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ABSTRACT

Bargaining with Informational Externalities in a Market Equilibrium*

This paper studies a dynamic bargaining model with informational externalities between bargaining pairs. Two principals bargain with their respective agents about the price they will pay for their work while its cost is agents' private information and correlated between them. The principals benchmark their agents against each other by making the same offers in the equilibrium even if this involves delaying or advancing the agreement compared to the autarky. When principals compete in complements this pattern is reinforced while under competition in substitutes the principals trade off the benefits of differentiation in the product market against the cost of the agents' rent.

JEL Classification: C78, D82, D83 and L10
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1 Introduction

How should principals bargain with their agents when there are informational linkages between the negotiations? Consider manufacturers that want their exclusive retailers to undertake a demand-increasing activity which has costs unknown to manufacturers but correlated between the retailers. A manufacturer can wait and see what other retailers do while a retailer anticipates what they will do and might agree earlier. When is the agreement reached? Is it inefficiently delayed or advanced? How does the nature of competition between the manufacturers affect the timing?

A bargaining game with informational externalities is a feature of quite a few situations in real life. Consider unions bargaining with firms. The firms are privately informed of their profits, their market prospects, etc. This information is correlated across firms in the same industry. Then, each union may delay the agreement in order to learn the outcome of negotiations in other firms, see Gu and Kuhn (1998). Waiting too long, however, can be costly as the suppliers and consumers may switch to other firms. Another example is an acquiring company buying a target company in which case the target may try to exaggerate its value. Among other tools, acquirers typically use comparable transactions method to infer the target value which consists of looking at as similar and recent M&As as possible (Rosenbaum and Pearl (2009)). This is particularly important if the acquirers are new to the market, for example, foreign companies entering a local market.

Regulators routinely use international benchmarking to understand better the scope for cost reduction, efficiency improvements and reform.[1] The regulated companies may try inflate the cost or resist the reform.[2] This information might be correlated between the regulated firms in similar countries. Then, each regulator might wish to change or postpone the reform in order to be able to draw lessons from the experience of the other country. Finally, firms procuring a new cost-decreasing technology from their suppliers may not know its cost. Each firm can then wait and observe if other firms obtain a similar technology which helps in estimating its cost.

[1] See, for example, Productivity Commission (1999) for telecoms, Jamasb and Pollitt (2003) for electricity distribution utilities, Civity (2012) for train operators.

[2] For example, in 1998 Telstra, the Australian incumbent telecom firm, claimed its cost of the Universal Service Obligation to be A$1.8 billion which represented about 9% of the industry revenues while in the US it is about 1%. An independent study found it about twice lower, see Productivity Commission (1999) and Gibson Quai & Associates and Ovum (2000).

In our model, there are two identical principal-agent pairs, or hierarchies, A and B. Each agent deals only with his respective principal. The agent's cost is the agent's private information; principals know only its cost distribution. Each agent works at most once during the relationship by which we mean exerting the demand-increasing effort, selling the new technology, agreeing to the acquirer's offer, etc. There are two periods, and all participants share a common discount factor. Inside each hierarchy, the bargaining proceeds à la Sobel and Takahashi (1983) and our autarky benchmark (a single hierarchy) is thus exactly their solution. In period 1, each principal offers a price to her agent that she is ready to pay for his work in that period. If the agent accepts it, she works; the principal obtains her revenues and the game ends for this hierarchy. If the agent rejects the offer, the game moves to period 2. The principal observes whether the other agent has worked or not and makes another offer to her agent that he is free to accept or reject. The principals do not have commitment power and, therefore, the second-period offer is always ex post optimal.

In any equilibrium, there is a cut-off level of cost at which the agent is indifferent between accepting the first-period offer and rejecting it in order to get a second-period offer. Thus in period 1, by his decision each agent reveals whether his cost is above or below this cut-off. Then, each principal has an incentive to delay the agreement in order to obtain information about the cost of the other agent and, because they are correlated, about the cost of her own agent. However, the delay is costly because of discounting.

