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ABSTRACT

Channeling the Say in Political Decision Bodies*

We examine optimal procedures for public project provision, financing, and redistribution in democracy. We consider a large and heterogeneous decision body and show that first-best outcomes are obtained by a procedure that involves two proposal-making rounds, the right of the minority to move first, and a ban on subsidies for the agenda-setters. We explore the robustness of the result and consider applications of our rules. For instance, the result rationalizes those rules of democracies that grant minorities in the electorate or in parliament the right to initiate collective decisions on new project proposals. We further show that the above procedure constitutes the unique minimal form of political competition that ensures first-best outcomes.

JEL Classification: D72 and H40
Keywords: constitutional design, majority rule, public project provision and subsidies

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1 Introduction

Rules and procedures may have a strong impact on collective decisions. Since the classic results provided by McKelvey (1976, 1979), this fact has been emphasized for policy spaces that include flexible redistributional politics. For such policy spaces, a Condorcet winner does not typically exist. Agenda-setters generally have a great power in such circumstances. The last proposer, in particular, may often effectively dictate the outcome, which may lead to socially undesirable results.

In this paper, we examine how the final say in a sequence of proposals within large decision bodies can be channeled towards socially desirable outcomes. More specifically, we study public project provision, financing, and flexible redistribution in a stylized environment that consists of a large electorate with risk-neutral citizens. Individual utilities of the public project are heterogeneous with some citizens benefiting from the public project – henceforth called *winners* – and the rest losing – henceforth called *losers* – relative to per-capita costs. Our analysis encompasses the two possible scenarios: either losers account for a majority of the citizenry or winners do. We further assume that there are small but positive deadweight costs of redistribution. Accordingly a socially optimal allocation prescribes public project provision if and only if aggregate benefits exceed production costs and there are no subsidies, i.e., taxes are solely raised for the financing of the public project.

When either all citizens are winners or all are losers, the problem whether to carry out the public project or not is trivial. Otherwise, simple voting will lead to inefficiencies, as the majority will impose its will on the entire citizenry, no matter whether the sum of total benefits associated with the public project is positive or negative. Since transfers are distortionary, social efficiency cannot be achieved by using transfers.

To determine whether in our setting, there exist procedures that yield socially optimal outcomes, we consider agenda-setting sequences in which, first, a member of one group – be it the majority or the minority – makes a proposal and, second, a member of the other group makes an amendment. A proposal sets out whether or not to provide the public project, the tax rate, and an arbitrary distribution of (nonnegative) subsidies among the electorate. Except in Section 5.4, we assume that expenditures are financed through uniform taxation and thus that the tax rate determines aggregate tax revenues. Once the two proposals have been made, all individuals cast a vote to choose one of the proposals. The decision is taken according to the simple majority rule.

Our major insight is that the right of the minority to make the first proposal, coupled with a ban on agenda-setters to channel subsidies to themselves – but not to other citizens – yields first-best outcomes and breaks the dictatorial power of the last agenda-setter. A significant part of the paper will be devoted to the proof of this result and to its importance. We provide an extended intuition for the result in Section 4. Here, we give a shorter intuition for the case when project losers constitute the majority. In that case, according to the above rules, a project winner – being a member of the minority – makes the first proposal.

Suppose that the first agenda-setter, a winner, makes a proposal that includes the public project and subsidization to a fraction of citizens that renders such a proposal attractive for a particular majority of citizens. The second agenda-setter, a loser, would like to avoid both adoption of the public project and subsidization, since he is a project loser and his tax burden would increase under subsidization due to uniform taxation. On the one hand, if the project is socially inefficient – and thus (aggregate) project losses are relatively large compared to (aggregate) project benefits –, the second agenda-setter's best counter-proposal focuses on avoiding his loss associated with the project. Yet, attracting a majority requires unavoidable subsidies. The first agenda-setter, anticipating that he will end up with a higher tax burden because of subsidies but without the provision of public project, will propose the status quo, i.e., no public project and no subsidies – the socially efficient solution. On the other hand, if the project is socially efficient – and thus (aggregate) project benefits are relatively large compared to (aggregate) project losses –, it is best for the second agenda-setter to focus on the avoidance of subsidies, as forming a majority against the first proposal would require very large subsidies and would impose a tax burden that is larger than the second agenda-setter's loss associated with the project. Hence, the second agenda-setter's best counter-proposal is to match the project proposal and to eliminate all subsidies. Anticipating this counter-proposal, the first agenda-setter, a winner, sticks to his original idea. Hence, the project is adopted without subsidies – again, the socially efficient solution.

That is, at the final stage of the procedure and regardless of the identity of the majority, we obtain that in equilibrium

- socially efficient public projects are proposed and accepted,

- socially inefficient public projects are not proposed,
- no subsidies are proposed.

We stress that to achieve the first-best outcome, socially costly subsidization of citizens to form majorities has to be allowed.[1] The ban on subsidies to agenda-setters – which we call *Agenda-Setter Rule* – together with the right of the minority to take the initiative, ensures that the first proposal typically contains subsidization, but subsidies do not occur in the winning proposals. Without the Agenda-Setter Rule, the agenda-setter making the second proposal benefits from a second-mover advantage and always channels the say towards his preferred decision. When losers constitute a majority, for instance, the second agenda-setter, a loser, never proposes the public project.

In the second part of the paper, we show that the above two-round procedure – proposal by a member of the minority, counter-proposal by a member of the majority, no subsidies to agenda-setters but subsidies to other citizens, and uniform taxation – is the unique minimal form of political competition that yields first-best outcomes for any parameter constellation in any equilibrium of the game. Hence, each feature of the two-round procedure is necessary and together, they are sufficient for optimality. This result is developed in four steps.

First, if no amendment is possible and only one agenda-setter – be it a member of the majority or the minority – makes a proposal which is pitted against the status quo –, inefficiencies will occur for some constellation of parameters, even if subsidies for the agenda setter are not allowed or, more generally, even if the set of possible subsidy schemes is limited. Thus, two-round proposal-making constitutes a *minimal* form of political competition that always yields first-best outcomes.

Second, allowing a member of the majority to move first and a member of the minority to make amendments does not always produce first-best outcomes. For instance, under the reversed-order two-shot procedure when the majority consists of losers, the second agenda-setter, a winner, can often impose his preferred policy, regardless of whether the project is socially desirable or not. Thus, the two-round proposal-making in which a member of the minority proposes first constitutes the *unique* form of political competition that yields first-best outcomes among the set of two-round procedures of the same

[1]This is in contrast with Gersbach (2009), where it is necessary that subsidies are forbidden to guarantee the efficiency of sophisticated majority rules.

type. This result rationalizes those rules of direct democracies that grant parliamentary minorities the right to initiate collective decisions about new project proposals.[2] Similar rules can be found in the parliamentary regulations of representative democracies.[3]

Third, we show that in the absence of agenda-setter rules and leaving all other features unchanged, the procedure will fail to invariably yield the socially efficient outcome. Without the Agenda-Setter Rule, in particular, the second agenda-setter will make a proposal with his preferred project choice and appropriate subsidies that ensure winning over the first proposal.

Fourth, giving full flexibility to the tax schedule destroys the optimality of the mechanism.[4]

Finally, in the third part of the paper, we explore the robustness of our results in more general settings, departing from the stylized environment in which they are derived. We find that the suggested procedure remains appealing in richer environments. In particular, we show that our two-round procedure still yields first-best outcomes or is at least welfare-improving when we depart from binary characteristics of public goods, allow for heterogeneity among winners and losers or consider a large but finite electorate.

The paper is organized as follows. In Section 2 we situate the paper in the context of the literature on the subject and analyze the applicability of our procedure in real environments. Section 3 is devoted to presenting the formal model and the main assumptions, and to describing the two-round procedure. Section 4 contains the main finding of the paper, which is that the two-round procedure yields social efficiency, i.e. when subsidies to agenda-setters are banned, the tax schedule is uniform, and the right to propose is given to the minority, the final say yields the social optimum. In Section 5 we investigate the uniqueness and minimality of the two-round procedure as a form of political competition. Section 6 is devoted to a discussion of a number of extensions of the stylized environment in which our results are proved. Section 7 concludes. Proofs of all results can be found in Appendices A, B, and C.

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[2] Famous examples are California or Switzerland.

[3] See e.g. in Germany `http://www.bundestag.de/service/glossar/I/initiativrecht.html`. This page was retrieved on 18-Oct-2013.

[4] This issue is further discussed in Section 5.

2 Relation to the Literature and Applications

2.1 Links to the literature

Our model is related to several strands of the literature. First, it is in the spirit of the incomplete social contract introduced by Aghion and Bolton (2003) and further developed by Gersbach (2009). Procedures based on this approach have to satisfy basic democratic requirements. One strand of this literature (Gersbach, 2009) explores whether sophisticated proposal-dependent decision rules, together with a set of constitutional rules, can achieve first-best outcomes. In the present paper, we explore whether there is a set of rules which, together with the simple majority rule, can achieve first-best allocations. The simple majority rule being by far the most common and well-justified collective decision rule (see May, 1952; and Aghion and Bolton, 2003), we show that there is indeed a set of rules that can achieve the social optimum together with the simple majority rule.

Second, our model can be interpreted as one of electoral competition with policy-motivated candidates: two candidates or parties propose binding platforms, and citizens vote for one of them.[5] The question whether political competition will lower or eliminate rents of competing political parties has been a prominent issue. The literature has emphasized that competition lowers rents, but that in general some rents will be appropriated by political actors (see Myerson, 1993; Polo, 1998; and Gersbach and Schneider, 2012). In the context of public project provision, we show that a subsidy rule for the agenda-setters, coupled with the right of the minority to take the initiative in proposal-making and uniform taxation, eliminates all inefficient rents when candidates/parties offer competing platforms. Our results can be viewed as a constitutional implementation of the classic perspective put forward in seminal papers (Stigler, 1972; Wittman, 1989, 1995; and Becker, 1958) that competition among political candidates/parties enhances or even maximizes citizens' welfare, as socially desirable policies are undertaken and inefficient political rents are reduced or eliminated. We stress that the interpretation of our model as one of electoral competition requires that there is one dominating issue in the election, which mirrors our project decision. Examples could be a fundamental health care reform (e.g. "Obamacare"), exiting nuclear power, joining the EU or the Eurozone, or limiting immigration.

[5] With the possibility of flexible and costly redistribution, there typically exist only equilibria in which politicians move sequentially as we assume in this paper.

Third, we consider legislative policy-making in which public project provision and redistribution interact. As the financing of public project provision and redistribution occurs solely through a uniform tax rate, our setup features a public policy with concentrated benefits and costs spread out over the whole population. Such fiscal common problems and the associated excessive spending have been the focus of a large literature developed by Buchanan and Tullock (1962) and first formalized by Weingast et al. (1981). A variety of institutional provisions have been suggested that might help to remedy the fiscal commons problem – see Feld and Schaltegger (2009) for a recent overview. We complement this literature by offering a set of rules – two-round proposal-making coupled with uniform taxation, a ban on subsidies to agenda-setters and the right of the minorities to propose – that simultaneously induces optimal public project provision and avoids excessive taxation.

Fourth, we assume that agenda-setters can, however, channel subsidies flexibly to different constituencies. The combination of tax rules and flexible subsidization in actual legislative policy-making is discussed and justified in Gersbach, Hahn, and Imhof (2013). While we take these characteristics of legislative policy-making as given for most of our analysis, in Section 5 we provide a further justification why it is socially desirable that flexibility in taxation and subsidization differs. Specifically, we show that first-best solutions are not attainable if agenda-setters can be fully flexible with respect to taxation and subsidization.

Fifth, our model and results may have some bearing on the well-developed and rich literature on legislative bargaining pioneered by Baron and Ferejohn (1989), which we will not summarize here.[6] Our procedure can be interpreted as a simple, yet modified version of a dynamic legislative bargaining game within large committees where agents take a decisive vote between a proposal and an amendment. We show that such a procedure, coupled with the Agenda-Setter Rule, uniform taxation and the initiative right in proposal-making for minorities, yields first-best allocations.

[6]See Bernheim, Rangel, and Rayo (2006) for a summary of the literature and a comprehensive analysis of the power of the last agenda-setter in real-time agenda-setting with evolving default points.

2.2 Broader applications

A first fundamental feature of the two-round procedure proposed in the paper is that members of the minority have the right to take the initiative and to make a proposal before members of the majority. Rules that give priority to minorities in proposal-making exist in direct democracies and parliamentary democracies, as we saw in the Introduction. We note that the application of a minority-priority rule in real-world settings requires the status of "minority" and "majority" to be verifiable in court. Let us now examine a couple of examples where such a verification is feasible and thus may help to broaden the application of the two-round procedure. In a parliamentary system, on the one hand, where coalitions of parties form a majority government, the remaining parties constitute the minority and are recognized as such. On the other hand, our model can also be interpreted as a collective decision of two separate jurisdictions (see e.g. Alesina et al., 2004). For example, consider a country with two local jurisdictions of different size that decide on a project that will mainly benefit one of the communities only, but is financed by the federal budget. Such projects could be local infrastructure projects or the organization of an international event in the capital city of one of the two regions. Our main result reveals that the bargaining problem between the two communities can always be resolved in a socially optimal way via a two-round procedure, as long as the right to propose is granted to the small jurisdiction while the right to amend is given to the large jurisdiction. We note that no matter which community benefits from the project, the two-round procedure maximizes utilitarian welfare.[7]

There may be other circumstances in which beneficiaries and losers from public projects can easily be ascertained, and majorities and minorities can be identified. If a project benefits either only older or only younger people, for instance, winners and losers can be identified by their age.[8] Similarly, policies that are targeted at specific groups (such as families with two or more children) or sectors (such as the agricultural sector) will typically permit a verifiable winner/loser division of the population.[9]

A second critical feature of our procedure is that agenda-setters cannot propose subsidies for themselves. We have two comments. First, what is needed for our procedure to

[7] The results in Section 6.1 reveal how our results extend to situations with several jurisdictions.

[8] This holds as long as altruistic motives are not so strong that they overcompensate the direct impact of a project on the utility of individuals.

[9] We note that the two-round procedure cannot rely on self-selection of winners and losers, as individuals might have an incentive to manipulate the procedure by misrepresenting their preferences.

yield first-best outcomes is that subsidies to agenda-setters cannot be determined by the agenda-setters themselves. Accordingly, the Agenda-Setter Rule could prescribe that the two agenda-setters receive a very large subsidy, hence reducing the need of agenda-setters to bargain off-procedure in exchange for side-payments. Second, a very harsh penalty could be determined for agenda-setters found guilty of infringing the ban on subsidies. This again would reduce the incentives for agenda-setters to bypass the ban.

In practice, there are different ways how the Agenda-Setter Rule could be applied or could manifest itself. In the US Congress, for instance, when a group of senators introduces a bill, the agenda-setters could be banned from adding earmarks benefiting their constituency to this bill. Earmarks are a common practice in US legislative decision-making (see e.g. Lazarus and Steigerwalt (2009) or Doyle (2011)). Since 2006, there have been congressional efforts aiming at reforming earmarks, including some attempts to ban earmarks altogether.[10] Our paper suggests that it might suffice to ban earmarks for the group of Congress members who introduce or support a bill.

In direct-democracy settings, the Agenda-Setter Rule would imply that an interest group could not take the initiative for a public vote on a project prescribing subsidies to the group of citizens represented by this same interest group: such initiatives would be declared unconstitutional.

3 Model

3.1 Set-up

We consider a society facing the standard problem of local public project provision and financing. Citizens are indexed by either i or j and are uniformly arranged on the unit interval. The provision of a public project yields benefits v_j for all $j \in [0,1]$ and involves per-capita costs $k \geq 0$. In the main body of the paper, we assume $v_j \in \{V_w, V_l\}$ with $V_w > k > V_l$, and we allow V_l to be higher or lower than zero. Accordingly, we refer to individuals obtaining V_w from the public project as *winners* and to individuals receiving V_l as *losers*. Without loss of generality, we assume winners to be located on the interval $[0,p]$ and losers on the interval $(p,1]$, with $p \in (0,1)$. If $p < \frac{1}{2}$, i.e. if the project

[10]See http://www.nytimes.com/2010/11/13/us/politics/13earmark.html?_r=0, retrieved on 6-11-2013.

beneficiaries account for less than half of the population, the set of winners constitutes the *minority*, while the set of losers constitutes the *majority*. Alternatively, if $p \geq \frac{1}{2}$, i.e. if the project beneficiaries account for at least half of the population, the set of winners constitutes the *majority*, while the set of losers constitutes the *minority*. The set of all possible public projects comprises all quadruples (V_w, V_l, k, p) that satisfy the assumptions introduced so far, i.e.[11]

$$\mathcal{P} := \{(V_w, V_l, k, p) \in \mathbb{R}_{++} \times \mathbb{R} \times \mathbb{R}_{+} \times (0,1) \mid V_w > k > V_l\}.$$

For subsequent analysis, we consider an arbitrary public project from the set $\mathcal{P}$. The project parameters and the identities of winners and losers are common knowledge.

A *proposal*, denoted by π, comprises a subsidy scheme and a decision on the public project. Subsidies are constrained to be non-negative and bounded from above by some value $\hat{s} \in \mathbb{R}_{++}$. We assume that $\hat{s}$ is very high, so agents can be subsidized by a large amount.[12] Let $\mathbb{S}$ be the set of all non-negative, Lebesgue-measurable and Lebesgue-integrable functions on the unit interval. Each subsidy scheme involved in a proposal π is a function $s(\pi) \in \mathbb{S}$. We use $s_j(\pi)$ to denote the subsidy given to individual $j \in [0,1]$. The variable $g(\pi) \in \{0,1\}$ indicates whether the public project is suggested ($g(\pi) = 1$) or not ($g(\pi) = 0$) under proposal π. In the main body of the paper, we impose that accruing costs are financed through a uniform tax, i.e. that all individuals pay the same tax amount. In Section 5, however, we explore the significance of the presence of tax rules by allowing arbitrary taxation of individuals. Under uniform taxation, the society's budget constraint amounts to:

$$g(\pi)k + (1+\varepsilon)\int_0^1 s_j(\pi)dj = t(\pi). \tag{1}$$

In equation (1), $t(\pi)$ represents the per-capita tax, which is uniquely determined by $g(\pi)$ and the subsidy scheme $s(\pi)$. We further assume that there are deadweight costs of taxation. Deadweight costs are extensively discussed in the literature (see Aghion and Bolton 2003 or Gersbach 2009). Such costs can be interpreted in a narrow sense, i.e. as resources used for collecting and transferring funds. In a broader sense, they may represent distortions, say when individuals reduce labor supply because income is

[11] We denote $\mathbb{R}_+ := \{x \in \mathbb{R} : x \geq 0\}$ and $\mathbb{R}_{++} := \{x \in \mathbb{R} : x > 0\}$.

[12] In the main body of the paper, it suffices to assume that $\hat{s} > \max\{V_w - k, k - V_l\}$. Limiting the maximal subsidy ensures that optimal proposals always exist. If subsidies were unbounded, a small subset of agents could fully exploit the entire electorate. Studying this possibility lies beyond the scope of the present paper.

taxed. We assume that deadweight costs associated with public project provision are incorporated in k, while deadweight costs of redistribution are captured by ε in (1). We also assume that ε is small but positive.

To sum up, the set of feasible proposals is

$$\Pi := \left\{ \pi = (g(\pi), s(\pi)) \in \{0,1\} \times \mathbb{S} \;\middle|\; \begin{array}{r} g(\pi)k + (1+\varepsilon)\displaystyle\int_0^1 s_j(\pi)dj = t(\pi) \\ \text{and } s_j(\pi) \leq \hat{s} \text{ for all } j \in [0,1] \end{array} \right\}.$$

Adoption of a proposal $\pi \in \Pi$ yields to a citizen $j \in [0,1]$ a net utility change $u_j(\pi)$ given by

$$u_j(\pi) = g(\pi)v_j + s_j(\pi) - t(\pi) \text{ for } j \in [0,1].$$[13]

Accordingly, the utilitarian welfare measure associated with $\pi \in \Pi$ amounts to

$$W(\pi) = g(\pi)\left[pV_w + (1-p)V_l - k\right] - \varepsilon \int_0^1 s_j(\pi)dj.$$

For the remainder of the paper, it is sufficient to make ε arbitrarily small, but to keep in mind that paying out subsidies lowers aggregate welfare. This will greatly simplify the presentation.

