac{\tilde{c}_i^{r2} - \tilde{c}_i^{r1}}{\tilde{c}_i^{r1}} \quad (\text{D.4})$$

When  $\lambda_{r1,r2,i} > 0$ , agent  $i$  is better off from regime  $r1$  to regime  $r2$  in the long run.

## D.2 Conditional welfare evaluation

In order to properly evaluate the gains from financial integration, it is necessary to calculate welfare conditional on initial conditions.<sup>17</sup> Starting from an common initial state, including initial portfolios for all agents and exogenous shocks, assume that there is an unanticipated change in regime from  $r1$  to  $r2$ . Accordingly, agents in the economy optimize their consumption and portfolio paths in each integration regime after they observe the realization of shocks. Specifically, assume that the economy starts at period 0 with end-of-period portfolio  $\{k_{i,0}^j, b_{j,0}\}_{i=1,2}^{j=1,2,3}$  and exogenous state  $A_0$ . The switch of regimes happens unexpectedly at period 1 and the economy stays in that regime from period 1 onwards. Nevertheless, the exogenous shocks in period  $t = 1, 2$  are unknown for agents in period 0. Let the welfare in period 0 be  $V_{0,j}^r$  for integration regime  $r$  and agent  $j$ ,

$$V_{j,0}^r(\{k_{i,0}^j, b_{j,0}\}_{i=1,2}^{j=1,2,3}, A_0) \equiv E_0 \left\{ U(\tilde{c}_j^*(\{k_{i,0}^j, b_{j,0}\}_{i=1,2}^{j=1,2,3}, A_1)) + \beta_j E_1[V_j^r(\{k_{i,1}^j, b_{j,1}\}_{i=1,2}^{j=1,2,3}, A_2)] \right\} \quad (\text{D.5})$$

where  $\tilde{c}_j^*(\{k_{i,0}^j, b_{j,0}\}_{i=1,2}^{j=1,2,3}, A_1)$  is agent  $j$ 's optimal effective consumption given endogenous state  $\{k_{i,0}^j, b_{j,0}\}_{i=1,2}^{j=1,2,3}$  and exogenous state  $A_1$  at period 1,  $V_j^r$  denotes the value function and  $E_t$  represents

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<sup>17</sup>This is because unconditional measures of welfare ignore the transitory gains in utility that investors get from early consumption.

conditional expectations over exogenous state  $A_{t+1}$  with  $t = 0, 1$ .

Once obtaining the effective consumption  $\tilde{c}_j^*$  and value function  $V_j^r$ , we can calculate the conditional welfare for agent  $j$ ,  $V_{j,0}^r$ , based on the transition matrix  $\Pi$ . Similarly, the certainty equivalence of effective consumption in the short run  $\tilde{c}_j^{r,s}$  is defined as,

$$\tilde{c}_j^{r,s} = [V_{j,0}^r(1 - \sigma)(1 - \beta_j) + 1]^{\frac{1}{1-\sigma}}. \quad (\text{D.6})$$