We first consider the model without competition between the principals as the model with only informational externalities is new and of an independent interest. In the simplest case the costs of the two agents are the same. The only feature of the contracting in each hierarchy that is relevant for the other principal is the information generated in period 1, i.e., the cut-off level of cost and we look for the equilibria in these cut-offs. We find that there are multiple symmetric equilibria, that is, the same cut-off is used in both hierarchies. The principals benchmark their agents against each other: if one agent deviates from the equilibrium behavior and rejects a first-period offer which is above his cost, the other agent will still accept the offer in period 1 and, therefore, the principal will detect the deviation and will not improve her offer in period 2. The equilibrium cut-off may be higher or lower than in the benchmark case of autarky. Intuitively, a small deviation from the autarky cut-off in either direction in order to match the cut-off of the other hierarchy brings a first-order gain for the principal as it allows to benchmark the agent but otherwise creates only a second-order loss.[3]

Efficiency in our model is monotonically increasing in the cut-off since the delay in contracting occurs only for rent extraction reasons. It is affected by informational externalities via two effects. The first effect is a standard one: the possibility to learn in the future makes the principals to delay the agreement, which harms efficiency. The second effect is strategic and increases efficiency. Since offers in period 2 are made under better information, they give a lower rent to the agents in period 2 and, therefore, the agents become "softer", i.e., more likely to accept a given offer, in period 1. As discussed above, the equilibrium cut-off may be higher or lower than the autarky one and, therefore, informational externalities may increase or decrease efficiency. In the profit-maximizing equilibrium, i.e., the equilibrium on which the two principals would like to coordinate, the comparison with the autarky depends on how fast the principal's value of contracting falls from period 1 to period 2.

The competition between the two principals introduces a new effect in their relationship. Under complete information about the agent, when the competition is in substitutes, the principals prefer to differentiate which in our model means setting different cut-offs, that is, an asymmetric equilibrium. When the competition is in complements, the opposite holds and the principals coordinate on the same cut-off in any equilibrium. Informational externalities, as we explained above, make the principals benchmark their agents against each other. Thus, in the substitutes case the principals face a trade-off between benchmarking the agents in order to save on their rent and differentiating to increase profits in the market. The resolution of this trade-off depends on the strength of competition: when it is weak, the benchmarking effect dominates while the differentiation effect dominates when it is strong. In the complements case the two effects are aligned and in any equilibrium the same cut-off is set.

We then analyze the effect of substitutability/complementarity in competition on

[3] We then introduce imperfect correlation which we model as the probability of the two types being the same while being independent with the complementary probability. We show that our model is continuous in that probability, that is, moves smoothly in all the respects from the autarky to the perfect correlation case as that probability goes from 0 to 1. Interestingly, we find that there might be less delay with higher correlation.

the efficiency of equilibria relative to the case of complete information. The distortion caused by the asymmetric information is due to the fact that principals contract with some types in the second period instead of the first one and with some types they do not contract at all. The profit-maximizing equilibrium is determined by the joint contracting value, that is, the difference in the payoffs when both principals contract with the agent and when both do not. This value increases fast as competition changes from strong substitutes to strong complements and the profit-maximizing equilibrium thus becomes relatively more efficient as competition becomes less in substitutes or more in complements. Also, asymmetric equilibria are in general more efficient than symmetric ones. The reason is that in an asymmetric equilibrium principals learn more about their agents since one of them does not delay the contracting in order to learn from the other. If none of the agents agreed in the first period, the second-period offers are then high since they incorporate the fact that a high first-period offer was rejected. Other effects of substitutability/complementarity (for example, the result that the autarky in terms of efficiency is between the cases of substitutes and complements) are more ambiguous.

The rest of the paper is organized as follows. Below we discuss the related literature. Section 2 describes the model and the autarky benchmark. In Section 3 the principals do not compete but the types may not be perfectly correlated. Competition is studied in Section 4. A few alternative models and extensions are informally discussed in Section 5 such as the full-commitment case, a model with a single principal contracting with two agents and the case of more than two bargaining pairs. Section 6 concludes. The proofs are in the Appendix.

1.1 Related literature

This paper belongs to the literature on dynamic adverse selection problems with some outside information. The two most closely related papers are Gu and Kuhn (1998) and Drugov (2007). Gu and Kuhn (1998) study simultaneous bargaining of several firms with their unions. They drastically simplify the model by assuming that the principal (union) can make an offer only once (but can choose when) and, therefore, the agent (firm) cannot strategically reject the offer which is the heart of any real bargaining problem.[4] In Drugov (2007) the agent can strategically reject the principal's offer in an anticipation of a higher offer in the future. The model differs quite a lot as there are only two types of agent, but the quantity produced is continuous. The presence of both time and quantity distortions makes the model rather difficult. Neither of these papers studies competition between the principals.