3.2 Socially efficient solution

We next characterize the socially efficient solution chosen by a social planner who maximizes the utilitarian welfare for any given realization of parameters $(V_w, V_l, k, p) \in \mathcal{P}$. Such a social planner will implement the public project if and only if it is *socially efficient*, i.e., if and only if $pV_w + (1-p)V_l \geq k$. Moreover, he will never choose a positive level of total subsidies as it would reduce welfare. These observations are summarized in the following definition.

Definition 1

A socially optimal proposal $\pi^{soc} \in \Pi$ has the following characteristics:[14]

$$g(\pi^{soc}) = \begin{cases} 1 & \text{if } pV_w + (1-p)V_l \geq k, \\ 0 & \text{if } pV_w + (1-p)V_l < k, \end{cases} \quad \text{and} \quad s_j(\pi^{soc}) = 0 \text{ q.e. } j \in [0,1].$$

[13] We assume that individuals' income is sufficient to pay taxes under any proposal considered.

[14] Our analysis usually involves $s_j(\pi^{soc}) = 0$ for all $j \in [0,1]$. However, it is sufficient that $s_j(\pi^{soc}) \neq 0$ only for a set of individuals with zero measure. We note that *q.e.* stands for quasi-everywhere.

As we mentioned in Section 2, in this paper we adopt the incomplete social contract perspective. In particular, although project parameters and valuations are observable, they are not verifiable in court. If parameters were verifiable, complete social contracts guaranteeing socially optimal solutions could be drafted. As a consequence, constitutions can only contain rules on how to decide on public projects. As we are considering the democratic provision of public projects (Gersbach 2009), we are looking at collective choice processes in which agenda setters make proposals on which citizens vote according to majority rule.

3.3 A procedure with two proposal-making rounds

In our setting, a *procedure* is a set of rules whose application yields a proposal that will be implemented. The ensuing – and relevant – political problem is how to design a procedure that always induces implementation of a welfare-maximizing proposal.

In the present paper, we consider procedures with either one or two proposal-making rounds. Henceforth, whenever we deal with procedures with two proposal-making rounds, we often refer to the proposal raised in the second round as the *counter-proposal* to the proposal raised in the first round. More precisely, the main procedure that we suggest in the paper specifies the following sequence of events:

(1) A member of the minority makes a first proposal $\pi' \in \Pi$.

(2) A member of the majority makes a second proposal (or counter-proposal) $\pi'' \in \Pi$.

(3) All individuals cast their vote on either π' or π'', and no abstention occurs.[15] The proposal that receives more votes is adopted.[16] Ties between proposals are broken by design of the procedure in favor of the counter-proposal.

Without loss of generality, we assume that the agenda-setter who is a winner is $j = 0$, and correspondingly use $\pi^0 \in \{\pi', \pi''\}$ to denote his proposal. Similarly, we let the

[15]A weaker assumption for which all the results in this paper hold is that abstention exogenously affects losers and winners in the same proportion. More generally, it would suffice to require the share of "active" losers to be larger than the share of "active" winners if and only if the share of all losers is larger than the share of all winners, "active" here referring to individuals who vote according to an exogenous decision.

[16]Such a setting corresponds to the legal system of direct democracy or parliamentary democracy with perfect representation.

agenda-setter who is a loser be $j = 1$, and correspondingly use $\pi^1 \in \{\pi', \pi''\}$ to denote his proposal. For notational clarity we will also refer to $j = 0$ as AS_w (agenda-setter who is a winner) and to $j = 1$ as AS_l (agenda-setter who is a loser). Throughout the paper, we will use the indicator function $I(\pi', \pi'')$, which adopts a value of 0 if the first proposal, π', is implemented and a value of 1 if the counter-proposal, π'', is implemented.

Additionally, we impose that the above procedure is supplemented with a very simple subsidization rule, which we call *Agenda-Setter Rule*, that will turn out to be sufficient to achieve socially efficient solutions.

Agenda-Setter Rule

A feasible proposal $\pi \in \{\pi^0, \pi^1\}$ has to comprise $s_0(\pi) = s_1(\pi) = 0$.

This rule bans subsidies for agenda-setters. We note, however, that it will suffice for our procedure to yield first-best outcomes that subsidies to agenda-setters cannot be determined by the agenda-setters themselves, while it will not actually be necessary that subsidies to agenda-setters be zero.[17] Finally, we note that every procedure that we study in the paper will consist of a proposal-making stage (with a number of rounds) and a voting stage. Throughout the paper, we will only explicitly indicate the number of rounds in the proposal-making stage. Accordingly, we will refer to the above procedure as a *two-round procedure*.

3.4 Equilibrium concept and tie-breaking rules

To analyze any two-round procedure – and in particular the one we just introduced in the previous section –, we apply the standard concept of subgame-perfect Nash equilibrium. Hence, an *equilibrium* consists of a proposal made by the first agenda-setter – be it a member of the majority or the minority –, a counter-proposal made by the second agenda-setter – be it a member of the majority or the minority –, and a voting decision for each possible pair of proposal/counter-proposal taken by all voters. To facilitate the analysis, given a proposal $\pi' \in \Pi$ by the first agenda-setter and a proposal $\pi'' \in \Pi$ by the second agenda-setter, we impose the following tie-breaking rules:

[17] Moreover, for our results to hold, it is sufficient that the Agenda-Setter Rule bans the possibility that any agenda-setter can discretionary choose the subsidy given to any preceding agenda-setter in the sequence of proposal-making.

Tie-Breaking Rule 1

If $u_j(\pi') = u_j(\pi'')$, individual j will vote for the counter-proposal π''.

Tie-Breaking Rule 2

If the second agenda-setter, j, is indifferent between a proposal $\tilde{\pi} \in \Pi$ involving $g(\tilde{\pi}) = 1$ and a proposal $\hat{\pi} \in \Pi$ involving $g(\hat{\pi}) = 0$, i.e., $u_j(\tilde{\pi}) = u_j(\hat{\pi})$, he will always suggest $\tilde{\pi}$ over $\hat{\pi}$.

Tie-Breaking Rule 3

If, given the first proposal π', the second agenda-setter is indifferent between the voting outcome when making a proposal $\tilde{\pi} \in \Pi$ with $I(\pi', \tilde{\pi}) = 1$ and the voting outcome when making a proposal $\hat{\pi} \in \Pi$ with $I(\pi', \hat{\pi}) = 0$, he will always suggest $\tilde{\pi}$ over $\hat{\pi}$.

Several remarks are in order. First, Tie-Breaking Rule 1 enables the second agenda-setter to ensure that the counter-proposal is adopted if at least half of the electorate is not worse off than they would be with the first proposal. Although it simplifies the presentation, it does not condition our results. Without Tie-Breaking Rule 1, we would need to discretize the spaces of subsidies, forcing the agenda-setter to pay the smallest possible amount of subsidies to receive the support of voters who would be indifferent otherwise. Then, we would have to describe the subgame perfect equilibrium of this game. If the smallest, yet positive amount of subsidies becomes arbitrarily small, we would approximate the equilibria considered in this paper.[18] Due to Tie-Breaking Rule 1, we can thus write

$$I(\pi', \pi'') = \begin{cases} 1 & \text{if } u_j(\pi'') \geq u_j(\pi') \text{ holds for at least half of the society,} \\ 0 & \text{otherwise,} \end{cases}$$

where π' and π'' denote the proposal by the first agenda-setter and the counter-proposal by the second agenda-setter, respectively. Second, Tie-Breaking Rule 2 ensures that the second agenda-setter will choose to implement the public good in case of indifference. We could dispense with Tie-Breaking Rule 2 if the second agenda-setter obtained an arbitrarily small utility gain from being active (changing the status quo), compared to being passive (keeping the status quo). We note that Tie-Breaking Rule 2 is actually only used in the proof of our main result – Theorem 1 in Section 4 – in the case of a particular combination of project parameters.[19] Hence, except for this very particular parameter

[18]The same considerations apply when there is a tie in the number of votes between the proposal and the counter-proposal, and the latter is assumed to be implemented.

[19]More specifically, Tie-Breaking 2 is needed only in the case where $p = \frac{k - V_l}{V_w - V_l}$.

constellation, we can interpret Tie-Breaking Rule 2 as a feature of the equilibria of the game rather than as a tie-breaking rule per se. Third and last, Tie-Breaking Rule 3 simply ensures that the second agenda-setter does not make proposals that are bound to be rejected (see Lemma 1 in Appendix A.). Since the second agenda-setter can always repeat the first proposal, Tie-Breaking Rule 3 does not constrain the set of equilibria. Note that in fact, Tie-Breaking Rule 3 follows immediately if the second agenda-setter obtains a very small disutility whenever his proposal is defeated.

3.5 Types of inefficiency

Given an arbitrary procedure, there are three types of inefficiency that may arise in the process of choosing a proposal:

1. Socially inefficient public projects may be implemented.
2. Socially efficient public projects may not be implemented.
3. Socially harmful subsidies may arise.

As we show in this paper, avoiding all three types of inefficiency can be achieved by the procedure introduced in this section. Moreover, as soon as the design features of this procedure are changed, inefficiencies will emerge (see Section 5).

4 Channeling the Say

The main result of this paper – Theorem 1 below – shows that, together with the Agenda-Setter Rule, uniform taxation and political competition ensure that the social optimum is always implemented, as long as a member of the minority proposes first and a member of the majority proposes second.

Theorem 1
The equilibrium outcome under the two-round procedure – proposal by a member of the minority, counter-proposal by a member of the majority and voting, with the Agenda-Setter Rule and uniform taxation – is π^{soc}.[20]

[20] We note that proposals in equilibrium may differ with regard to a set of agents with measure zero.

Accordingly, the equilibrium outcome equals the socially efficient outcome, so the above simple, sequential mechanism inhibits all three types of inefficiency. The proof of the theorem is given in Appendix A. Here we provide an intuitive explanation in several stages when $p < \frac{1}{2}$.[21] Recall that in this case, the two-round procedure prescribes that a winner proposes first and then a loser counter-proposes. Now, consider a specific proposal defined as follows: The public project is suggested and all winners, as well as a certain fraction of losers, obtain the same utility a winner would obtain if the project were provided without subsidies.

Next, suppose that this proposal is made by the first agenda-setter. Given that all citizens have to pay the same tax amount, a loser as second agenda-setter can choose between two paths. On the one hand, to avoid implementation of the public project, he has to offer subsidies that are high enough to some of the losers subsidized under the first proposal, to motivate them to vote for the counter-proposal. By doing so, the second agenda-setter forms a majority consisting of subsidized and non-subsidized losers.

On the other hand, the second agenda-setter can simply reduce his own tax burden by matching the first public project proposal and by eliminating all subsidies. Such a counter-proposal will be preferred to the first proposal by all winners and all non-subsidized losers of the first proposal.

With the Agenda-Setter Rule, the first agenda-setter – a winner – prefers that the second agenda-setter – a loser – chooses the second path, as it yields higher utility. The first agenda-setter faces the following dilemma: Making the coalition of subsidized losers in his proposal large may motivate the second agenda-setter to choose the second path and propose the public project that is coupled with zero subsidies. However, making the set of subsidized losers too large will trigger the second path if winners, together with all non-subsidized losers, account for less than half of the population.

It turns out that, if and only if the public project is socially efficient, the first agenda-setter can actually subsidize a set of losers large enough, such that for the second agenda-setter, the second path turns out to be more attractive than the first one. When the public project is socially inefficient, any counter-proposal that suggests the implementation of the public project is worse for the second agenda-setter than the following counter-proposal: "no public project", the same subsidies as in the counter-proposal

[21] A similar intuition can be given for the case $p \geq \frac{1}{2}$.

just considered, and additional subsidies to all winners that were subsidized in the counter-proposal just considered. The extra subsidies equal the benefit that winners derive from the public project. If and only if the public project is socially inefficient, the second agenda-setter will always prefer to make a counter-proposal which excludes the implementation of the public project.

5 Alternative Procedures

We have shown that socially desirable outcomes can always be achieved through the two-shot procedure analyzed so far in this paper. Recall that, together with the use of the majority rule at the voting stage, the defining features of the procedure are:

- Subsidies to agenda-setters are restricted by rules.
- There are two rounds of proposal-making.
- Subsidies to non-agenda-setter voters are only restricted to be non-negative and are bounded from above.
- Taxes are imposed uniformly across voters.
- A member of the minority proposes first, and a member of the majority makes an amendment.

In this section, we examine whether these features are critical for the optimality of the procedure. For this purpose, we analyze whether the social optimum can be achieved if one or several of the features are changed. We discuss what happens if characteristics of the procedure are changed. We will show that the two-round procedure in Theorem 1 is a unique, minimal procedure. The proofs of all the results presented in this section are given in Appendix B. Note that some of the negative results in this section are derived only in the case where $p < \frac{1}{2}$. Similar negative results could be derived for $p \geq \frac{1}{2}$.

5.1 Minimal form of political competition

We consider first a procedure that does not permit political competition.[22] More precisely, only one agent can make a proposal. When this agent is a member of the majority,

[22] See Gersbach, Hahn, and Imhof (2013) for such procedures.

he uses the fact that his own constituency is a majority of the citizenry. Hence, he is able to impose his preferred policy regarding the provision of the public project. Accordingly, we now only analyze the case when a member of the minority makes a unique proposal, giving the majority no power of amendment. More precisely, we deal with a one-shot procedure with uniform taxation and arbitrary subsidy schemes plus the Agenda-Setter Rule. As in the case of two-round procedures, we apply the standard concept of subgame-perfect Nash equilibrium, so that an *equilibrium* consists of a proposal made by the only agenda-setter and of a voting decision for each proposal taken by all voters against the status quo. Ties in the voting stage are broken in favor of the proposal. We obtain the following result.

Proposition 1
If $p < \frac{1}{2}$, under the one-round procedure – proposal by a winner and voting, with the Agenda-Setter Rule and uniform taxation – inefficiencies of types 1 and 3 arise in equilibrium for some parameter constellations.[23]

In the next step we prove that limiting flexibility in the subsidy scheme does not prevent social inefficiencies. Let $\mathcal{A} \subseteq \mathbb{S}$ be any non-empty set of Lebesgue-measurable and Lebesgue-integrable functions on the unit interval bounded from above by $\hat{s}$. We assume that subsidy schemes must be chosen from $\mathcal{A}$.

Proposition 2
If $p < \frac{1}{2}$, under the modified one-round procedure – proposal by a winner and voting, with the Agenda-Setter Rule, uniform taxation and limited subsidy schemes – inefficiencies of types 1 and 3 or inefficiencies of type 2 arise in equilibrium for some parameter constellations.

Results that are similar to the above two propositions can be obtained in the case where $p \geq \frac{1}{2}$. To sum up, we have seen that with one-round procedures social inefficiencies may arise even if the Agenda-Setter Rule is imposed. The results in Propositions 1 and 2 imply that the two-shot procedure analyzed in Theorem 1 is *minimal*, where minimality refers to the number of rounds needed to ensure that the socially optimal outcome is always achieved. Minimality of the procedure is a desirable property as long as there

[23] We note that inefficiencies of type 2 do not occur in the one-round procedure.

are (monetary or opportunity) costs associated with the preparation of proposals and voting procedures.

5.2 Reverse order of proposal-making

One might be tempted to think that any kind of political competition in the proposal-making stage will preclude all types of inefficiency. In our setting, there are four different possibilities of implementing a two-shot procedure depending on who proposes first (either a member of the majority or a member of the minority) and who makes a counter-proposal (either a member of the majority or a member of the minority). For obvious reasons, the only two significant sequences are minority/majority and majority/minority.[24] Whereas the former option is efficient under the Agenda-Setter Rule and uniform taxation, the second option – which we call *reversed two-shot procedure* – fails to accomplish the same objectives. Accordingly, it is socially unprofitable to give political initiative to the majority instead of the minority. This follows from the two propositions below. As in the two-round procedure where a member of the minority proposes first, ties in the voting stage are broken in favor of the counter-proposal.

Proposition 3
If $p < \frac{1}{2}$, under the reversed two-shot procedure – proposal by a loser, counter-proposal by a winner and voting, with the Agenda-Setter Rule and uniform taxation – inefficiencies of types 1 and 3 arise in equilibrium for some parameter constellations.

The intuition for this result is as follows. For some combinations of V_w and V_l and when per capita costs of implementing a project are not very large, it is a best response for the second agenda-setter, AS_w, to subsidize some losers so as to gain their vote in favor of the implementation of the project. By doing so, he leaves the first agenda-setter, AS_l, who anticipates AS_w's behavior, with no option but to propose project implementation in exchange for minimizing the amount of subsidies, which have to be paid according to a uniform tax.

[24]The sequences majority/majority and minority/minority may obviously yield socially inefficient outcomes. Moreover, they disallow one group from making a proposal and thus violate the principle of equal rights for proposal-making.

Proposition 4

If $p \geq \frac{1}{2}$, under the two-round procedure – proposal by a winner, counter-proposal by a loser and voting, with Agenda-Setter Rule and uniform taxation – inefficiencies of type 2 arise in equilibrium for some parameter constellations.

The intuition for this second result is as follows. Suppose that aggregate benefits of providing the public project outweigh the aggregate costs, but the per-capita loss of losers exceeds the per-capita benefit of winners. Then, the second agenda-setter, a loser, is better off bearing the tax burden of subsidizing a sufficiently large subset of winners, so that these winners renounce the provision of the public project, than by incurring the losses from project adoption.

Theorem 1 and Propositions 3 and 4 indicate that in order to attain the social optimum, it is crucial that the initiative to propose be given to a member of the minority group (i.e., to a winner when $p < \frac{1}{2}$ and to a loser when $p \geq \frac{1}{2}$). Our emphasis on minorities having the right to take the initiative is a well-known feature in legislative decision-making, as noted in the Introduction. The literature has discussed several other ways how initial agenda-setters might be chosen, such as auctioning the agenda-setting right or initiative group formation (see Gersbach, Hahn and Imhof, 2013, for a recent discussion). How such rules can be integrated in our framework is left to further research.

As a consequence of the results of this section, we can say that the procedure analyzed in Theorem 1 is *unique* among the set of minimal procedures that always yield the social optimum.

5.3 No Agenda-Setter Rule and the (undesirable) say

Next we establish the need for rules that restrict subsidies to agenda-setters in order to attain efficiency. More precisely, in Proposition 5 below we show that without further rules on agenda-setters' subsidization, the second agenda-setter can credibly condition subsidies to the first agenda-setter on a specific first proposal and in this way impact the choice of the latter in his favor.

Proposition 5

Under the simple two-round procedure without any agenda-setter rule – proposal by a member of the minority, counter-proposal by a member of the majority and uniform *taxation – there are equilibrium outcomes that yield*

(a) *inefficiencies of type 2 if $p < \frac{1}{2}$ and the project is socially efficient,*

(b) *inefficiencies of type 1 if $p \geq \frac{1}{2}$ and the project is not socially efficient.*

Proposition 5 shows that the absence of any rule restricting the subsidies to agenda-setters allows the second agenda-setter, a member of the majority, to exploit his power in the following way: If and only if the first agenda-setter makes a particular proposal will he be granted the maximum amount of subsidies. Otherwise, he will be given a zero subsidy. As the cap on individual subsidies is large and there are (infinitely) many agents with the same traits as the first agenda-setter and who can be subsidized instead of the latter, this threat channels the say towards the last agenda-setter's interest. Importantly, the negative result in the above proposition does not hinge on the assumption that there is a continuum of voters – see Section 6 for the case where the electorate is finite –, but only hinges on whether the first agenda-setter will be given a positive subsidy by the second agenda-setter or not.

More specifically, part (a) in the above proposition reveals that when $p < \frac{1}{2}$, the second agenda-setter, AS_l, has an incentive to offer the first agenda-setter, AS_w, a subsidy $\hat{s}$ if the latter does not propose implementation of the public project and does propose zero taxes. As a consequence, preserving the status quo and subsidization solely of agenda-setters is an equilibrium. The interpretation of part (b) is similar to that of part (a): for any parameter constellation, there is always an equilibrium in which the public project is not implemented.

Finally, it is worth mentioning that although, without the Agenda-Setter Rule, other outcomes might arise in equilibrium, such equilibria could be excluded by the following refinement: Suppose agenda-setters can publicly announce anything regarding the strategies they will adopt in the procedure before the game starts. This is a realistic assumption, as political procedures are preceded by debates. In such an environment, the announcement of the second agenda-setter not to subsidize the first agenda-setter – unless the latter suggests a certain proposal – is credible under every proposal, because implementation of the announcement is costless and singles out the best-possible outcome for the second agenda-setter. Hence, the first agenda-setter has every reason to believe the first agenda-setter.[25]

5.4 Flexible taxation

For the main result of the paper we have assumed that subsidies can be distributed arbitrarily, whereas taxes are uniform.[26] This reflects the widespread observation that there is much more flexibility in subsidy schemes than in tax systems. Nevertheless, from a conceptual point of view, restricting tax rules has to be justified. In Proposition 6 below we investigate the robustness of our results regarding the possibility of choosing arbitrary tax schemes.

Proposition 6
If $p < \frac{1}{2}$, under the flexible taxation two-shot procedure – proposal by a winner, counter-proposal by a loser, flexible taxation and voting, with the Agenda-Setter Rule – inefficiencies of types 1 and 3 arise in equilibrium for some parameter constellations.

Gersbach, Hahn, and Imhof (2013) show that restrictions on tax systems are advantageous for the society in one-shot procedures. Proposition 6 extends this insight to competitive political environments. This provides a further important justification for the use of tax rules in public project provision.

6 General Environments

The optimality of the two-round mechanism analyzed in Section 4 has been proved in a specific, stylized environment. We explore in this section whether this result translates into other, more general frameworks. In particular, we examine the following variants of our model:

- The decision on the public project is not binary.
- Individual benefits/costs from the public project can take more than two values.

[25] On refinements in cheap-talk games see e.g. Chen et al. (2009).

[26] Restrictions on tax distribution can be imposed in many other ways than uniform taxation, e.g. making the tax burden of agenda-setters dependent on the aggregate tax burden. Nevertheless, investigation of such rules is beyond the scope of the present paper and represents a subject for further research.

- The set of voters is finite.

In the following, we provide the results for each variant. All proofs of the results presented in this section are contained in Appendix C, which is attached as a separate appendix after the References. For the sake of brevity, we focus only on the case of local public goods, i.e., we assume throughout this section that $p < \frac{1}{2}$, so the set of losers accounts for more than half of the citizenry. Recall that our procedure then prescribes that the first agenda-setter be a winner and the second agenda-setter a loser. [27]

6.1 Divisible public project

Assume now that instead of simply proposing a choice between implementing the public project or not, agenda-setters can suggest the level ρ of the public project, where ρ varies continuously in $[0, 1]$. The associated per-capita costs are $k(\rho)$, where $k(\cdot)$ is a strictly increasing function with $k(0) = 0$ and $k(1) = k$. Each voter $i \in [0, 1]$ values public project ρ either with $V_w(\rho)$ if $i \in [0, p]$ or with $V_l(\rho)$ if $i \in (p, 1]$, where $V_w(0) = 0$, $V_w(1) = V_w$, $V_l(0) = 0$, and $V_l(1) = V_l$. We further assume that $V_l(\rho) < k(\rho) < V_w(\rho)$ for each $\rho \in [0, 1]$, that $V_w(\rho) - k(\rho)$ and $k(\rho) - V_l(\rho)$ are increasing functions in $\rho \in [0, 1]$, and that $\frac{k(\rho)-V_l(\rho)}{V_w(\rho)-V_l(\rho)}$ is a non-increasing function in $\rho \in (0, 1]$. This latter assumption implies that if $pV_w(\rho) + (1 - p)V_l(\rho) \geq k(\rho)$, then $pV_w(\rho') + (1 - p)V_l(\rho') \geq k(\rho')$ for all $\rho' \in (\rho, 1]$. Note that the sets of winners and losers remain invariant for all positive levels of public good provision.

The following result demonstrates that in this modified setting, the outcome of our two-round procedure is always (weakly) welfare-improving – according to our notion of social utility – with respect to the status quo.

Proposition 7
If $p < \frac{1}{2}$ and the decision on the public project is not binary, the outcome in equilibrium under the two-round procedure – proposal by a winner, counter-proposal by a loser and voting, with the Agenda-Setter Rule and uniform taxation – yields a level of provision of the public good that is the maximal $\rho \in [0, 1]$ for which $pV_w(\rho) + (1 - p)V_l(\rho) \geq k(\rho)$. Moreover, no subsidies occur in equilibrium.

[27] Similar results as those presented in this section can be derived for the case $p \geq \frac{1}{2}$.

In particular, note that if $pV_w(\rho) + (1-p)V_l(\rho) < k(\rho)$ for all $\rho \in (0,1]$, i.e., if there is no positive level of public good provision that is socially welfare-improving, the outcome of the two-shot procedure in equilibrium is the status quo.

6.2 Heterogeneous winners and losers

In this section we assume that winners (losers) are heterogeneous regarding their benefits (losses) from the public project. Specifically, given a project (V_w, V_l, k, p), suppose that the valuation v_i of voter $i \in [0,1]$ of the public project is described by the following expression:

$$v_i = \begin{cases} V_w + x_i & \text{if } i \in [0,p], \\ V_l + x_i & \text{if } i \in (p,1]. \end{cases}$$

We assume that x_i is drawn from a uniform distribution on $[-\delta, \delta]$, with $\delta \geq 0$ and $V_w - \delta > k > V_l + \delta$.[28] With a continuum of voters, the expected utility that a random citizen derives from the project remains equal to $pV_w + (1-p)V_l$. Thus, a project is socially efficient or not regardless of δ.[29]

With heterogeneous winners and losers, a new problem arises in our two-step procedure: which type of winner (loser) should be selected to be agenda-setter? In the result below, we show that as long as the second agenda-setter, a loser, has a valuation large enough relative to V_l, this does not matter: Under the latter assumption, the exact individual valuation of winners and losers proves irrelevant for the outcome and, moreover, our two-step procedure continues to yield first-best outcomes. Specifically, we consider the condition

$$v_1 \geq V_l - \max\left\{0, \frac{p-\hat{p}}{1-p}\right\}(V_w - V_l) - H(\hat{p} - p)\delta, \tag{2}$$

where $\hat{p} = \frac{k - V_l}{V_w - V_l}$, $H(x) = 1$ if $x \geq 0$ and $H(x) = 0$ if $x < 0$.

Proposition 8
If $p < \frac{1}{2}$ and winners and losers are heterogeneous, the equilibrium outcome under the two-round procedure – proposal by a winner, counter-proposal by a loser whose valuation satisfies (2) and voting, with the Agenda-Setter Rule and uniform taxation – is π^{soc}.

[28] It suffices to impose that the random variable from which x_i is drawn can vary among voters, has bounded support and that the worst possible valuation for a winner is higher than the cost k, and the best-possible valuation of a loser is smaller than the cost k.

[29] As a technical assumption, we impose that the valuation distribution across voters admits the use of the law of large numbers for a continuum of random variables.

Additionally, we can show that regardless of the exact valuation of the second agenda-setter, the probability that our two-step procedure does not produce a first-best outcome for an arbitrary project converges to zero as δ converges to zero. This is proved in the result below.

Proposition 9
If $p < \frac{1}{2}$ and winners and losers are heterogeneous, the probability that the equilibrium outcome under the two-round procedure – proposal by a winner, counter-proposal by a loser and voting, with the Agenda-Setter Rule and uniform taxation – is π^{soc}, converges to one as δ converges to zero.

A further interpretation of the modified model introduced in this section runs as follows: Suppose that while it is common knowledge whether every voter is a winner or a loser, every citizen receives a private signal, x_i, accounting for idiosyncratic differences among voters regarding the exact valuation of the public project. Proposition 8 reveals that this private information cannot be exploited by any agenda-setter whose valuation is at least as high as V_l, whereas Proposition 9 proves that the probability that an arbitrary agenda-setter can exploit this information tends to zero as the noise level of the private signal, δ, becomes arbitrarily small.[30]

6.3 Finite number of voters

For convenience, we have worked throughout the paper with a continuum of voters. In this section we show that the main thrust of our results continues to hold when there is a finite (but large) number of voters, say $n + 1$, with n an even natural.

First, without the Agenda-Setter Rule, inefficiencies occur in the same way as when there are infinitely many voters. More specifically, if the number of voters is finite but large enough, the outcome in equilibrium under the two-round procedure – *proposal by a winner, counter-proposal by a loser and uniform taxation* – may yield inefficiencies of type 2. The intuition for this result is the same as before. Without the Agenda-Setter Rule, the second agenda-setter can choose between subsidizing the first agenda-setter or not in his best response to a first proposal. By anticipation of the second agenda-setter's

[30]Note that for any value of δ, no agenda-setter can exploit his private information when the project is socially inefficient.

discretionary power in choosing the subsidies in his second proposal, the first agenda-setter will renounce proposing the public project, in order to secure subsidies for himself in the finally-approved proposal.

Proposition 10
If $p < \frac{1}{2}$ and there is a finite but large number of voters, under the simple two-round procedure – proposal by a winner, counter-proposal by a loser and uniform taxation – there are equilibrium outcomes that yield inefficiencies of type 2 if the project is socially efficient.

Second, the proof of Theorem 1 can be immediately adapted to a setting with a finite number of voters, as long as the parameters of the project are such that they permit to construct the proposals (and counter-proposals) used in the proof of the theorem. These proposals require subsidizing a certain fraction of agents which might be impossible to build due to the fact that there is only a finite number of agents.[31] Hence, the probability for finite decision bodies that a random project does not allow the construction of the needed proposals is positive. However, this probability converges to zero as the number of citizens becomes large, hence implying that our procedure is bound to efficiency in large decision bodies.[32]

7 Conclusion

In this paper, we have examined a set of (parliamentary) decision-making procedures, and identified among them a procedure that guarantees efficiency for the public-good provision considered: only welfare-enhancing public goods are selected and wasteful cross-subsidies are avoided in equilibrium. Although subsidies are not paid as part of the efficient outcome, they are critical for efficiency along and off the equilibrium path. The procedure itself is simple and imposes few exogenous constraints: a member of the minority makes a first proposal, followed by a counter-proposal made by a member of the

[31] Specifically, it requires that $\hat{p} \cdot (m+1)$ and $\varepsilon \cdot (m+1)$ be natural numbers for some very small $\varepsilon > 0$ and $m \in \left\{\frac{n}{2}+1, \dots, n+1\right\}$ – see the proof of Theorem 1 –, where $\hat{p} = \frac{k-V_l}{V_w-V_l}$.

[32] More precisely, this convergence is true if $p \neq \hat{p}$. This can be verified easily in the proof of Theorem 1, as the relevant inequalities where the second agenda-setter compares two proposals – one implementing the public project and the other not implementing it – are strict, hence allowing for some room to slightly change one of the proposals while keeping the sign of the inequality, as the number of voters is very large.

majority, with the constraint that subsidies to the two agenda-setters are disallowed and that along with subsidies, the public project is financed through a uniform tax. In the robustness analysis, we have additionally shown that the conditions for the procedure are "tight": the number of proposals is critical – at least two – and so is their sequence with the right of the minority to make the first proposal. The constraint on subsidies for the proposers and the uniformity in the tax schedule are also necessary for the procedure to always yield first-best outcomes.

The decision environment considered in the main part of the paper consists of an exogenous public good project that can either be accepted or rejected. The analysis in the previous section has revealed, however, that our results extend to various ramifications of our basic framework. Numerous other extensions deserve further scrutiny. Multiple public projects and bundling of projects are particularly important avenues for future research.[33] The present paper may be a useful starting point for such endeavors.

[33] We note that for a sequence of independent projects, the mechanism can be used repeatedly, separately for each project, and can induce socially efficient outcomes.

Appendix A

We start by proving an auxiliary result – see Lemma 1 below –, which will be used in all the appendices. This result simplifies the analysis throughout the paper, as counter-proposals that are not seconded by at least half of the society are ruled out in equilibrium.

Lemma 1

If a counter-proposal $\pi'' \in \Pi$ is a best response to proposal $\pi' \in \Pi$, then

$$I(\pi', \pi'') = 1.$$

Proof:

Given an arbitrary proposal $\pi' \in \Pi$, the second agenda-setter can always match the first agenda-setter's proposal, i.e., choose $\pi''_C := \pi'$. Since $u_j(\pi''_C) = u_j(\pi')$ for all $j \in [0, 1]$, it holds by Tie-Breaking Rule 1 that $I(\pi', \pi''_C) = 1$. Moreover, due to Tie-Breaking Rule 3, the second agenda-setter will prefer making proposal π''_C to any proposal $\pi'' \neq \pi''_C$ such that $I(\pi', \pi'') = 0$. As a consequence, either the second agenda-setter will choose π''_C or he will choose a proposal $\pi'' \neq \pi''_C$ such that $I(\pi', \pi''_C) = 1$ and $u_k(\pi'') \geq u_k(\pi''_C) = u_k(\pi')$, where k denotes the second agenda-setter.[34]

□

As a consequence of Lemma 1, the best-response for the second agenda-setter, j, can be described by the correspondence $R : \Pi \to \Pi$ defined for any $\pi' \in \Pi$ by[35]

$$R(\pi') = \underset{\pi'' \in \Pi, I(\pi', \pi'')=1}{\operatorname{argmax}} \left\{ s_j(\pi'') + g(\pi'')(V_j - k) - \int_0^1 s_i(\pi'')di \geq u_j(\pi') \right\}. \tag{3}$$

However, whenever the best response is uniquely defined up to a relabeling of individuals and redistribution within sets of Lebesgue-measure zero, we will slightly abuse our notation and consider $R(\cdot)$ to be a function rather than a correspondence.

[34] There often exist more sophisticated counter-proposals π'' than a simple copy of π' that yield $u_k(\pi'') > u_k(\pi')$ and $I(\pi', \pi'') = 1$, with k denoting the second agenda-setter. Investigation of such counter-proposals is part of our subsequent analysis.

[35] For the sake of brevity, we slightly abuse notation in equation (3).

The rest of this appendix is devoted to the proof of Theorem 1, which follows immediately from the two propositions below.

Proposition 11
If $p < \frac{1}{2}$, the equilibrium outcome under the two-round procedure – proposal by a winner, counter-proposal by a loser and voting, with the Agenda-Setter Rule and uniform taxation – is π^{soc}.

Proposition 12
If $p \geq \frac{1}{2}$, the equilibrium outcome under the two-round procedure – proposal by a loser, counter-proposal by a winner and voting, with the Agenda-Setter Rule and uniform taxation – is π^{soc}.

It thus only remains to prove Propositions 11 and 12. Accordingly, we distinguish two cases.

CASE I: $p < \frac{1}{2}$

Recall that in this case, the set of losers account for more than half of the population. For the purpose of proving Proposition 11, we first examine individual voting behavior and the second agenda-setter's best response. Then, we find the optimal strategy for the first agenda-setter.

A.1 Individual voting behavior

Let π^0 and π^1 be arbitrary proposals in Π made by $j = 0$ and $j = 1$ respectively. Under Tie-Breaking Rule 1, individual $i \in [0, 1]$ will vote in favor of proposal π^1 if and only if $u_i(\pi^1) \geq u_i(\pi^0)$, which can be rewritten as

$$s_i(\pi^1) \geq s_i(\pi^0) + (g(\pi^0) - g(\pi^1))(v_i - k) - \left(\int_0^1 s_j(\pi^0)dj - \int_0^1 s_j(\pi^1)dj\right).$$

Given a proposal $\pi^0 = (g(\pi^0), (s_j(\pi^0))_{j\in[0,1]})$ and subsidies of the counter-proposal $(s_j(\pi^1))_{j\in[0,1]\setminus\{i\}}$ to all individuals except i, it will prove very useful to define

$$\sigma_i(g(\pi^1)) := s_i(\pi^0) + (g(\pi^0) - g(\pi^1))(v_i - k) - \left(\int_0^1 s_j(\pi^0)dj - \int_0^1 s_j(\pi^1)dj\right). \quad (4)$$

Note that $\sigma_i(g(\pi^1))$ represents the smallest subsidy that must be paid to individual i so that he will vote for proposal π^1 given $g(\pi^1)$ and subsidies for all individuals $j \in [0,1]$, $j \neq i$, under π^1. In particular, if $\sigma_i(g(\pi^1)) \leq 0$, individual i will vote for proposal π^1 irrespective of the size of the subsidy he receives under π^1.

Whereas $\sigma_i(g(\pi^1))$ for each $i \in [0,1]$ depends on the entire proposals π^0 and π^1, we notice that, for any $i, j \in [0,1]$ with $i \neq j$, $\sigma_i(g(\pi^1)) - \sigma_j(g(\pi^1))$ depends solely on $s_i(\pi^0)$, $s_j(\pi^0)$, $g(\pi^0)$, and $g(\pi^1)$. Let $i, j \in [0,1]$ be two arbitrary different individuals. Then,

$$\sigma_j(g(\pi^1)) - \sigma_i(g(\pi^1)) = s_j(\pi^0) - s_i(\pi^0) + (g(\pi^0) - g(\pi^1))(v_j - v_i). \tag{5}$$

In particular, given $\pi^0 = (g(\pi^0), s(\pi^0)) \in \Pi$ and $g(\pi^1) \in \{0,1\}$, the sign of $\sigma_j(g(\pi^1)) - \sigma_i(g(\pi^1))$ for any two individuals $i, j \in [0,1]$, $i \neq j$, is not affected by the choice of the subsidy scheme $s(\pi^1)$.

A.2 The best response of the second agenda-setter

According to (3), the best response function for AS_l is described by a correspondence $R(\cdot)$ from Π into itself. However, when the best response is uniquely defined up to a relabeling of individuals and redistribution within sets of Lebesgue-measure zero, it will turn out to be more convenient to consider $R(\cdot)$ to be a function rather than a correspondence.

For the analysis of the second mover's strategy, it is useful to introduce the following sets:[36]

- $N := \{j \in (p,1) \mid s_j(\pi^0) = 0\}$,
- $\bar{N} := \{j \in (0,1) \mid j \notin N\}$.

Observe that N is the set of all losers, except AS_l, who receive zero subsidies under the proposal π^0, whereas $\bar{N}$ is the set of all individuals, except AS_w and AS_l, who do not belong to N. For an arbitrary set $M \subseteq [0,1]$, let the corresponding lower-case letter, m, denote its Lebesgue measure. It is obvious that $\bar{n} = 1 - n$.

It is useful[37] to distinguish two cases: $n \geq \frac{1}{2}$ and $n < \frac{1}{2}$.

[36] We omit the dependence on π^0.

[37] Mathematically speaking, it is not necessary to consider the two cases. However, we introduce them here for the sake of completeness.

A.2.1 Case $\mathbf{n \geq \frac{1}{2}}$

Here, we introduce the proposal $\pi_*^1 \in \Pi$ defined as follows:

$$g(\pi_*^1) = 0 \text{ and } s_j(\pi_*^1) = 0 \text{ for all } j \in [0,1]\,. \tag{6}$$

In the result below, we prove that π_*^1 is always AS_l's best response whenever AS_w suggests zero subsidies to a set of losers of at least measure $\frac{1}{2}$.

Lemma 2

If $n \geq \frac{1}{2}$, then $R(\pi^0) = \pi_*^1$.

Proof:

First, we show that $I(\pi^0, \pi_*^1) = 1$. Indeed, for all $i \in N$,

$$\sigma_i(g(\pi_*^1)) = g(\pi^0)(V_l - k) - \int_0^1 s_j(\pi^0)dj \leq 0.$$

As a consequence, $u_i(\pi_*^1) \geq u_i(\pi^0)$ for all $i \in N$, and these individuals vote for π_*^1 because of Tie-Breaking Rule 1. Since $n \geq \frac{1}{2}$, proposal π_*^1 would be implemented against π^0, that is $I(\pi^0, \pi_*^1) = 1$.