It is well known that the principal can use correlation among agents' types or an exogenous signal about the agent's type to extract rents (see Crémer and McLean (1985), Shleifer (1985) and Riordan and Sappington (1988) for early papers) and that agents' liability and risk attitude are crucial for the exact scope of what the principal can achieve (Robert (1991)). However, only few papers consider a dynamic problem. Fuchs and Skrzypacz (2010) analyze an infinite-horizon bargaining model and show that the delay is non-monotonic in the likelihood of a signal that eliminates the informational asymmetry. In Drugov (2010), in a two-period model, we show a related result that the delay is non-monotonic in the quality of the signal.[5] In Strausz (2006) the model is also dynamic but the agent has unlimited liability which is crucial for Crémer-McLean mechanisms. Contracting under an exogenous signal about agent's type is the first step in analyzing our model since each hierarchy cannot influence the information revealed by the other hierarchy.[6]

The literature on competing vertical structures with correlated private information of agents is also very related. In Martimort (1996) and Martimort and Piccolo (2010) the types are perfectly correlated, however, the principals do not have access to the report of the other agent report before offering the contract nor can they contract on the choice of the other agent. Bertoletti and Poletti (1997) apply Crémer-McLean mechanism in a Cournot oligopoly setting. Piccolo and Pagnozzi (2013) study the incentives of the principals to share the information about their agents. When they share, the contract of each principal depends on the reports of both agents. Otherwise, the game proceeds as in Martimort (1996).

[4] In a related paper, Kuhn and Gu (1999) study the same game with the possibility of a strategic rejection, but impose a sequential order of moves and, therefore, the revelation of information is exogenous. The first bargaining occurs in the autarky, while the second starts with more information.

[5] We mention there that a general analysis of the effects of an exogenous general signal is intractable in the Sobel and Takahashi (1983) model because even for a uniform prior the posterior might be very complicated. What allows us to conduct the analysis here is that the signal takes a particular form, namely, a partition of the type space.

[6] Demougin and Garvie (1991), Boyer and Laffont (2003) and Gary-Bobo and Speigel (2006) study a static adverse selection problem with an exogenous contractible signal.

The crucial feature of this literature is a static setting. As a result, the principals have somehow to share the information when the contracts are offered (and be able to commit to it) or be able to contract on it ex post when it becomes observable. In a dynamic setting no commitment is needed for the informational leakages between the hierarchies. The information revealed early becomes observable later through the agents' choices and the principals use it by modifying their subsequent offers. Another consequence of the static setting is that, obviously, the issues of timing do not arise. The focus of analysis there is on the distortions in quantities. In our model, as in any dynamic bargaining model, the focus of analysis is on the timing of agreement.

2 Setup

There are two principal-agent pairs, that we call hierarchies, A and B. In each of them the agent (he) may work for the principal (she), that is, a retailer may undertake a demand-increasing effort, a supplier may sell the new technology or a target may agree to the acquirer's offer, etc. The principals negotiate with their respective agents about the price they will pay for the work. There are two periods, and all parties share a common discount factor $\delta < 1$. Each agent works at most once, either in period 1 or in period 2 or does not work at all. The two agents have the same cost θ which is uniformly distributed on $[0, 1]$.[7,8] They know θ, but principals know only its distribution.

If an agent works in period 1, the principal's revenues are $v_1 < 1$; working in period 2 yields revenues v_2 for the principal. Working in period 1 yields higher revenues even for $\delta = 1$, $v_1 \geq v_2$, since the principal gets benefits from the agent's work for two periods.[9] Both the agent and the principal have the outside option of zero.[10]

The timing of the game is as follows. At the beginning of period 1 each principal proposes a contract to her respective agent. If an agent agrees to work in period 1, he

[7] Assuming uniform distribution allows us to obtain closed-form solutions. Qualitative results do not depend on this assumption. For other distributions we need an assumption analogous to the one in Sobel and Takahashi (1983) on p. 416 that guarantees that the problem $\max_p (F(p) - F(s))(v - p)$ has a unique solution for each v and s.

[8] We consider imperfect correlation in Section 3.3.

[9] Our model thus incorporates both cases of $v_1 = v_2$ and $v_1 = (1 + \delta) v_2$.