Second, observe that, for any arbitrary $\pi \in \Pi$,

$$u_1(\pi) = g(\pi)(V_l - k) - \int_0^1 s_j(\pi)dj \leq 0 = u_1(\pi_*^1). \tag{7}$$

So π_*^1 yields the highest utility AS_l can obtain under the Agenda-Setter Rule within the set of possible choices $\pi \in \Pi$. Moreover, notice that, if either $g(\pi) = 1$ or $\int_0^1 s_j(\pi)dj > 0$, the inequality in (7) is strict and, hence, up to changes in sets of zero measure, π_*^1 is the unique best response when $n \geq \frac{1}{2}$.

$\square$

In words, Lemma 2 makes it clear that the public project will never be adopted if AS_w proposes π^0 such that $n \geq \frac{1}{2}$.

A.2.2 Case $\mathbf{n < \frac{1}{2}}$

First we split the set $\bar{N}$ into further subsets. For that purpose we define[38]

[38]Note that, given $\pi^0, \pi^1 \in \Pi$, the function $f : [0,1] \to \mathbb{R}_+$ defined by $f(i) = \max\{0, \sigma_i(g(\pi^1))\}$ is Lebesgue-measurable and Lebesgue-integrable since $s(\pi^0)$ is Lebesgue-measurable and Lebesgue-integrable. Rigorously speaking, for the definition of Σ_0 and Σ_1 the minimization must be taken over a maximal compact set of sets $Q \subseteq \bar{N}$, so that both minimization problems have a solution, i.e., $\Sigma_0, \Sigma_1 \neq \emptyset$.

- $\Sigma_0 := \arg\min_{Q_0 \subseteq \bar{N}} \left\{ \int_{Q_0} \sigma_j(0)dj \mid q_0 = \frac{1}{2} - n \right\}$,
- $\Sigma_1 := \arg\min_{Q_1 \subseteq \bar{N}} \left\{ \int_{Q_1} \sigma_j(1)dj \mid q_1 = \frac{1}{2} - n \right\}$.

where $\sigma_j(0)$ and $\sigma_j(1)$, for all $j \in [0,1]$, have been defined in (4) for proposals π^0 and π^1 such that $g(\pi^1) = 0$ and $g(\pi^1) = 1$ respectively.[39] In words, Σ_0 is composed of all sets $Q_0 \subseteq \bar{N}$ of measure $\frac{1}{2} - n$ containing individuals j with smallest aggregate $\sigma_j(0)$. Similarly, Σ_1 is composed of all sets $Q_1 \subseteq \bar{N}$ of measure $\frac{1}{2} - n$ containing individuals j with smallest aggregate $\sigma_j(1)$. We point out that $Q_0 \in \Sigma_0$ and $Q_1 \in \Sigma_1$ independently of the exact design of the subsidy scheme $s(\pi^1)$ since, according to (5), the sign of $\sigma_j(g(\pi^1)) - \sigma_i(g(\pi^1))$ for any $i, j \in [0,1]$ with $i \neq j$ depends only on π^0 and $g(\pi^1) \in \{0,1\}$. We also note that for some choices of π^0, different elements $Q_0, Q_0' \in \Sigma_0$ (or $Q_1, Q_1' \in \Sigma_1$) may differ even in a subset of individuals of non-zero measure.[40] It turns out that outcomes of the game will be independent of a particular choice of Q_0 and Q_1. We select two arbitrary elements $Q_0 \in \Sigma_0$ and $Q_1 \in \Sigma_1$.

Second, we define two specific proposals. On the one hand, we define $\tilde{\pi}^1 \in \Pi$, where

$$g(\tilde{\pi}^1) = 0, \quad s_i(\tilde{\pi}^1) = \begin{cases} \max\{0, \sigma_i(0)\} & \text{for } i \in Q_0, \\ 0 & \text{for all other } i. \end{cases} \tag{8}$$

Notice that according to the definition in (4), $\sigma_i(g(\tilde{\pi}^1))$ depends on $\int_0^1 s_j(\sigma_j(g(\tilde{\pi}^1)))dj$ for all $i \in [0,1]$. That implies that (8) defines $s_i(\tilde{\pi}^1)$ only implicitly. We next show that, in fact, $s_i(\tilde{\pi}^1)$ is well-defined for all $i \in [0,1]$.

Remark 1

The proposal $s(\tilde{\pi}^1)$ implicitly defined in (8) is well-defined.

Proof:

Given π^0 and $\tilde{\pi}^1$, as defined in (8), let $b_i := s_i(\pi^0) + (g(\pi^0) - g(\tilde{\pi}^1))(v_i - k) - \int_0^1 s_j(\pi^0)dj$ for all $i \in [0,1]$. By (8),

$$\int_0^1 s_j(\tilde{\pi}^1)dj = \int_{Q_0} \max\left\{0, b_j + \int_0^1 s_j(\tilde{\pi}^1)dj\right\} dj.$$

[39] Notice that, for each $Q_0 \in \Sigma_0$ and $Q_1 \in \Sigma_0$, it holds that $q_0 = q_1 = \frac{1}{2} - n$, and that AS_w and AS_l do not belong to $Q_0 \cup Q_1$ since, by definition, they do not belong to $\bar{N}$.

[40] For instance, if $s_i(\pi^0) = s > s_j(\pi^0)$ for all $i \in A$ and all $j \notin A$, with $s > 0$ and $A \subseteq [p, 1]$ a measurable set of measure larger than $\frac{1}{2}$, then $\sigma_i(g(\pi^1))$ is the same for all $i \in A$, so any subset of A of measure $\frac{1}{2}$ belongs to $\Sigma_{g(\pi^1)}$.

Rearranging terms, we obtain

$$\int_0^1 s_j(\tilde{\pi}^1)dj = \frac{1}{1 - \left|Q_0^{\int_0^1 s_j(\tilde{\pi}^1)dj}\right|} \cdot \int\limits_{Q_0^{\int_0^1 s_j(\tilde{\pi}^1)dj}} b_j dj, \tag{9}$$

where, for any $x \in \mathbb{R}_+$, we define $Q_0^x := \{j \in Q_0 \mid b_j + x \geq 0\}$. If we additionally define

$$f(x) := x - \frac{1}{1 - |Q_0^x|} \int_{Q_0^x} b_j dj,$$

the existence of a solution $s(\tilde{\pi}^1)$ to equation (9) such that $\int_0^1 s_j(\tilde{\pi}^1)dj = x$ is equivalent to the existence of a solution x to the equation

$$f(x) = 0. \tag{10}$$

Under the assumptions of the paper, $f(\cdot)$ is a continuous function. Moreover, $f(0) \leq 0$ and $\lim_{x \to +\infty} f(x) = +\infty$. As a consequence, equation (10) has a (maybe not unique) solution. If there is only one solution x to equation (10), we choose $s(\tilde{\pi}^1)$ as the unique solution to equation (9). If there are multiple solutions to equation (10), we choose $s(\tilde{\pi}^1)$ as the unique solution to equation (9) such that $\int_0^1 s_j(\tilde{\pi}^1)dj = x^*$, where x^* is the minimum[41] solution to equation (10). The reason for this latter choice is that – *ceteris paribus* – the second agenda-setter will always prefer smaller aggregate subsidies. As a consequence, $s(\tilde{\pi}^1)$ is uniquely defined up to a relabeling of agents and changes in sets of measure zero.

□

On the other hand, we define $\tilde{\tilde{\pi}}^1 \in \Pi$, where

$$g(\tilde{\tilde{\pi}}^1) = 1, \quad s_i(\tilde{\tilde{\pi}}^1) = \begin{cases} \max\{0, \sigma_i(1)\} & \text{for } i \in Q_1, \\ 0 & \text{for all other } i. \end{cases} \tag{11}$$

We note that, like $\tilde{\pi}^1$, $\tilde{\tilde{\pi}}^1$ is well-defined.

Next we prove that we can present the reaction function of AS_l in a simple way.[42]

Lemma 3

If $n < \frac{1}{2}$, then

$$R(\pi^0) = \begin{cases} \tilde{\tilde{\pi}}^1 & \text{if } u_1(\tilde{\tilde{\pi}}^1) \geq u_1(\tilde{\pi}^1), \\ \tilde{\pi}^1 & \text{otherwise.} \end{cases}$$

[41] By the above reasoning it is obvious that x^* exists.

[42] As we prove in Lemma 3, $R(\cdot)$ has just one element up to a relabeling of individuals and changes in sets of Lebesgue-measure zero. Hence, the best response for AS_l is described by a function rather than a correspondence.

Proof:

We first prove that $s_i(R(\pi^0)) = 0$ for all $i \in N$. Indeed, since $s_i(\pi^0) = 0$ for all $i \in N$, we obtain

$$\begin{aligned}\sigma_i(g(R(\pi^0))) &= [g(\pi^0) - g(R(\pi^0))](V_l - k) - \left(\int_0^1 s_j(\pi^0)dj - \int_0^1 s_j(R(\pi^0))dj\right)\\ &= u_1(\pi^0) - u_1(R(\pi^0)) \leq 0\end{aligned}$$

for all $i \in N$, where the inequality holds by definition of best response.

Second, we show that $s_i(R(\pi^0)) = \max\{0, \sigma_i(g(R(\pi^0)))\}$ for all $i \in Q_{g(R(\pi^0))}$, where $g(R(\pi^0)) \in \{0, 1\}$. Indeed, since $u_1(\pi^1)$ is decreasing as a function of aggregate subsidies of the counter-proposal π^1, AS_l has a strict incentive to form the "cheapest" majority that ensures $I(\pi^0, \pi^1) = 1$. Thus, he only pays subsidies to half of the electorate, and he pays them the minimal amount that will prompt them to vote for $R(\pi^0)$. Because any AS_l's best response will secure the votes of individuals in N, he needs the additional support of a fraction of $\frac{1}{2} - n$ individuals. The tax burden is smallest if those individuals in $\bar{N}$ for which $\sigma_j(g(R(\pi^0)))$ is smallest, i.e., $Q_{g(R(\pi^0))}$, are subsidized. Moreover, it cannot be profitable to pay higher subsidies than $\max\{0, \sigma_i(g(R(\pi^0)))\}$ to each individual $i \in Q_{g(R(\pi^0))}$. Note that depending on whether $g(R(\pi^0)) = 0$ or $g(R(\pi^0)) = 1$, the chosen coalitions may differ, so Q_0 is in general different from Q_1.[43]

Third, we observe that, since the above subsidy scheme already guarantees the support of half of the society for $R(\pi^0)$, AS_l will not pay further subsidies, that is $s_i(R(\pi^0)) = 0$ for all other individuals i. Therefore the aggregate amount of subsidies under $R(\pi^0)$ is

$$\int_0^1 s_j(R(\pi^0))dj = \int_{Q_{g(R(\pi^0))}} \max\{0, \sigma_j(g(R(\pi^0)))\}dj.$$

Fourth, given the subsidy scheme $s(R(\pi^0))$, only one alternative for the best response, $R(\pi^0)$, remains. Either it is optimal for AS_l to choose $g(R(\pi^0)) = 0$ or it is optimal to choose $g(R(\pi^0)) = 1$. That is, in each respective case either $\tilde{\pi}^1$ is a best response or $\tilde{\tilde{\pi}}^1$ is.

Lastly, we note that $g(R(\pi^0)) = 1$ holds if and only if $u_1(\tilde{\tilde{\pi}}^1) \geq u_1(\tilde{\pi}^1)$ as

$$u_1(\pi^0) = u_j(\pi^0), u_1(\tilde{\pi}^1) = u_j(\tilde{\pi}^1) \text{ and } u_1(\tilde{\tilde{\pi}}^1) = u_j(\tilde{\tilde{\pi}}^1) \text{ for all } j \in N, \tag{12}$$

[43] The argument in this paragraph depends on whether such a subsidy scheme is well-defined. This question was answered in the affirmative in Remark 1.

and

$$\max\left\{u_1(\tilde{\pi}^1), u_1(\tilde{\tilde{\pi}}^1)\right\} \geq u_1(\pi^0), \tag{13}$$

and Tie-Breaking Rule 2 applies. We stress that the latter tie-breaking rule is only needed if $u_1(\tilde{\pi}^1) = u_1(\tilde{\tilde{\pi}}^1)$. Expression (12) is straightforward, whereas (13) is a consequence of Lemma 1 and the fact that $\tilde{\pi}^1$ (resp. $\tilde{\tilde{\pi}}^1$) is the best response if $g(R(\pi^0)) = 0$ (resp. $g(R(\pi^0)) = 1$). We further note that (12) shows that the interest of AS_l coincides exactly with the interests of voters in N regarding the choice between $\tilde{\pi}^1$, $\tilde{\tilde{\pi}}^1$, and π^0.

□

A.3 The optimal strategy of the first agenda-setter

Once the best response of the second agenda-setter has been analyzed, we use backward induction to study the optimal strategy for the first agenda-setter, AS_w. For the analysis it is useful to introduce the parameter $\hat{p} \in (0,1)$, where

$$\hat{p} = \frac{k - V_l}{V_w - V_l}. \tag{14}$$

The existence and uniqueness of $\hat{p} \in (0,1)$ is guaranteed since $V_w - k > 0$, $V_l - k < 0$ and $pV_w + (1-p)V_l - k$ is linear in p. We note that projects are socially efficient if and only if $p \geq \hat{p}$.

In the following, we introduce two specific proposals that will turn out to be optimal proposals for the first agenda-setter in some particular circumstances. On the one hand, for $p \geq \hat{p}$, consider the proposal π^0_* defined as follows:[44]

$$g(\pi^0_*) = 1 \text{ and } \quad s_j(\pi^0_*) = \begin{cases} \frac{(\frac{1}{2}-p+\hat{p}^\tau)[V_w - V_l]}{1-(\frac{1}{2}+\hat{p}^\tau)} & \text{for } j \in (0,p]\,, \\ \frac{(1-p)[V_w-V_l]}{1-(\frac{1}{2}+\hat{p}^\tau)} & \text{for } j \in \left(p, \frac{1}{2}+\hat{p}^\tau\right]\,, \\ 0 & \text{for } j \in \left(\frac{1}{2}+\hat{p}^\tau, 1\right) \text{ and } j \in \{0,1\}, \end{cases} \tag{15}$$

where $\hat{p}^\tau := \hat{p} + \tau$, with $0 \leq \tau \leq p - \hat{p}$. Observe that, in order for π^0_* to be well-defined, it is sufficient that $\frac{1}{2} - p + \hat{p}^\tau \geq 0$ and $1 - (\frac{1}{2} + \hat{p}^\tau) > 0$. Since $0 < p < \frac{1}{2}$, it follows that both conditions hold if $\hat{p}^\tau \leq p$. On the other hand, let $\pi^0_{**} \in \Pi$ be any proposal that satisfies $n \geq \frac{1}{2}$. It will transpire that the details of such a proposal do not matter.

[44] We assume that $\hat{s}$ is sufficiently large for $s_j(\pi^0_*)$, as defined in (15), not to violate the upper bound on subsidies.

The following lemma proves that the optimal strategy for the first agenda-setter, AS_w, depends solely on whether p is larger or smaller than $\hat{p}$.

Lemma 4

For AS_w it is optimal to choose

$$\pi^0 = \begin{cases} \pi^0_{**} & \text{if } p < \hat{p}, \\ \pi^0_{*} & \text{if } p \geq \hat{p}. \end{cases}$$

Proof:

In the first stage, AS_w chooses $\pi^0 \in \boldsymbol{\pi}^0_*$, where

$$\boldsymbol{\pi}^0_* = \underset{\pi^0 \in \Pi}{\operatorname{argmax}} \left\{ u_0(R(\pi^0)) = g(R(\pi^0))(V_w - k) - \int_0^1 s_j(R(\pi^0))dj \right\}.$$

We distinguish two cases depending on whether $\hat{p}$ is larger than p or not.

Case 1: $p < \hat{p}$

We claim (and prove below) that if $n < \frac{1}{2}$, regardless of the exact strategy chosen by AS_w, AS_l's best response is $\tilde{\pi}^1$ and not $\tilde{\tilde{\pi}}^1$, as defined in (8) and (11) respectively. According to Lemma 2, whenever $n \geq \frac{1}{2}$ the best response for AS_l is π^1_*, as defined in (6). As a consequence, AS_w faces the following choice:

$$u_0(R(\pi^0)) = \begin{cases} u_0(\pi^1_*) = 0 & \text{if } n \geq \frac{1}{2}, \\ u_0(\tilde{\pi}^1) = -\int_0^1 s_j(\tilde{\pi}^1)dj \leq 0 & \text{if } n < \frac{1}{2}. \end{cases} \tag{16}$$

From (16) it follows that AS_w will select any proposal π^0 such that $n \geq \frac{1}{2}$, i.e., π^0_{**}.

It only remains to prove the above claim, i.e., that AS_l's best response is $\tilde{\pi}^1$ and not $\tilde{\tilde{\pi}}^1$. We assume that

$$u_i(\tilde{\tilde{\pi}}^1) \geq u_i(\pi^0) \tag{17}$$

for all $i \in Q_1$.[45] We note that if (17) does not hold for a subset of non-zero measure of Q_1, it must necessarily be the case that $I(\pi^0, \tilde{\tilde{\pi}}^1) = 0$, due to the way $\tilde{\tilde{\pi}}^1$ is constructed. In that case, by definition of best response, $\tilde{\tilde{\pi}}^1$ cannot be the best response, and so, by Lemma 3 AS_l's best response is $\tilde{\pi}^1$, as we claimed.

Let $\pi^0 \in \Pi$ be an arbitrary strategy chosen by AS_w so that $n < \frac{1}{2}$. First, we split the set Q_1 into two disjoint subsets Q_1^w and Q_1^l, where

$$Q_1^w := \{j \in Q_1 \mid j \in (0, p]\},$$
$$Q_1^l := \{j \in Q_1 \mid j \in (p, 1)\}.$$

[45] It would be equivalent to require inequality (17) to hold for q.e. $i \in Q_1$.

Set Q_1^w comprises all winners from Q_1 and set Q_1^l all losers from Q_1. As $Q_1 = Q_1^w \cup Q_1^l$ and $Q_1^w \cap Q_1^l = \emptyset$, we obtain $q_1^w + q_1^l = q_1 = \frac{1}{2} - n$. Observe that, by definition, $q_1^w \leq p$ and hence, by assumption,

$$q_1^w < \hat{p}. \tag{18}$$

Second, starting from $\tilde{\tilde{\pi}}^1$ it is helpful to define an auxiliary proposal[46] $\tilde{\pi}'^1 \in \Pi$ to be considered for the second agenda-setter, AS_l. Specifically, let $\tilde{\pi}'^1 \in \Pi$ be a proposal that comprises $g(\tilde{\pi}'^1) = 0$ and the following subsidy scheme:

$$s_j(\tilde{\pi}'^1) = \begin{cases} \max\{0, \sigma_j(1)\} + \dfrac{V_w - k}{1 - q_1^w} & \text{for } j \in Q_1^w, \\ \max\{0, \sigma_j(1)\} & \text{for } j \in Q_1^l, \\ 0 & \text{for all other } j. \end{cases} \tag{19}$$

The aggregate amount of subsidies under proposal $\tilde{\pi}'^1$ is

$$\int_0^1 s_j(\tilde{\pi}'^1)dj = \int_{j \in Q_1} \max\{0, \sigma_j(1)\}\, dj + q_1^w \frac{V_w - k}{1 - q_1^w}. \tag{20}$$

Third, we show that $u_i(\tilde{\pi}'^1) \geq u_i(\pi^0)$ for all $i \in Q_1$, so by Tie-Breaking Rule 1, $\tilde{\pi}'^1$ would win a vote against π^0. On the one hand, consider $i \in Q_1^w$. Then

$$\begin{aligned} u_i(\tilde{\pi}'^1) &= \max\{0, \sigma_i(1)\} + \frac{V_w - k}{1 - q_1^w} - \int_{j \in Q_1} \max\{0, \sigma_j(1)\}\, dj - q_1^w \frac{V_w - k}{1 - q_1^w} \\ &= V_w - k + \max\{0, \sigma_i(1)\} - \int_{j \in Q_1} \max\{0, \sigma_j(1)\}\, dj = u_i(\tilde{\tilde{\pi}}^1) \geq u_i(\pi^0), \end{aligned}$$

where the last inequality holds by (17) since $i \in Q_1$. On the other hand, consider $i \in Q_1^l$. Then

$$\begin{aligned} u_i(\tilde{\pi}'^1) &= \max\{0, \sigma_i(1)\} - \int_{j \in Q_1} \max\{0, \sigma_j(1)\}\, dj - q_1^w \frac{V_w - k}{1 - q_1^w} \\ &> V_l - k + \max\{0, \sigma_i(1)\} - \int_{j \in Q_1} \max\{0, \sigma_j(1)\}\, dj = u_i(\tilde{\tilde{\pi}}^1) \geq u_i(\pi^0), \end{aligned}$$

where the last inequality again holds by (17) since $i \in Q_1$ and the strict inequality holds using the definition of $\hat{p}$ since (18) implies that

$$q_1^w \frac{V_w - k}{1 - q_1^w} < k - V_l. \tag{21}$$

[46]Notice that $s(\tilde{\pi}'^1)$ is again defined only implicitly. However, the fact that this proposal is well-defined follows from a similar argument as for proposals $\tilde{\pi}^1$ and $\tilde{\tilde{\pi}}^1$ defined in (8) and (11) respectively (see Remark 1).

Fourth, AS_l will prefer the implementation of $\tilde{\pi}'^1$ to $\tilde{\tilde{\pi}}^1$. Indeed,

$$u_1(\tilde{\pi}'^1) = -q_1^w \frac{V_w - k}{1 - q_1^w} - \int_{j \in Q_1} \max\{0, \sigma_j(1)\}\, dj$$
$$> (V_l - k) - \int_{j \in Q_1} \max\{0, \sigma_j(1)\}\, dj = u_1(\tilde{\tilde{\pi}}^1),$$

where the inequality holds by (18).

Finally, since $\tilde{\pi}^1$ is the optimal response for AS_l provided that the project is not implemented and AS_l prefers $\tilde{\pi}'^1$ to $\tilde{\tilde{\pi}}^1$, it must be the case that the electorate chooses $\tilde{\pi}^1$ over π^0 and that $u_1(\tilde{\pi}^1) \geq u_1(\tilde{\pi}'^1) > u_1(\tilde{\tilde{\pi}}^1)$. Therefore it is optimal for AS_l to choose $\tilde{\pi}^1$, as we claimed.

Case 2: $p \geq \hat{p}$

We show that proposal π^0_* introduced in (15), which is well-defined if $p \geq \hat{p}^\tau$, induces $g(R(\pi^0_*)) = 1$ and $s_j(R(\pi^0_*)) = 0$ for all $j \in [0, 1]$. Indeed, it is not difficult to check that π^0_* yields

$$t(\pi^0_*) = k + \frac{(\frac{1}{2} - p + \hat{p}^\tau)[V_w - V_l]}{1 - (\frac{1}{2} + \hat{p}^\tau)}$$

and

$$u_j(\pi^0_*) = V_w - k \text{ for all } j \in \left(0, \frac{1}{2} + \hat{p}^\tau\right].$$

Moreover, $N = \left(\frac{1}{2} + \hat{p}^\tau, 1\right)$, which implies $n = \frac{1}{2} - \hat{p}^\tau$. Since $\hat{p}^\tau > 0$, then $n < \frac{1}{2}$, so both Q_0 and Q_1 are sets of positive measure. By Lemma 3 we know that AS_l will choose $\tilde{\tilde{\pi}}^1$ if $u_1(\tilde{\tilde{\pi}}^1) \geq u_1(\tilde{\pi}^1)$ and will choose $\tilde{\pi}^1$ otherwise. On the one hand, proposal $\tilde{\pi}^1$ is given by $g(\tilde{\pi}^1) = 0$ and[47]

$$s_j(\tilde{\pi}^1) = \begin{cases} 0 & \text{for } j \in \left[0, \frac{1}{2}\right) \cup \left(\frac{1}{2} + \hat{p}^\tau, 1\right], \\ V_w - V_l & \text{for } j \in Q_0 = \left[\frac{1}{2}, \frac{1}{2} + \hat{p}^\tau\right]. \end{cases}$$

Therefore,

$$u_1(\tilde{\pi}^1) = -\hat{p}^\tau (V_w - V_l).$$

On the other hand, it is also easy to check that proposal $\tilde{\tilde{\pi}}^1$ is given, if $p \geq \hat{p}^\tau$, by $g(\tilde{\tilde{\pi}}^1) = 1$ and $s_j(\tilde{\tilde{\pi}}^1) = 0$ for all $j \in [0, 1]$. Thus,

$$u_1(\tilde{\tilde{\pi}}^1) = V_l - k = -\hat{p}(V_w - V_l) \geq \hat{p}^\tau (V_w - V_l) = u_1(\tilde{\pi}^1),$$

[47] Note again that there are other possible choices of $\tilde{\pi}^1$ and Q_0, as long as $q_0 = \hat{p}$ and $Q_0 \subseteq \left(0, \frac{1}{2} + \hat{p}^\tau\right)$. However, the equilibrium outcome will not depend on the specific choice.

where the second equality holds by the definition of $\hat{p}$. Since for AS_l proposal $\tilde{\tilde{\pi}}^1$ yields the same utility that proposal $\tilde{\pi}^1$, by Tie-Breaking Rule 2 the equilibrium proposal is $\tilde{\tilde{\pi}}^1$. We stress that the inequality is strict except if $p = \hat{p}$, which is the only case where Tie-Breaking Rule 2 is actually needed. Thus $g(R(\pi_*^0)) = 1$, as claimed.

Lastly, for any $\pi \in \Pi$ such that either $g(\pi) = 0$ or $\int_0^1 s_j(\pi)dj > 0$ we have

$$u_0(R(\pi_*^0)) = V_w - k > u_0(\pi).$$

As a consequence, it is optimal for AS_w to choose π_*^0.

□

The preceding results (Lemmas 2, 3, and 4) establish Proposition 11. On the one hand, when $p < \hat{p}$ the first agenda-setter, AS_w, will propose π_{**}^0, the second agenda-setter will counter-propose π_*^0, and the latter proposal will win the voting and be implemented. Since π_*^0 prescribes no public project implementation and no subsidies, it is the socially efficient solution. On the other hand, when $p \geq \hat{p}$ the first agenda-setter, AS_w, will choose π_*^0 and then AS_l will respond with a proposal that prescribes public project implementation coupled with no subsidies, which constitutes the socially efficient solution.

CASE II: $p \geq \frac{1}{2}$

To prove Proposition 12, we follow the same steps as for the proof of Proposition 11, with the expositional difference that we now incorporate all steps into a single, more compact proof.

Proof of Proposition 12:

Recall that when $p \geq \frac{1}{2}$, the set of winners accounts for more than half of the population, so that our procedure prescribes that losers move first. Let $\pi^1 \in \Pi$ be an arbitrary proposal suggested by AS_l and let $\pi^0 \in \Pi$ be the amendment proposed by AS_w. Analogously to the case where AS_w proposes first, we can define, for each $i \in [0, 1]$,

$$\sigma_i(g(\pi^0)) := s_i(\pi^1) + (g(\pi^1) - g(\pi^0))(v_i - k) - \left(\int_0^1 s_j(\pi^1)dj - \int_0^1 s_j(\pi^0)dj \right),$$

which amounts to the minimum subsidy that has to be given to individual i to make him vote for π^0 instead of π^1, for given π^1 and $(s_j(\pi^0))_{j\in[0,1]\setminus\{i\}}$.

Following the argument in the proof of Lemma 2, we next split the set $(0,1)$ of all individuals except the two agenda-setters into two subsets. For that purpose, we define[48]

- $\Gamma_0 := \arg\min_{Q_0 \subseteq (0,1)} \left\{ \int_{Q_0} \sigma_j(0) dj \mid q_0 = \frac{1}{2} \right\}$,
- $\Gamma_1 := \arg\min_{Q_1 \subseteq (0,1)} \left\{ \int_{Q_1} \sigma_j(1) dj \mid q_1 = \frac{1}{2} \right\}$.

In words, Γ_0 is composed of all sets $Q_0 \subseteq (0,1)$ of measure $\frac{1}{2}$ containing individuals j with smallest aggregate $\sigma_j(0)$, whereas Γ_1 is composed of all sets of $Q_1 \subseteq (0,1)$ of measure $\frac{1}{2}$ containing individuals j with smallest aggregate $\sigma_j(1)$.[49] Analogously to the proof of Lemma 2 in Appendix A, $Q_0 \in \Gamma_0$ and $Q_1 \in \Gamma_1$ independently of the exact design of the subsidy scheme $s(\pi^1)$. Also, for some choices of π^0, elements $Q_0, Q_0' \in \Gamma_0$ (or $Q_1, Q_1' \in \Gamma_1$) may differ even in a subset of individuals of positive measure. However, as before, outcomes of the game will be independent of a particular choice of Q_0 and Q_1. We select two arbitrary elements $Q_0 \in \Gamma_0$ and $Q_1 \in \Gamma_1$.

This time, we further split set Q_0 into two disjoint subsets Q_0^w and Q_0^l, where

$$\begin{aligned} Q_0^w &:= \{j \in Q_0 \mid j \in (0,p]\}, \\ Q_0^l &:= \{j \in Q_0 \mid j \in (p,1)\}. \end{aligned}$$

Set Q_0^w comprises all winners from Q_0, and set Q_0^l all losers from Q_0. By construction, $Q_0 = Q_0^w \cup Q_0^l$ and $Q_0^w \cap Q_0^l = \emptyset$, so $q_0^w + q_0^l = q_0 = \frac{1}{2}$.

Analogously to Lemma 1, it can be proved that due to Tie-Breaking Rules 1 and 3, an optimal counter-proposal $R(\pi^1)$ will always yield $I(\pi^1, R(\pi^1)) = 1$. Moreover, analogously to Lemma 3, the reaction function of AS_w, who is now the second agenda-setter, can be summarized as follows:

$$R(\pi^1) = \begin{cases} \tilde{\tilde{\pi}}^0 & \text{if } u_0(\tilde{\tilde{\pi}}^0) \geq u_0(\tilde{\pi}^0), \\ \tilde{\pi}^0 & \text{otherwise,} \end{cases}$$

where we use Tie-Breaking Rule 2 and where

$$g(\tilde{\pi}^0) = 0, \quad s_j(\tilde{\pi}^0) = \begin{cases} \max\{0, \sigma_j(0)\} & \text{for } j \in Q_0, \\ 0 & \text{for all other } j, \end{cases} \tag{22}$$

[48] Analogous precisions to those made in Footnote 38 need to be made to properly define Γ_0 and Γ_1.

[49] Notice that now, for each $Q_0 \in \Gamma_0$ and $Q_1 \in \Gamma_1$, it holds that $q_0 = q_1 = \frac{1}{2}$ and that, as before, AS_w and AS_l do not belong to $Q_0 \cup Q_1$ by definition.

and

$$g(\tilde{\tilde{\pi}}^0) = 1, \quad s_j(\tilde{\tilde{\pi}}^0) = \begin{cases} \max\{0, \sigma_j(1)\} & \text{for } j \in Q_1, \\ 0 & \text{for all other } j. \end{cases} \tag{23}$$

We point out that although the subsidy schemes of $\tilde{\pi}^0$ and $\tilde{\tilde{\pi}}^0$ are only defined implicitly, they are, in fact, well-defined.[50]

The proof proceeds now by distinguish two cases, depending on whether it is socially optimal to implement the public project or not.[51]

Case 1: $p < \hat{p}$

We further distinguish two subcases, depending on whether p is strictly larger than $\frac{1}{2}$ or not.

- **Case 1.A:** $p > \frac{1}{2}$

  Let the proposal π_*^1 be defined as follows:[52]

$$g(\pi_*^1) = 0 \text{ and } \quad s_j(\pi_*^1) = \begin{cases} 0 & \text{for } j \in \left(0, p - \frac{1}{2}\right] \text{ and } j \in \{0, 1\}, \\ p\frac{V_w - V_l}{p - \frac{1}{2}} & \text{for } j \in \left(p - \frac{1}{2}, p\right], \\ \frac{V_w - V_l}{2p-1} & \text{for } j \in (p, 1). \end{cases} \tag{24}$$

  The subsidies in (24) guarantee that the uniform tax under proposal π_*^1 coincides with the subsidies to the losers, i.e.,

$$\int_0^1 s_j(\pi_*^1) dj = \frac{p}{2p-1}(V_w - V_l) + \frac{1-p}{2p-1}(V_w - V_l) = \frac{V_w - V_l}{2p-1}.$$

  The utilities amount to

$$u_j(\pi_*^1) = \begin{cases} -\frac{V_w - V_l}{2p-1} & \text{for } j \in \left(0, p - \frac{1}{2}\right] \text{ and } j \in \{0, 1\}, \\ V_w - V_l & \text{for } j \in \left(p - \frac{1}{2}, p\right], \\ 0 & \text{for } j \in (p, 1). \end{cases}$$

  Next, we show that the best response for AS_w is $\tilde{\pi}^0$ and not $\tilde{\tilde{\pi}}^0$. First, it is straightforward to verify that $Q_0 = \left(0, p - \frac{1}{2}\right] \cup (p, 1)$ and $s_j(\tilde{\pi}^0) = 0$ for all $j \in (0, 1)$. Thus,

$$u_0(\tilde{\pi}^0) = 0. \tag{25}$$

[50] See Remark 1 for the analogous definition problem solved in the proof of Lemma 3.

[51] We stress that all tie-breaking rules apply.

[52] We assume that $\hat{s}$ is sufficiently large for $s_j(\pi_*^1)$, as defined in (24), not to violate the upper bound on subsidies.

Second, for $i \in \left(p - \frac{1}{2}, p\right]$ and $j \in (p, 1)$, we have

$$\sigma_i(1) = \sigma_j(1) \geq \frac{\hat{p}}{p}(V_w - V_l).$$

Therefore, $Q_1 = \left(0, p - \frac{1}{2}\right] \cup (p, 1)$ and

$$s_j(\tilde{\tilde{\pi}}^0) \geq \frac{\hat{p}}{p}(V_w - V_l) \text{ for } j \in (p, 1)\,,$$

which implies

$$u_0(\tilde{\tilde{\pi}}^0) \leq V_w - k - (1-p)\frac{\hat{p}}{p}(V_w - V_l) = \frac{p - \hat{p}}{p}(V_w - V_l) < 0. \tag{26}$$

Finally, from (25) and (26), it follows that the best response of AS_w is $\tilde{\pi}^0$, which implies that the public project will not be implemented in equilibrium.[53] By anticipating AS_w's response, the first agenda-setter, AS_l, will propose π^1_*, in order that subsidies are completely eliminated in the proposal chosen by the voters.

- **Case 1.B:** $p = \frac{1}{2}$

  Let $\hat{p}^* = \hat{p} - \varepsilon$ for $\varepsilon > 0$ arbitrarily small. We now consider the proposal π^1_*, which is defined as follows:[54]

$$g(\pi^1_*) = 0 \text{ and } \quad s_j(\pi^1_*) = \begin{cases} \frac{2\hat{p}^*}{2\hat{p}^*-1}(V_w - V_l) & \text{for } j \in \left(0, \frac{1}{2}\right]\,, \\ 0 & \text{for } j \in \left(\frac{1}{2}, \hat{p}^*\right] \text{ and } j \in \{0,1\}, \\ \frac{1}{2\hat{p}^*-1}(V_w - V_l) & \text{for } j \in (\hat{p}^*, 1)\,. \end{cases}$$

  Note that

$$\int_0^1 s_j(\pi^1_*)dj = \frac{\hat{p}^*}{2\hat{p}^* - 1}(V_w - V_l) + \frac{1 - \hat{p}^*}{2\hat{p}^* - 1}(V_w - V_l) = \frac{1}{2\hat{p}^* - 1}(V_w - V_l).$$

  Therefore,

$$u_j(\pi^1_*) = \begin{cases} V_w - V_l & \text{for } j \in \left(0, \frac{1}{2}\right]\,, \\ -\int_0^1 s_j(\pi^1_*)dj & \text{for } j \in \left(\frac{1}{2}, \hat{p}^*\right] \text{ and } j \in \{0,1\}, \\ 0 & \text{for } j \in (\hat{p}^*, 1)\,. \end{cases}$$

  On the one hand, we trivially obtain $Q_0 = \left(\frac{1}{2}, 1\right)$ and $s_j(\tilde{\pi}^0) = 0$ for all $j \in (0, 1)$. On the other hand, for $i \in \left(0, \frac{1}{2}\right]$ and $j \in (\hat{p}^*, 1)$, we have $\sigma_i(1) = \sigma_j(1) \geq \frac{\hat{p}}{\hat{p}^*}(V_w - V_l)$. Hence, $\left(\frac{1}{2}, \hat{p}^*\right] \subseteq Q_1$ and

$$s_j(\tilde{\tilde{\pi}}^0) \geq \frac{\hat{p}}{\hat{p}^*}(V_w - V_l) \quad \text{ for } j \in (\hat{p}^*, 1).$$

[53] Recall that Lemma 1 holds throughout the paper.

[54] We assume that $\hat{s}$ is sufficiently large for $s_j(\pi^1_*)$, as defined in (24), not to violate the upper bound on subsidies.

As a consequence,

$$\begin{aligned}u_0(\tilde{\tilde{\pi}}^0) &\leq V_w - k - (1-\hat{p}^*)\frac{\hat{p}}{\hat{p}^*}(V_w - V_l) = (1-\hat{p})(V_w - V_l) - (1-\hat{p}^*)\frac{\hat{p}}{\hat{p}^*}(V_w - V_l)\\ &= \frac{\hat{p}^* - \hat{p}}{\hat{p}^*}(V_w - V_l) < 0 = u_0(\tilde{\pi}^0),\end{aligned}$$

leading to the same conclusions as in the above case.

Case 2: $p \geq \hat{p}$

In this case, it will be useful to consider an auxiliary proposal $\tilde{\tilde{\pi}}'^0 \in \Pi$ that comprises $g(\tilde{\tilde{\pi}}'^0) = 1$ and the following subsidy scheme:

$$s_j(\tilde{\tilde{\pi}}'^0) = \begin{cases} \max\{0, \sigma_j(0)\} + \frac{k-V_l}{1-q_0^l} & \text{for } j \in Q_0^l, \\ \max\{0, \sigma_j(0)\} & \text{for } j \in Q_0^w, \\ 0 & \text{for all other } j. \end{cases}$$

The aggregate amount of subsidies under $\tilde{\tilde{\pi}}'^0$ is

$$\int_0^1 s_j(\tilde{\tilde{\pi}}'^0)dj = \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj + q_0^l \frac{k - V_l}{1 - q_0^l}. \tag{27}$$

Next, we show that $u_i(\tilde{\tilde{\pi}}'^0) \geq u_i(\pi^1)$ for all $i \in Q_0$, so by Tie-Breaking Rule 1, $\tilde{\tilde{\pi}}'^0$ would win a vote against π^1, the proposal by AS_l. On the one hand, consider $i \in Q_0^l$. Then

$$\begin{aligned}u_i(\tilde{\tilde{\pi}}'^0) &= V_l - k + \max\{0, \sigma_i(0)\} + \frac{k - V_l}{1 - q_0^l} - \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj - q_0^l \frac{k - V_l}{1 - q_0^l}\\ &= \max\{0, \sigma_i(0)\} - \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj = u_i(\tilde{\pi}^0) \geq u_i(\pi^1),\end{aligned}$$

where the last inequality holds since $i \in Q_0$. On the other hand, consider $i \in Q_0^w$. Then

$$\begin{aligned}u_i(\tilde{\tilde{\pi}}'^0) &= V_w - k + \max\{0, \sigma_i(0)\} - \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj - q_0^l \frac{k - V_l}{1 - q_0^l}\\ &\geq \max\{0, \sigma_i(0)\} - \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj = u_i(\tilde{\pi}^0) \geq u_i(\pi^1),\end{aligned}$$

where the last inequality holds since $i \in Q_0$. The first inequality holds since $q_0^l \leq 1 - p \leq 1 - \hat{p}$, which yields

$$\frac{q_0^l}{1 - q_0^l} \leq \frac{1-p}{p} \leq \frac{1-\hat{p}}{\hat{p}} = \frac{V_w - k}{k - V_l}. \tag{28}$$

Finally, AS_w will prefer the implementation of $\tilde{\tilde{\pi}}'^0$ to $\tilde{\pi}^0$. Indeed,

$$u_0(\tilde{\tilde{\pi}}'^0) = V_w - k - q_0^l \frac{k - V_l}{1 - q_0^l} - \int_{j \in Q_0} \max\{0, \sigma_j(0)\}\, dj$$
$$\geq - \int_{j \in Q_0} \max\{0, \sigma_j(0)\}\, dj = u_0(\tilde{\pi}^0),$$

where the inequality holds by (28). Note that the inequality is tight if and only if $p = \hat{p}$.

Since $\tilde{\tilde{\pi}}^0$ is the optimal response for AS_w, provided that the project is implemented and AS_w prefers $\tilde{\tilde{\pi}}'^0$ to $\tilde{\pi}^0$, it must be the case that the electorate chooses $\tilde{\tilde{\pi}}^0$ over π^1 and that $u_0(\tilde{\tilde{\pi}}^0) \geq u_0(\tilde{\tilde{\pi}}'^0) \geq u_0(\tilde{\pi}^0)$. Due to Tie-Breaking Rule 2, it is optimal for AS_w to choose $\tilde{\tilde{\pi}}^0$.[55] Finally, in anticipation, the first agenda-setter, AS_l, will propose π^1 such that $g(\pi^1) = 1$ and $s_j(\pi^1) = 0$ for all $j \in (0, 1)$, so that subsidies in $\tilde{\tilde{\pi}}^0$ are completely eliminated.

□

Appendix B

Here we prove Propositions 1, 2, 3, 4, 5, and 6.

B.1 When amendments are not allowed

Throughout this section, we assume that $p < \frac{1}{2}$, i.e., losers account for more than half of the population. We consider a one-round procedure defined as follows: There is only one agenda-setter, a winner denoted by AS_w. He makes a proposal π^0 which is either accepted or rejected by the citizens. In the latter case, the status quo prevails, yielding zero utility for all individuals. The decision is taken according to the simple majority rule. The utility functions of all individuals are exactly the same as in the rest of the paper. Likewise, taxes are uniform, and subsidies to all individuals other than the agenda-setter are only constrained to be non-negative and bounded from above. The Agenda-Setter Rule prevails and is now only applied to AS_w. To examine one-round procedures, Tie-Breaking Rules in Section 3 are modified as follows:

Tie-Breaking Rule 4
If $u_i(\pi^0) = 0$, individual i will vote for the proposal π^0.

[55] As in the case where $p < \frac{1}{2}$, Tie-Breaking Rule 2 is, in fact, only needed when $\hat{p} = p$.

Tie-Breaking Rule 5

If AS_w is indifferent between a proposal $\pi^0 \in \Pi$ involving $g(\pi^0) = 1$ and a proposal π'^0 involving $g(\pi'^0) = 0$, he will always suggest π^0.

As only one proposal is made, the Tie-Breaking Rules favor this proposal over the status quo. To show how particular inefficiencies occur, we considerer parameter constellations that fulfill

$$V_w \geq V_w^* := \frac{k - (\frac{1}{2} - p)V_l}{1 - (\frac{1}{2} - p)}. \tag{29}$$

This assumption is equivalent to $\hat{p} \leq p + \frac{1}{2}$. We also assume that $p < \hat{p}$, i.e., it is not socially efficient to implement the project.

In the proof of Proposition 1 below we show that, under the above assumptions, it is always optimal for AS_w to propose to implement the public project and get his proposal approved. Moreover, the optimal proposal for the agenda-setter includes strictly positive subsidies. Consequently, inefficiencies of type 1 and 3 arise.

Proof of Proposition 1:

The agenda-setter AS_w chooses a proposal according to

$$\underset{\pi^0 \in \Pi}{\operatorname{argmax}} \left\{ u_0(\pi^0) = g(\pi^0)(V_w - k) - \int_0^1 s_j(\pi^0) dj \right\}.$$

Let $\pi_*^0 \in \Pi$ be such that $g(\pi_*^0) = 0$ and $s_j(\pi_*^0) = 0$ for all $j \in [0, 1]$. Since $u_0(\pi_*^0) = 0 = u_j(\pi_*^0)$ for all $j \in [0, 1]$, by Tie-Breaking Rule 4 π_*^0 would be unanimously supported. As the agenda-setter cannot obtain subsidies, π_*^0 is his optimal proposal provided he does not want to propose the public project.

We next consider proposals that involve project adoption. We claim that AS_w cannot do better than with π_{**}^0, which is defined by $g(\pi_{**}^0) = 1$ and the following subsidy scheme:

$$s_j(\pi_{**}^0) = \begin{cases} V_w^* - V_l & \text{for } j \in (p, \frac{1}{2}], \\ 0 & \text{for all other } j. \end{cases}$$

Notice that $V_w^* - V_l > 0$ because $k > V_l$. Thus, the above proposal is feasible, since it involves non-negative subsidies and is bounded from above by $\hat{s}$ for a sufficiently large $\hat{s}$. For all $j \in [0, p]$, we have

$$u_j(\pi_{**}^0) = V_w - k - \left(\frac{1}{2} - p\right)(V_w^* - V_l) = \left[\frac{1 - \hat{p}}{\hat{p}} - \frac{1 - \left(\frac{1}{2} + p\right)}{\left(\frac{1}{2} + p\right)}\right](k - V_l) \geq 0, \tag{30}$$

where the inequality holds since $\hat{p} \leq \frac{1}{2} + p$ and $f(x) = \frac{1-x}{x}$ is decreasing in $x \in (0, 1]$. Moreover, for all $j \in (p, \frac{1}{2}]$

$$u_j(\pi^0_{**}) = V_l - k + V^*_w - V_l - \left(\frac{1}{2} - p\right)(V^*_w - V_l) = 0, \tag{31}$$

whereas, for all $j \in (\frac{1}{2}, 1]$

$$u_j(\pi^0_{**}) = V_l - k - \left(\frac{1}{2} - p\right)(V^*_w - V_l) < 0.$$

By Tie-Breaking Rule 4, all individuals $j \in [0, \frac{1}{2}]$ will vote for π^0_{**}, and the proposal will be carried out.

Lastly, by (30), under π^0_{**}, the utility of the agenda-setter AS_w is non-negative. Moreover, a smaller (and non-negative) level of subsidies would not gain the support of 50% of the citizens. Therefore among the proposals that involve project adoption AS_w's optimal proposal is π^0_{**}. Finally, due to Tie-Breaking Rule 5, we have π^0_{**} proposed and implemented. Hence, inefficiencies of type 3 will occur. As the project is socially inefficient, inefficiencies of type 1 will also occur.

□

Next, we consider the case where subsidy schemes are restricted to belonging to some given $\mathcal{A} \subseteq \mathbb{S}$ with $\mathcal{A} \neq \emptyset$. As before, individual subsidies are also bounded from above by $\hat{s}$. We further assume that, if $s(\pi)$ and $s'(\pi)$ differ only on a set of measure zero, then $s(\pi) \in \mathcal{A}$ if and only if $s'(\pi) \in \mathcal{A}$.[56]

Proof of Proposition 2:

Let π^0_{**} be the proposal defined in the proof of Proposition 1 and let V^*_w be as defined in (29). Assume that $\hat{p} \leq \frac{1}{2} + p$ or, equivalently, $V_w \geq V^*_w$. We claim that, up to changes on sets of measure zero and a relabeling of losers, a necessary condition for AS_w to propose $\pi^0 \in \Pi$ such that $g(\pi^0) = 1$ is

$$s_j(\pi^0) \geq s_j(\pi^0_{**}) \text{ for all } j \in [0, 1]\,. \tag{32}$$

Indeed, if AS_w wants π^0 to be approved, due to Tie-Breaking Rule 4, π^0 must give non-negative utility to at least half of the society. Hence, at least a fraction of $\frac{1}{2} - p$ losers

[56]This assumption is not necessary for the result, but it simplifies the proof.

must have non-negative utility under π^0. We can assume without loss of generality that these individuals are located in $H := \left(p, \frac{1}{2}\right]$. Then,

$$
\begin{aligned}
0 \leq \int_H u_i(\pi^0)di &= \int_H \left[V_l - k + s_i(\pi^0) - \int_0^1 s_j(\pi^0)dj\right] di \\
&= \left(\frac{1}{2} - p\right)(V_l - k) - \left(\frac{1}{2} - p\right)\int_0^1 s_j(\pi^0)dj + \int_H s_j(\pi^0)dj \\
&\leq \left(\frac{1}{2} - p\right)(V_l - k) + \left(1 - \left(\frac{1}{2} - p\right)\right)\int_0^1 s_j(\pi^0)dj
\end{aligned}
$$

which implies

$$
\int_0^1 s_j(\pi^0)dj \geq \frac{1 - \left(\frac{1}{2} + p\right)}{\frac{1}{2} + p}(k - V_l) = \left(\frac{1}{2} - p\right)(V_w^* - V_l).
$$

As a consequence, we have for all $j \in \left(p, \frac{1}{2}\right]$

$$
\begin{aligned}
0 \leq & u_j(\pi^0) \leq (V_l - k) + s_j(\pi^0) - \frac{1 - \left(\frac{1}{2} + p\right)}{\frac{1}{2} + p}(k - V_l) \\
&= s_j(\pi^0) - \frac{(k - V_l)}{\frac{1}{2} + p} = s_j(\pi^0) - (V_w^* - V_l),
\end{aligned}
$$

so (32) holds. Hence we can write $s_j(\pi^0) = s_j(\pi^0_{**}) + \delta_j$, where $\delta_j \geq 0$ for all $j \in [0, 1]$. We claim that, up to changes on subsets of measure zero,

$$
\delta_i \geq \int_0^1 \delta_j dj, \text{ for all } i \in H. \tag{33}
$$

Suppose otherwise, i.e., $\delta_i < \int_0^1 \delta_j dj$ for all i belonging to a subset of non-zero measure $B \subseteq H$. Then for all $i \in B$, we have

$$
\begin{aligned}
u_i(\pi^0) = V_l - k + s_i(\pi^0) - \int_0^1 s_j(\pi^0)dj &= V_l - k + s_i(\pi^0_{**}) + \delta_i - \int_0^1 s_j(\pi^0_{**})dj - \int_0^1 \delta_j dj \\
&< V_l - k + s_i(\pi^0_{**}) - \int_0^1 s_j(\pi^0_{**})dj = 0,
\end{aligned}
$$

where the last equality holds by (31). This contradicts the fact that all voters $i \in B$ vote for π^0 and not for the status quo. Now let $i \in [0, p]$. Then, $u_i(\pi^0) \geq 0$ if and only if

$$
0 \leq V_w - k + s_i(\pi^0) - \int_0^1 s_j(\pi^0)dj = \left[\frac{1 - \hat{p}}{\hat{p}} - \frac{1 - \left(\frac{1}{2} + p\right)}{\left(\frac{1}{2} + p\right)}\right](k - V_l) + \delta_i - \int_0^1 \delta_j dj,
$$

where the equality holds by (30). By the non-negativity of δ_i and the Agenda-Setter Rule – which implies $\delta_0 = 0$ – we obtain that $u_i(\pi^0) \geq 0$ for all $i \in [0, p]$ is equivalent to

$$
\int_0^1 \delta_j dj \leq \left[\frac{V_w - k}{k - V_l} - \frac{1 - \left(\frac{1}{2} + p\right)}{\left(\frac{1}{2} + p\right)}\right](k - V_l). \tag{34}
$$

In conclusion, for a proposal π^0 with $g(\pi^0) = 1$ to be proposed by AS_w and approved by at least half of the society, it is necessary that $s_j(\pi^0) = s_j(\pi^0_{**}) + \delta_j$ for all $j \in [0,1]$, where (33) and (34) hold.

For a given project $(V_w, V_l, k, p) \in \mathcal{P}$, let $A(V_w, V_l, k, p) \subseteq \mathcal{A}$ be the (maybe empty) set of subsidy schemes that satisfy (33) and (34), and let $D(V_w, V_l, k, p)$ be the subset of $A(V_w, V_l, k, p)$ for which $\int_0^1 \delta_j dj$ is minimum. Repeating exactly the same arguments as in the proof of Proposition 1, we find that the optimal choice for AS_w is[57]

$$\pi^0 = \begin{cases} \pi \text{ with } g(\pi) = 1 \text{ and } s(\pi) \in D(V_w, V_l, k, p) & \text{if } D(V_w, V_l, k, p) \neq \emptyset, \\ \pi^0_* & \text{otherwise,} \end{cases}$$

where π^0_* has been defined by $g(\pi^0_*) = 0$ and $s_j(\pi^0_*) = 0$ for all $j \in [0,1]$.

Finally, we can define two different projects (V_w, V_l, k, p) and (V'_w, V'_l, k', p') such that[58]

$$p < \hat{p} < \frac{1}{2} + p, \quad \hat{p'} < p' \quad \text{and} \quad D = D(V_w, V_l, k, p) = D(V'_w, V'_l, k', p').$$

On the one hand, if $D \neq \emptyset$, then project (V_w, V_l, k, p) will be implemented, although it is not socially efficient. Moreover, aggregate subsidies are strictly positive. On the other hand, if $D = \emptyset$, then project (V'_w, V'_l, k', p') will not be undertaken, although its implementation is socially efficient. In the first case, inefficiencies of type 1 and 3 arise, whereas inefficiencies of type 2 arise in the second case.

$\square$

B.2 When the political initiative is given to a member of the majority

We consider the same two-shot procedure studied in Section 4, with the modification that now, a member of the majority is the first agenda-setter and a member of the minority is the second agenda-setter. We next prove Propositions 3 and 4.

Proof of Proposition 3:

When winners are a minority, the best-response correspondence for the second agenda-setter, a winner, is exactly the same as in the proof of Proposition 12 given in Appendix

[57] We assume that $\mathcal{A}$ satisfies some regularity conditions so that $A(V_w, V_l, k, p) \neq \emptyset$ if and only if $D(V_w, V_l, k, p) \neq \emptyset$.

[58] It is always possible to choose parameter values so that two projects with the desired properties exist.

A, where losers were a minority. Let $\tilde{\pi}^0$, $\tilde{\tilde{\pi}}^0$, Q_0, Q_1, Q_0^l and Q_0^w be the proposals and sets defined there.

Next, we study the optimal strategy for the first agenda-setter, AS_l. We claim (and prove below) that it is always optimal for AS_w to choose $\tilde{\tilde{\pi}}^0$ instead of $\tilde{\pi}^0$ as a counter-proposal to π^1 whenever

$$\hat{p} \leq \frac{1}{2}. \tag{35}$$

Since AS_l can advance AS_w's behavior, we now apply backward induction to analyze the optimal choice for AS_l. Under $\tilde{\tilde{\pi}}^0$, the net utility change for AS_l is

$$u_1(\tilde{\tilde{\pi}}^0) = (V_l - k) - \int_{Q_1} \max\{0, \sigma_j(1)\}\, dj \leq V_l - k. \tag{36}$$

As a consequence, AS_l's optimal strategy is to propose π^1 such that $\max\{0, \sigma_j(1)\} = 0$ for all $j \in Q_1$. This can be achieved by choosing π^1 with $g(\pi^1) = 1$ and $s_j(\pi^1) = 0$ for all $j \in [0,1]$. For this particular choice of π^1, we obtain $Q_1 = [\frac{1}{2}, 1)$, $\int_{Q_1} \max\{0, \sigma_j(1)\}\, dj = 0$ and $u_1(\tilde{\tilde{\pi}}^0) = V_l - k$. In particular, we conclude that the public project will be implemented if (35) holds, even if doing so is socially inefficient. More specifically, observe that inefficiencies of type 1 and possibly type 3 may arise.

It only remains to prove the above claim, i.e., that it is always optimal for AS_w to choose $\tilde{\tilde{\pi}}^0$ instead of $\tilde{\pi}^0$. We assume that

$$u_i(\tilde{\pi}^0) \geq u_i(\pi^1) \tag{37}$$

for all $i \in Q_0$. We note that if (37) does not hold for a subset of non-zero measure of Q_0, it must necessarily be the case that $I(\pi^1, \tilde{\pi}^0) = 0$ due to the way $\tilde{\pi}^0$ is constructed. In that case, $\tilde{\pi}^0$ cannot be the best response, so AS_w's best response is $\tilde{\tilde{\pi}}^0$, as claimed. We distinguish two cases, $q_0^w > 0$ and $q_0^w = 0$.[59]

- **Case 1:** $q_0^w > 0$

  Let $\tilde{\tilde{\pi}}'^0 \in \Pi$ be a proposal that comprises $g(\tilde{\tilde{\pi}}'^0) = 1$ and the following well-defined subsidy scheme:[60]

$$s_j(\tilde{\tilde{\pi}}'^0) = \begin{cases} \max\{0, \sigma_j(0)\} + \dfrac{\frac{1}{2} + q_0^w - \hat{p}}{q_0^w}(V_w - V_l) & \text{for } j \in Q_0^w, \\ \max\{0, \sigma_j(0)\} + V_w - V_l & \text{for } j \in Q_0^l, \\ 0 & \text{for all other } j. \end{cases}$$

[59] In this Proposition we do not need to distinguish between $n \geq \frac{1}{2}$ and $n < \frac{1}{2}$.

[60] An argument similar to that in Remark 1 proves that this subsidy scheme is indeed well-defined.

Notice that if (35) holds the above scheme defines non-negative subsidies for all citizens. Note also that in general $\tilde{\tilde{\pi}}'^{0}$ need not coincide with $\tilde{\tilde{\pi}}^{0}$. The aggregate amount of subsidies under proposal $\tilde{\tilde{\pi}}'^{0}$ is

$$\begin{aligned}\int_0^1 s_j(\tilde{\tilde{\pi}}'^{0})dj &= \int_{j\in Q_0} \max\{0,\sigma_j(0)\}\,dj + (V_w - V_l)(1-\hat{p}) \\ &= \int_{j\in Q_0} \max\{0,\sigma_j(0)\}\,dj + V_w - k, \end{aligned} \tag{38}$$

where the second equality holds by the definition of $\hat{p}$.

Next we show that $u_i(\tilde{\tilde{\pi}}'^{0}) \geq u_i(\pi^1)$ for all $i \in Q_0$. Since by construction Q_0 includes half of the society, Tie-Breaking Rule 1 implies that $\tilde{\tilde{\pi}}'^{0}$ would win a vote against π^1. On the one hand, consider $i \in Q_0^w$. Then,

$$\begin{aligned} &u_i(\tilde{\tilde{\pi}}'^{0}) \\ &= V_w - k + \frac{\frac{1}{2} + q_0^w - \hat{p}}{q_0^w}(V_w - V_l) + \max\{0,\sigma_i(0)\} - \int_{j\in Q_0} \max\{0,\sigma_j(0)\}\,dj - (V_w - k) \\ &= \frac{\frac{1}{2} + q_0^w - \hat{p}}{q_0^w}(V_w - V_l) + \max\{0,\sigma_i(0)\} - \int_{j\in Q_0} \max\{0,\sigma_j(0)\}\,dj \\ &> \max\{0,\sigma_i(0)\} - \int_{j\in Q_0} \max\{0,\sigma_j(0)\}\,dj = u_i(\tilde{\pi}^0) \geq u_i(\pi^1), \end{aligned}$$

where the last inequality holds by (37) since $i \in Q_0$. On the other hand, consider $i \in Q_0^l$. Then,

$$\begin{aligned} u_i(\tilde{\tilde{\pi}}'^{0}) &= (V_l - k) + (V_w - V_l) + \max\{0,\sigma_i(0)\} - \int_{j\in Q_0} \max\{0,\sigma_j(0)\}\,dj - (V_w - k) \\ &= \max\{0,\sigma_i(0)\} - \int_{j\in Q_0} \max\{0,\sigma_j(0)\}\,dj = u_i(\tilde{\pi}^0) \geq u_i(\pi^1), \end{aligned}$$

where the last inequality holds by (37) since $i \in Q_0$.

Lastly, notice that AS_w is indifferent between $\tilde{\tilde{\pi}}'^{0}$ and $\tilde{\pi}^0$. Indeed,

$$u_0(\tilde{\tilde{\pi}}'^{0}) = -\int_{j\in Q_0} \max\{0,\sigma_j(0)\}\,dj = u_0(\tilde{\pi}^0).$$

- **Case 2:** $q_0^w = 0$

In this second case, let $\tilde{\tilde{\pi}}'^{0} \in \Pi$ be a proposal that comprises $g(\tilde{\tilde{\pi}}'^{0}) = 1$ and the following subsidy scheme:

$$s_j(\tilde{\tilde{\pi}}'^{0}) = \begin{cases} (V_w - V_l) + \max\{0,\sigma_j(0)\} & \text{for } j \in Q_0^l, \\ 0 & \text{for all other } j. \end{cases}$$

Notice again that in general $\tilde{\tilde{\pi}}'^0$ need not coincide with $\tilde{\tilde{\pi}}^0$. The aggregate amount of subsidies under proposal $\tilde{\tilde{\pi}}'^0$ is now

$$\int_0^1 s_j(\tilde{\tilde{\pi}}'^0)dj = \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj + \frac{1}{2}(V_w - V_l). \tag{39}$$

As in Case 1, we show that $u_i(\tilde{\tilde{\pi}}'^0) \geq u_i(\pi^1)$ for all $i \in Q_0$, so, given that Q_0 includes half of the society, Tie-Breaking Rule 1 implies that $\tilde{\tilde{\pi}}'^0$ would be chosen by the electorate over π^1. Let $i \in Q_0$. Then,

$$\begin{aligned}
&u_i(\tilde{\tilde{\pi}}'^0)\\
&= (V_l - k) + (V_w - V_l) + \max\{0, \sigma_i(0)\} - \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj - \frac{1}{2}(V_w - V_l)\\
&= (\frac{1}{2} - \hat{p})(V_w - V_l) + \max\{0, \sigma_i(0)\} - \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj\\
&\geq \max\{0, \sigma_i(0)\} - \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj = u_i(\tilde{\pi}^0) \geq u_i(\pi^1),
\end{aligned}$$

where the second equality holds by definition of $\hat{p}$, the first inequality holds since $\frac{1}{2} - \hat{p} \geq 0$, and the last inequality holds since $i \in Q_0$.

Lastly, notice that AS_w weakly prefers the implementation of $\tilde{\tilde{\pi}}'^0$ to $\tilde{\pi}^0$. Indeed,

$$\begin{aligned}
u_0(\tilde{\tilde{\pi}}'^0) &= (V_w - k) - \frac{1}{2}(V_w - V_l) - \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj\\
&\geq - \int_{j\in Q_0} \max\{0, \sigma_j(0)\}\, dj = u_0(\tilde{\pi}^0),
\end{aligned}$$

where the inequality holds by (35).

Finally, since $\tilde{\tilde{\pi}}^0$ is the optimal response for AS_w provided that the project is implemented and AS_w prefers $\tilde{\tilde{\pi}}'^0$ to $\tilde{\pi}^0$, it must be the case that the electorate chooses $\tilde{\tilde{\pi}}^0$ over π^1 and that $u_0(\tilde{\tilde{\pi}}^0) \geq u_0(\tilde{\tilde{\pi}}'^0) \geq u_0(\tilde{\pi}^0)$. Therefore it is optimal for AS_w to choose $\tilde{\tilde{\pi}}^0$, as we claimed.

□

Proof of Proposition 4:

When winners are a majority, the best-response correspondence for the second agenda-setter, a loser, is exactly the same as in the case where losers are a majority. It is therefore given by Lemma 3. Let $\tilde{\pi}^1$, $\tilde{\tilde{\pi}}^1$, Q_0, Q_1, Q_1^l and Q_1^w be the proposals and sets defined in Lemma 3, respectively. Let also $\hat{p}$ be given as in (14).

Suppose now that $p \geq \hat{p} \geq \frac{1}{2}$, so that it is socially efficient to carry out the public project. Given π^0, the proposal by AS_w, and the counter-proposal $\tilde{\tilde{\pi}}^1$, we also consider the auxiliary proposal $\tilde{\pi}'^1 \in \Pi$ defined in (19). By repeating the same steps as in the proof of Case 1 in Lemma 4, and since

$$\frac{q_1^w}{1-q_1^w} \leq \frac{\frac{1}{2}}{1-\frac{1}{2}} \leq \frac{\hat{p}}{1-\hat{p}} = \frac{k-V_l}{V_w-k},$$

it follows that it is optimal for AS_l to choose $\tilde{\pi}^1$. Hence, the public project will not be implemented, even if it is socially desirable to do so.

$\square$

B.3 Without the Agenda-Setter Rule

This section is entirely devoted to the proof of Proposition 5.

Proof of Proposition 5:

We prove both parts of the result.

Part (a): $p < \frac{1}{2}$

In this case, our procedure prescribes that a winner, AS_w, is the first proposer and a loser, AS_l, is the second proposer. Consider an arbitrary proposal $\pi^0 \in \Pi$ by AS_w. Suppose that $\pi_*^1 \in \Pi$ is a best response of AS_l to π^0. We claim that proposals

$$\tilde{\pi}^1 := (g(\pi_*^1), s(\tilde{\pi}^1)), \text{ where } s_j(\tilde{\pi}^1) = s_j(\pi_*^1) \text{ for all } j \in (0,1] \text{ and } s_0(\tilde{\pi}^1) = \hat{s},$$
$$\tilde{\tilde{\pi}}^1 := (g(\pi_*^1), s(\tilde{\tilde{\pi}}^1)), \text{ where } s_j(\tilde{\tilde{\pi}}^1) = s_j(\pi_*^1) \text{ for all } j \in (0,1] \text{ and } s_0(\tilde{\tilde{\pi}}^1) = 0$$

are also best responses to π^0.[61] To see this, note that $\tilde{\pi}^1$ and $\tilde{\tilde{\pi}}^1$ are feasible proposals, i.e, $\tilde{\pi}^1, \tilde{\tilde{\pi}}^1 \in \Pi$, and that, compared to π_*^1, only redistribution within measure zero groups is

[61] We note that Tie-Breaking Rule 1 implies that citizens support a counterproposal if it is identical to the original proposal plus subsidies paid to an arbitrary subset of citizens of measure zero. The reason is that such subsidies have no impact on the individual tax burden of citizens as the subset has zero measure. We stress that Tie-Breaking Rules merely simplify the exposition and avoid the need to discretize subsidy levels.

carried out, which implies $t(\tilde{\pi}^1) = t(\tilde{\tilde{\pi}}^1) = t(\pi^1_*)$ and $u_j(\tilde{\pi}^1) = u_j(\tilde{\tilde{\pi}}^1) = u_j(\pi^1_*)$ for all $j \in (0, 1]$. Consequently, $I(\pi^0, \tilde{\pi}^1) = I(\pi^0, \tilde{\tilde{\pi}}^1) = I(\pi^0, \pi^1_*) = 1$, where the last equality holds by Lemma 1. Therefore all three proposals yield the same utility for AS_l, which confirms the claim.

Now suppose that AS_l chooses π^1 according to the following rule:

$$\pi^1 = \begin{cases} \tilde{\pi}^1 & \text{if } \pi^0 \text{ is such that } g(\pi^1_*) = 0, s_1(\pi^1_*) = \hat{s}, \text{ and } t(\pi^1_*) = 0, \\ \tilde{\tilde{\pi}}^1 & \text{otherwise.} \end{cases} \tag{40}$$

For AS_w, this implies that, for any choice of π^0, the utility he obtains in the finally-voted proposal π^1 is

$$u_0(\pi^1) = \begin{cases} u_0(\tilde{\pi}^1) = \hat{s} & \text{if } g(\pi^1_*) = 0, s_1(\pi^1_*) = \hat{s}, t(\pi^1_*) = 0, \\ u_0(\tilde{\tilde{\pi}}^1) = g(\tilde{\tilde{\pi}}^1)V_w - t(\tilde{\tilde{\pi}}^1) & \text{otherwise.} \end{cases}$$

As $\hat{s} > V_w$, we obtain

$$u_0(\tilde{\pi}^1) > V_w \geq u_0(\tilde{\tilde{\pi}}^1).$$

Thus, if AS_l chooses his response according to the above rule, the public project will not be implemented, since AS_w will prefer to abandon the public project and receive high subsidies in return by proposing a suitable chosen π^0 such that π^1_*, the best response to π^0, satisfies $g(\pi^1_*) = 0$, $s_1(\pi^1_*) = \hat{s}$, $t(\pi^1_*) = 0$.

Finally observe that the rule in (40) is optimal for AS_l because it guarantees him the highest possible utility, that is

$$u_1(\tilde{\pi}^1) = \hat{s} \geq u_1(\pi) \text{ for all } \pi \in \Pi.$$

Hence, we have proved that for any parameter constellation, there always exists an equilibrium in which the public project is not implemented.

Part (b): $p \geq \frac{1}{2}$

In this case, our procedure prescribes that a loser, AS_l, is the first proposer and a winner, AS_w, is the second proposer. Consider an arbitrary proposal $\pi^1 \in \Pi$ by AS_l. Suppose that $\pi^0_* \in \Pi$ is a best response of AS_w to π^1. Analogously to part (a), the proposals

$$\tilde{\pi}^0 := (g(\pi^0_*), s(\tilde{\pi}^0)), \text{ where } s_j(\tilde{\pi}^1) = s_j(\pi^1_*) \text{ for all } j \in [0,1) \text{ and } s_1(\tilde{\pi}^0) = \hat{s},$$

$$\tilde{\tilde{\pi}}^0 := (g(\pi^0_*), s(\tilde{\tilde{\pi}}^0)), \text{ where } s_j(\tilde{\tilde{\pi}}^1) = s_j(\pi^1_*) \text{ for all } j \in [0,1) \text{ and } s_1(\tilde{\tilde{\pi}}^1) = 0$$

are also best responses to π^1. Now suppose that AS_w chooses π^0 according to the following rule:

$$\pi^0 = \begin{cases} \tilde{\pi}^0 & \text{if } \pi^1 \text{ is such that } g(\pi^0_*) = 1, s_0(\pi^0_*) = \hat{s}, \text{ and } t(\pi^0_*) = 0, \\ \tilde{\tilde{\pi}}^0 & \text{otherwise.} \end{cases} \tag{41}$$

For AS_l, this implies that for any choice of π^1, the utility he obtains from proposal π^0 is

$$u_1(\pi^0) = \begin{cases} u_1(\tilde{\pi}^0) = \hat{s} + V_l - k & \text{if } g(\pi^0_*) = 1, s_0(\pi^0_*) = \hat{s}, t(\pi^0_*) = 0, \\ u_1(\tilde{\tilde{\pi}}^0) = g(\tilde{\tilde{\pi}}^0)V_l - t(\tilde{\tilde{\pi}}^0) & \text{otherwise.} \end{cases}$$

As $\hat{s} > k - V_l$, we obtain

$$u_1(\tilde{\pi}^0) > 0 \geq u_1(\tilde{\tilde{\pi}}^0).$$

Thus, if AS_w chooses his response according to the above rule, the public project will always be implemented, since AS_l will prefer it and will receive high subsidies in return by proposing a suitably-chosen π^1 such that π^0_*, the best response to π^1, satisfies $g(\pi^0_*) = 1$, $s_0(\pi^0_*) = \hat{s}$, $t(\pi^0_*) = 0$. Finally, note that the rule in (41) is optimal for AS_w because it guarantees him the highest possible utility, that is

$$u_0(\tilde{\pi}^0) = \hat{s} + V_w - k \geq u_0(\pi) \text{ for all } \pi \in \Pi.$$

Hence, we have proved that for any parameter constellation, there always exists an equilibrium in which the public project is implemented.

□

B.4 When the tax system is flexible

In the last part of this appendix, the tax schedule is no longer limited to being uniform across voters but is only required to be non-negative. That is, we consider that $t(\pi)$ is a function on $[0, 1]$ such that $t_j(\pi) \geq 0$ for all $j \in [0, 1]$ and

$$\int_0^1 t_j(\pi)dj = g(\pi)k + \int_0^1 s_j(\pi)dj.$$

We assume that $p < \frac{1}{2}$, i.e., losers account for a majority of the population. Recall that in this case our procedure requires the first proposer to be a winner and the second proposer to be a loser.

We next prove the main result of the section.

Proof of Proposition 6:

When $p < \frac{1}{2}$, a winner is the first to propose, followed by an amendment by a loser. For a given proposal $\pi \in \Pi$ and a given individual $j \in [0, 1]$, we have the following net utility changes:

$$u_j(\pi) = g(\pi)v_j + s_j(\pi) - t_j(\pi).$$

We assume that

$$V_l > 0, \tag{42}$$

and we show that, when (42) holds, inefficiencies may arise. Indeed, consider the proposal π^* with $g(\pi^*) = 1$ and the subsidy and tax schemes given respectively by

$$s_j(\pi^*) = \begin{cases} \hat{s} & \text{for } j \in \left(0, \frac{1}{2}\right], \\ 0 & \text{for all other } j, \end{cases}$$

and[62]

$$t_j(\pi^*) = \begin{cases} 2k + \hat{s} & \text{for } j \in \left(\frac{1}{2}, 1\right), \\ 0 & \text{for all other } j. \end{cases}$$

With the above proposal, we obtain

$$u_j(\pi^*) = \begin{cases} V_w & \text{for } j = 0, \\ V_w + \hat{s} & \text{for } j \in (0, p], \\ V_l + \hat{s} & \text{for } j \in \left(p, \frac{1}{2}\right], \\ V_l - 2k - \hat{s} & \text{for } j \in \left(\frac{1}{2}, 1\right), \\ V_l & \text{for } j = 1. \end{cases}$$

Let $\pi \in \Pi$ be an arbitrary proposal. Then

$$u_j(\pi) = g(\pi)v_j + s_j(\pi) - t_j(\pi) \leq v_j + \hat{s} = u_j(\pi^*) \text{ for all } j \in \left(0, \frac{1}{2}\right].$$

Moreover,

$$u_0(\pi) = g(\pi)V_w - t_0(\pi) \leq V_w = u_0(\pi^*).$$

and

$$u_1(\pi) = g(\pi)V_l - t_1(\pi) \leq V_l = u_1(\pi^*).$$

That is, under π^* both agenda-setters and all voters $j \in \left(0, \frac{1}{2}\right]$ obtain their maximum possible utility. As a consequence of Tie-Breaking Rules 1 and 2, it immediately follows that there is an equilibrium in which both agenda-setters will propose π^*, leading to

[62] We assume that the cap for taxes is larger than $2k + \hat{s}$.

public project implementation regardless of whether it is socially efficient, coupled with strictly aggregate subsidies. That is, when (42) holds, inefficiencies of types 1 and 3 will arise.

□

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Appendix C

In this Appendix, we prove Propositions 7, 8, 9 and 10. For the sake of brevity, some formal details of the proofs are omitted, as they have already been discussed in Appendix A or Appendix B. For instance, although no tie-breaking rules are specified, we assume that they exist in the spirit of the rest of the paper. The multiplicity of best responses by the second agenda-setter is also neglected, as the final outcomes do not depend on specific choices. In this appendix, we assume that $p < \frac{1}{2}$, i.e., losers account for more than half of the population.[63]

C.1 Divisible public projects

Proof of Proposition 7:

Mimicking the argument in Appendix A, it can be shown that the best response of the second agenda-setter is characterized as follows. Let $\pi^{1,\rho}$ be the optimal counterproposal (for AS_l) among those that suggest a level $\rho \in [0,1]$ of provision of the public project when there is a binary decision between "no public project" and "public project ρ". Then, AS_l chooses the counter-proposal $\pi^{1,\rho}$ that maximizes his utility.[64]

Next, we focus on the optimal strategy of the first agenda-setter. We set $g(\pi) = \rho$ if he makes a proposal π involving the level ρ. Let $\sigma_i(\rho)$ be the minimal amount of resources that must be given to voter i to vote for a counter-proposal associated with level ρ, given that π^0 has been made initially. Also, for each $\rho \in [0,1]$, let Q_ρ be a set of measure $\frac{1}{2}$ composed of all voters $i \in (0,1)$ for which $\sigma_i(\rho)$ is minimal, and let

$$Q_\rho^w := \{j \in Q_\rho \mid j \in (0,p]\},$$
$$Q_\rho^l := \{j \in Q_\rho \mid j \in (p,1)\}.$$

Additionally, we define

$$\hat{p}(\rho) = \begin{cases} \frac{k(\rho)-V_l(\rho)}{V_w(\rho)-V_l(\rho)} & \text{if } \rho \in (0,1], \\ 1 & \text{if } \rho = 0. \end{cases}$$

First, suppose that AS_w proposes π^0 denoted by $g(\pi^0) = \rho^0$ with $pV_w(\rho^0)+(1-p)V_l(\rho^0) < k(\rho^0)$. Given $\rho \in (0,1]$ and $\pi^{1,\rho}$, let $\tilde{\pi}'^{1,\rho} \in \Pi$ be a proposal that comprises $g(\tilde{\pi}'^{1,\rho}) = 0$

[63] Analogous results as those shown in this appendix can be derived in the case $p \geq \frac{1}{2}$.
[64] Recall that Lemma 1 applies.

and the following subsidy scheme:

$$s_j(\tilde{\pi}'^{1,\rho}) = \begin{cases} \max\{0, \sigma_j(\rho)\} + \dfrac{V_w(\rho) - k(\rho)}{1 - q^w_\rho} & \text{for } j \in Q^w_\rho, \\ \max\{0, \sigma_j(\rho)\} & \text{for } j \in Q^l_\rho, \\ 0 & \text{for all other } j. \end{cases}$$

When $pV_w(\rho) + (1-p)V_l(\rho) < k(\rho)$, by replicating the steps carried out in the proof of Lemma 4 – see Case 1 –, it can be shown that $\tilde{\pi}'^{1,\rho}$ would win a vote against π^0, if $\pi^{1,\rho}$ would do so, and that $u_1(\pi^{1,0}) \geq u_1(\tilde{\pi}'^{1,\rho}) > u_1(\pi^{1,\rho})$. Therefore, if AS_w proposes π^0 that is socially welfare-decreasing, then either it is optimal for AS_l to choose $\pi^{1,0}$ as a counter-proposal or it is optimal for AS_l to chose a proposal $\pi^{1,\rho}$ that carries a level ρ of public project provision which is welfare-improving. We note that in either case, the counter-proposal chosen by AS_l is voted and carried out, and that it might entail positive subsidies for a set of citizens of non-zero measure.

Second, let now ρ^0 such that $pV_w(\rho^0) + (1-p)V_l(\rho^0) \geq k(\rho^0)$ and consider the proposal π^{0,ρ^0}_* defined as

$$g(\pi^{0,\rho^0}_*) = \rho^0 \text{ and } \quad s_j(\pi^{0,\rho^0}_*) = \begin{cases} \frac{(\frac{1}{2}-p+\hat{p}(\rho^0))[V_w(\rho^0)-V_l(\rho^0)]}{1-(\frac{1}{2}+\hat{p}(\rho^0))} & \text{for } j \in (0,p]\,, \\ \frac{(1-p)[V_w(\rho^0)-V_l(\rho^0)]}{1-(\frac{1}{2}+\hat{p}(\rho^0))} & \text{for } j \in \left(p, \frac{1}{2}+\hat{p}(\rho^0)\right]. \\ 0 & \text{for all other } j, \end{cases}$$

It is a matter of simple calculations to derive that

$$\int_0^1 s_i(\pi^{0,\rho^0}_*)di = \frac{(\frac{1}{2} - p + \hat{p}(\rho^0))[V_w(\rho^0) - V_l(\rho^0)]}{1 - (\frac{1}{2} + \hat{p}(\rho^0))}$$

and

$$u_j(\pi^{0,\rho^0}_*) = \begin{cases} V_w(\rho^0) - k(\rho^0) & \text{for } j \in (0,p]\,, \\ V_w(\rho^0) - k(\rho^0) & \text{for } j \in \left(p, \frac{1}{2}+\hat{p}(\rho^0)\right], \\ V_l(\rho^0) - k(\rho^0) - \int_0^1 s_i(\pi^{0,\rho^0}_*)di & \text{for all other } j, \end{cases}$$

If the first agenda-setter, AS_w, proposes π^{0,ρ^0}_*, the second agenda-setter, AS_l, chooses $\pi^{1,\rho}$, with $\rho \in [0,1]$, in response. We distinguish three cases, depending on ρ.

- **Case 1:** $\rho = \rho^0$

  By replicating the steps carried out in the proof of Lemma 4 – see Case 2 –, we observe that π^{1,ρ^0} is characterized by $g(\pi^{1,\rho^0}) = \rho^0$ and $s_j(\pi^{1,\rho^0}) = 0$ for all $j \in [0,1]$. Hence,

  $$u_1(\pi^{1,\rho^0}) = V_l(\rho^0) - k(\rho^0).$$

- **Case 2:** $\rho < \rho^0$

  In this case, it can be shown that $\pi^{1,\rho}$ is such that $g(\pi^{1,\rho}) = \rho$ and

  $$s_j(\pi^{1,\rho}) \geq \frac{V_w(\rho^0) - k(\rho^0) - (V_w(\rho) - k(\rho))}{1 - \hat{p}(\rho^0)} \qquad \text{for all } j \in (0, \hat{p}(\rho^0)].$$

  This implies

  $$\begin{aligned} u_1(\pi^{1,\rho}) &\leq V_l(\rho) - k(\rho) - \frac{\hat{p}(\rho^0)}{1 - \hat{p}(\rho^0)} \left[V_w(\rho^0) - k(\rho^0) - (V_w(\rho) - k(\rho))\right] \\ &= \left(-\hat{p}(\rho) + \frac{\hat{p}(\rho^0)}{1 - \hat{p}(\rho^0)}(1 - \hat{p}(\rho))\right)(V_w(\rho) - V_l(\rho)) - \frac{\hat{p}(\rho^0)}{1 - \hat{p}(\rho^0)} \left[V_w(\rho^0) - k(\rho^0)\right] \\ &\leq -\frac{\hat{p}(\rho^0)}{1 - \hat{p}(\rho^0)} \left[V_w(\rho^0) - k(\rho^0)\right] = V_l(\rho^0) - k(\rho^0) \leq u_1(\pi^{1,\rho^0}), \end{aligned}$$

  where the last inequality holds since $\hat{p}(\cdot)$ is non-increasing.

- **Case 3:** $\rho > \rho^0$

  In that case, it holds that

  $$u_1(\pi^{1,\rho}) \leq V_l(\rho) - k(\rho) < V_l(\rho^0) - k(\rho^0),$$

  where the strict inequality holds since $V_l(\cdot) - k(\cdot)$ is a strictly decreasing function.

Therefore, if we modify Tie-Breaking Rule 2 such that in case of tie, preference is given to those proposals with highest ρ, we conclude that if AS_w proposes π_*^{0,ρ^0}, it is optimal for AS_l to choose π^{1,ρ^0}, as defined in Case 1, as a counter-proposal, which yields

$$u_0(\pi^{1,\rho^0}) = V_w(\rho^0) - k(\rho^0). \tag{43}$$

Finally, let ρ^0 be a (weakly) welfare-improving level of public good provision and let ρ be a welfare-decreasing level of public good provision. Then, assume that $\pi^{1,\rho}$ is chosen by AS_l as a response to a proposal π^0 chosen by AS_w which prescribes the level ρ of public project. Since the same level of public good provision can be attained as an outcome if the first agenda-setter proposes π_*^{0,ρ^0} instead of π^0, and π_*^{0,ρ^0} induces a response by AS_l which carries no positive subsidies for certain, the first agenda-setter will always prefer in equilibrium to propose π_*^{0,ρ^0} instead of π^0.

$\square$

C.2 When winners and losers are heterogeneous

Proof of Proposition 8:

The present proof is a modification of the proof of Lemma 4 in Appendix A. Therefore, for the sake of brevity, we only describe the modifications necessary to establish the result. We note that $q_1^w + q_1^l = \frac{1}{2}$.[65] We distinguish two cases.

Case 1: $p < \hat{p}$

The proposal $\tilde{\pi}'^1 \in \Pi$ is now defined by $g(\tilde{\pi}'^1) = 0$ and the following subsidy scheme:

$$s_j(\tilde{\pi}'^1) = \begin{cases} \max\{0, \sigma_j(1)\} + v_j + \frac{1}{1-q_1^w}\left(\int_{Q_1^w} v_i di - k\right) + x^w & \text{for } j \in Q_1^w, \\ \max\{0, \sigma_j(1)\} + y^w & \text{for } j \in Q_1^l, \\ 0 & \text{for all other } j, \end{cases}$$

where

$$x^w = \frac{q_1^l}{1-q_1^w} 2\delta \text{ and } y^w := 2\delta$$

are the solutions of the following system of equations:

$$\begin{cases} 0 & = x^w - q_1^w x^w - q_1^l y^w \\ \frac{1}{1-q_1^w}\delta & = y^w - q_1^w x^w - q_1^l y^w. \end{cases}$$

We claim that $u_i(\tilde{\pi}'^1) \geq u_i(\pi^0)$ for all $i \in Q_1$. Recall that by construction, $u_i(\tilde{\tilde{\pi}}^1) \geq u_i(\pi^0)$ for all $i \in Q_1$. On the one hand, consider $i \in Q_1^w$. Then

$$\begin{aligned} & u_i(\tilde{\pi}'^1) - u_i(\tilde{\tilde{\pi}}^1) \\ = & \frac{1}{1-q_1^w}\left(\int_{Q_1^w} v_i di - k\right) - \int_{Q_1^w}\left(v_z + \frac{1}{1-q_1^w}\left(\int_{Q_1^w} v_i di - k\right)\right) dz + k = 0. \end{aligned}$$

[65] Because non-subsidized project-losers are not a homogeneous subgroup of the society anymore – as they were in the other parts of the paper – one has to pay more attention to their voting behavior. In particular, it is not guaranteed – as it was the case in the other parts of the paper – that they vote uniformly and similar as the second agenda-setter, AS_l.

On the other hand, consider $i \in Q_1^l$. Then

$$
\begin{aligned}
&u_i(\tilde{\pi}'^1) - u_i(\tilde{\tilde{\pi}}^1) \\
=&k - v_i - \int_{Q_1^w} \left(v_z + \frac{1}{1-q_1^w} \left(\int_{Q_1^w} v_i di - k \right) \right) dz + y^w - q_1^w x^w - q_1^l y^w \\
\geq&(k - V_l) - \delta - \frac{1}{1-q_1^w} \int_{Q_1^w} v_i di + \frac{q_1^w}{1-q_1^w} k + \frac{1}{1-q_1^w} \delta \\
\geq&(k - V_l) - \frac{q_1^w}{1-q_1^w} (V_w - k) > 0,
\end{aligned}
$$

where the strict inequality follows from (21), and the penultimate inequality holds since $\delta \geq 0$ and $q_1^w \leq p < \frac{1}{2}$. Similarly, it can be shown that AS_l will prefer the implementation of $\tilde{\pi}'^1$ to $\tilde{\tilde{\pi}}^1$, i.e., $u_1(\tilde{\pi}'^1) \geq u_1(\tilde{\tilde{\pi}}^1)$. The rest of the proof coincides with the proof of Lemma 4 for Case 1.

Case 2: $p \geq \hat{p}$

Let $\hat{p}^* := p$. Instead of the proposal defined in (15), we now consider the proposal π_*^0 defined such that $g(\pi_*^0) = 1$ and[66]

$$
s_j(\pi_*^0) = \begin{cases} \frac{\frac{1}{2}-p+\hat{p}^*}{1-(\frac{1}{2}+\hat{p}^*)}(V_w + \delta) - \frac{1}{1-(\frac{1}{2}+\hat{p}^*)} \int_p^{\frac{1}{2}+\hat{p}^*} v_i di & \text{for } j \in (0, p], \\ \frac{1-p}{1-(\frac{1}{2}+\hat{p}^*)}(V_w + \delta) - v_j - \frac{1}{1-(\frac{1}{2}+\hat{p}^*)} \int_p^{\frac{1}{2}+\hat{p}^*} v_i di & \text{for } j \in \left(p, \frac{1}{2} + \hat{p}^*\right], \\ 0 & \text{for all other } j. \end{cases} \tag{44}
$$

We note that π_*^0 is well defined. With simple algebraic manipulations, we can verify that

$$
\int_0^1 s_i(\pi_*^0) di = \frac{\frac{1}{2} - p + \hat{p}^*}{1 - (\frac{1}{2} + \hat{p}^*)} (V_w + \delta) - \frac{1}{1 - (\frac{1}{2} + \hat{p}^*)} \int_p^{\frac{1}{2}+\hat{p}^*} v_i di
$$

and

$$
u_j(\pi_*^0) = \begin{cases} v_j - k & \text{for } j \in (0, p], \\ V_w - k + \delta & \text{for } j \in \left(p, \frac{1}{2} + p\right], \\ v_j - k - \int_0^1 s_i(\pi_*^0) di & \text{for } j \in \left(\frac{1}{2} + p, 1\right). \end{cases}
$$

It follows that, for $i \in (0, p]$ and $j \in \left(p, \frac{1}{2} + p\right]$,

$$
\sigma_i(0) - \sigma_j(0) = u_i(\pi_*^0) - u_j(\pi_*^0) \leq V_w + \delta - k - (V_w - k + \delta) = 0.
$$

On the one hand, we obtain $(0, p] \subseteq Q_0$. Moreover, since $(0, p]$ consists of the continuum of all winners and project valuations of citizens in $(0, p]$ are independently drawn from

[66] We assume that $\hat{s}$ is sufficiently large for $s_j(\pi_*^0)$, as defined in (44), not to violate the upper bound on subsidies.

a uniform distribution with zero mean, by the law of large numbers we have

$$\int_0^p s_j(\tilde{\pi}^1)dj \geq p\frac{V_w - k}{1-p} = \frac{p}{1-p}(1-\hat{p})(V_w - V_l).$$

Hence,

$$u_1(\tilde{\pi}^1) \leq -\frac{p}{1-p}(1-\hat{p})(V_w - V_l). \tag{45}$$

On the other hand, it is straightforward to deduce that $Q_1 = (0, p] \cup (\frac{1}{2} + p, 1)$ and $s_j(\tilde{\tilde{\pi}}^1) = 0$ for all $j \in [0, 1]$, so

$$u_1(\tilde{\tilde{\pi}}^1) = v_1 - V_l + (V_l - k) = v_1 - V_l - \hat{p}(V_w - V_l). \tag{46}$$

Since we are assuming (2), we have

$$v_1 \geq V_l - \frac{p - \hat{p}}{1-p}(V_w - V_l). \tag{47}$$

Then, from (45), (46) and (47), it follows that $u_1(\tilde{\tilde{\pi}}^1) \geq u_1(\tilde{\pi}^1)$. The remaining steps necessary to show that it is optimal for AS_w to choose π_*^0 coincide with those in the proof of Lemma 4 for Case 2.

□

Proof of Proposition 9:

We focus only on the case $p \geq \hat{p}$, as the case $p < \hat{p}$ is exactly the same as in Proposition 8. Let us assume now that

$$\hat{p}^* := \hat{p} + \frac{1-\hat{p}}{V_w - V_l + \delta}\delta \leq p. \tag{48}$$

Note that if $p < \hat{p}$, the above inequality holds if δ is sufficiently small. Then, we can define the proposal π_*^0 as in (44) with $\hat{p}^*$ defined now as in (48). By following the same exact steps as in Case 2 in Proposition 8, we deduce that

$$\int_0^{\hat{p}^*} s_j(\tilde{\pi}^1)dj \geq \frac{V_w - k}{1-\hat{p}^*} = \frac{1-\hat{p}}{1-\hat{p}^*}(V_w - V_l).$$

Therefore,

$$u_1(\tilde{\pi}^1) \leq -\hat{p}^*\frac{1-\hat{p}}{1-\hat{p}^*}(V_w - V_l) \leq -\delta - \hat{p}(V_w - V_l) \leq v_1 - V_l - \hat{p}(V_w - V_l) = u_1(\tilde{\tilde{\pi}}^1), \tag{49}$$

where the second inequality holds by (48). We stress that (49) holds regardless of the exact project valuation of the second agenda-setter.

□

C.3 Finite number of agents

Proof of Proposition 10:

Let $\Omega = \{0, \dots, n\}$ be the finite set of voters, with $\{0, \dots, p\}$ and $\{p+1, \dots, n\}$ denoting the sets of winners and losers, respectively. The total number of voters, $n+1$, is assumed to be odd.[67] We also consider that $p < \frac{n}{2}$. The logic for the case $p \geq \frac{n}{2}$ is analogous. Without loss of generality, let 0 or equivalently AS_w, be the first agenda-setter and n, or equivalently AS_l, be the second agenda-setter. We denote by π^0 an arbitrary proposal by AS_w and by π^n an arbitrary proposal by AS_l. Let also π^0_* be the proposal defined by $g(\pi^0_*) = 0$ and

$$s_j(\pi^0_*) = \begin{cases} \hat{s} & \text{for } j \in \{0, \dots, \frac{n}{2}\} \cup \{n\}, \\ 0 & \text{for all other } j. \end{cases}$$

As in the case where there is a continuum of voters, any proposal π^n that is a best-response to π^0 for the second agenda-setter can be obtained as follows: *(i)* identify a proposal, denoted by $\tilde{\tilde{\pi}}^n(\pi^0)$, which yields the highest utility among all proposals that suggest the implementation of the public project, *(ii)* identify a proposal, denoted by $\tilde{\pi}^n(\pi^0)$, which yields the highest utility among all proposals that do not suggest the implementation of the public project, and *(iii)* choose a counter-proposal, $\pi^n \in \{\tilde{\tilde{\pi}}^n(\pi^0), \tilde{\pi}^n(\pi^0)\}$, depending on which of both yields a higher utility for the second agenda-setter.[68]

To calculate each of the two counter-proposals, $\tilde{\pi}^n(\pi^0)$ or $\tilde{\tilde{\pi}}^n(\pi^0)$, the second agenda-setter, AS_l, identifies a set of voters, Q_0 and Q_n, respectively. Together with AS_l, each of the two sets form a bare majority – i.e. they account for exactly $\frac{n}{2}+1$ citizens – and are made up of all voters to whom a (weakly) positive subsidy is given in the corresponding proposal. Moreover, the aggregate subsidies to motivate members of the sets to support the corresponding counter-proposal against π^0 are minimal. As in the continuum case, the minimal subsidy that must be given to each voter to make him vote for π^n depends only on $g(\pi^n)$ and π^0. We denote this minimum amount by $\sigma_j(g(\pi^n))$, $j \in \Omega$. Then, for all $j \in \Omega$,

$$\sigma_j(g(\pi^n)) - \sigma_0(g(\pi^n)) = s_j(\pi^0) - s_0(\pi^0) + (g(\pi^0) - g(\pi^n))(v_j - V_w),$$

[67] The proof can be adapted to the case where $n+1$ is even.

[68] Both proposals exist as the maximization problems always have a solution. We assume that Lemma 1 applies.

and $\sigma_j(g(\pi^n))$ increases for all $j \in \Omega$ with the aggregate level of subsidies in π^n, and decreases with the aggregate level of subsidies in π^0. Importantly, the choice of Q_0 (resp. Q_1) does not depend on the subsidy given to AS_l under his own proposal. Hence, from the point of view of AS_l, his own subsidy in π^n and the choice whom to subsidize are independent decisions. Moreover, it is easy to see that AS_l will always try to offer the possible maximum subsidy $\hat{s}$ to himself by increasing the subsidies to each of the citizens in $Q_{g(\pi^n(\pi^0))}$, so that they remain neutral with respect to the increase in AS_l's own subsidy.

For any arbitrary proposal by the first agenda-setter, π^0, let $R(\pi^0)$ denote the best-response correspondence of the second agenda-setter. Let also $\pi^1 \in R(\pi^0)$.

Given that AS_w anticipates the best-response correspondence of AS_l, the choice of π^0 by AS_w must satisfy the following property: conditional on $g(\pi^n)$, AS_w will choose a subsidy scheme in π^0 such that he belongs to $Q_{g(\pi^n)}$. This implies that in equilibrium it must be the case that *(i)* $\sigma_j(g(\pi^n)) - \sigma_0(g(\pi^n)) \geq 0$ for all $j \notin Q_{g(\pi^n)}$, *(ii)* $\sigma_0(g(\pi^n))$ is as high as possible, and *(iii)* the aggregate subsidies in π^n are as low as possible. Moreover, in any potential equilibrium AS_w will always choose a subsidy scheme such that[69]

$$\sigma_j(g(\pi^n)) = \sigma_0(g(\pi^n)) \text{ for all } j \notin Q_{g(\pi^n)}. \tag{50}$$

Next, suppose that AS_l chooses $\pi^n \in R(\pi^0)$ according to the following rule:

$$\pi^n = \begin{cases} \pi^1_* \in \arg\max_{\pi^n \in R(\pi^0)} u_0(\pi^n) & \text{if } \pi^0 = \pi^0_*, \\ \pi^1_* \in \arg\min_{\pi^n \in R(\pi^0)} u_0(\pi^n) & \text{otherwise.} \end{cases} \tag{51}$$

When $\pi^0 = \pi^0_*$, it follows from straightforward algebraic manipulations that

$$s_j(\tilde{\pi}^n(\pi^0)) = \begin{cases} \frac{1}{2} \cdot \hat{s} & \text{for } j = 0, \\ \hat{s} & \text{for } j = n, \\ 0 & \text{for all } j \in \{1, \ldots, n-1\}. \end{cases}$$

As a consequence, if n is finite but large enough,

$$u_n(\tilde{\pi}^n(\pi^0)) = \frac{n - \frac{1}{2}}{n+1} \cdot \hat{s} > \hat{s} + V_l - k \geq s_1(\tilde{\pi}^n(\pi^0)) + V_l - k \geq u_n(\tilde{\tilde{\pi}}^n(\pi^0)),$$

so that AS_l will choose $\tilde{\pi}^n(\pi^0_*)$ in response to π^0_*, leading to utilities of $u^*_0 := \frac{\frac{n}{2}-1}{n+1} \cdot \hat{s}$ for the first agenda-setter and $u^*_n := \frac{n-\frac{1}{2}}{n+1} \cdot \hat{s}$ for the second agenda-setter.

[69] We assume that AS_w does not choose weakly undominated strategies.

The selection rule in (51) together with Equation (50) implies that AS_w will never obtain a positive subsidy in the second proposal, π^n, unless he proposes π^0_* in the first round. Thus, for any choice of π^0, the best response π^n chosen by AS_l according to (51) satisfies

$$u_0(\pi^1) = \begin{cases} u_0(\pi^n) = u^* & \text{if } \pi^0 = \pi^0_*, \\ u_0(\pi^n) \leq V_w - k & \text{otherwise.} \end{cases}$$

Moreover, since AS_w can always guarantee himself u^*_0 by proposing π^0_*, AS_l cannot gain more than u^*_1 in any equilibrium where he chooses according to (51). Note that the preceding construction guarantees the existence of an equilibrium in which AS_l chooses according to (51).

Hence, we have proved that for any parameter constellation, there always exists an equilibrium in which the public project is not implemented.

□