[10] Under competition analyzed in Section 4, v_1, v_2 and the principal's outside option depend on what the other hierarchy does.

does so, the principal earns v_1 and the game is over for that hierarchy. If the agent does not work in period 1, the game continues to period 2. The principal observes whether the other agent worked in period 1 but she does not observe the contract between the other principal and the agent. She then makes a second-period offer which the agent accepts or rejects. Work and transfers, if any, take place and the game ends.

The principals cannot commit to two-period offers.[11] The strategy of principal j, $j = A, B$, is thus a sequence of prices $\{p_1^j, (p_2^{yj}, p_2^{nj})\}$, where p_2^{yj} and p_2^{nj} are ex post optimal and p_2^{yj} (p_2^{nj}) is offered when agent $-j$ did (not) work in period 1.[12] The strategy of agent j is a mapping from his type and the offered price into the acceptance/rejection decision. We consider only pure strategies for the principals while the agents may play mixed strategies.[13]

The equilibrium concept is Perfect Bayesian Equilibrium. As standard in the literature on private contracts, we also assume that the agents have passive beliefs: since principals move simultaneously and independently, a deviation from the equilibrium offer by one principal cannot be a signal about the behaviour of the other principal (see Martimort (1996), Martimort and Piccolo (2010) and Piccolo and Pagnozzi (2013) among others).

2.1 The benchmark: autarky

In the autarky each hierarchy acts in isolation or, equivalently, the private information of the agents is uncorrelated. This is then a standard bargaining model with one-sided private information and continuum of agent's types first studied by Sobel and Takahashi (1983). The principal offers p_1 in period 1 and, if rejected, she offers p_2 in

[11] The full-commitment case is explored in the working paper version Drugov (2011).

[12] In period 2 price posting is without loss of generality by the revelation principle. In period 1 this is less clear since the revelation principle does not apply due to imperfect commitment while the modified revelation principle of Bester and Strausz (2001) does not apply due to the infinite type space. Skreta (2006) shows that price posting is without loss of generality in any finite-horizon standard bargaining model with one-sided private information for any type space. Fuchs and Skrzypacz (2010) consider price posting and do not comment at all on its optimality. We conjecture that, because principals do not have any commitment power, observe the other agent's action only after period 1 and do not observe the contract/communication in the other hierarchy, price posting in period 1 is without loss of generality in our model too.

[13] A pure-strategy equilibrium always exists in our model. Mixed strategies of the principals introduce a significant complication since the posterior beliefs in period 2 are not uniform. Moreover, in a mixed-strategy equilibrium (if it exists) it is essentially impossible to benchmark the agents and, therefore, the principals are likely to do worse than in a pure-strategy equilibrium.

period 2. Deciding whether to accept the first-period offer, an agent of type θ chooses between the first-period rent $p_1 - \theta$ and the discounted second-period rent $\delta(p_2 - \theta)$. Thus, there is cut-off s given by

$$s = \frac{p_1 - \delta p_2}{1 - \delta} \tag{1}$$

such that all the types below it accept p_1 in period 1 while all the types above it reject it.[14] Since the principal does not have commitment, p_2 maximizes the principal's expected profits of period 2 given that the type of the agent is above s,[15]

$$\max_{p_2} (v_2 - p_2)(p_2 - s), \tag{2}$$

that is,

$$p_2 = \frac{s + v_2}{2}. \tag{3}$$

The principal solves the following problem

$$\begin{cases} \max\limits_{p_1, p_2, s} (v_1 - p_1) s + \delta (v_2 - p_2)(p_2 - s) \\ \text{s.t. } (1) \text{ and } (3) \end{cases} \tag{4}$$

This leads to the following proposition.

Proposition 1 *In the autarky there is a unique equilibrium. The principal offers prices* $p_1 = \frac{1}{2}\frac{4v_1 - 2\delta v_1 - \delta^2 v_2}{4 - 3\delta}$ *and* $p_2 = \frac{1}{2}\frac{2v_1 + 4v_2 - 5\delta v_2}{4 - 3\delta}$ *and the resulting cut-off is* $s = 2\frac{v_1 - \delta v_2}{4 - 3\delta}$.[16]

[14] The agent thus strictly prefers to play a pure strategy unless his type is equal to s. In this case, it is irrelevant what he does as it happens with probability of measure zero. For concreteness, we assume that he accepts p_1.

[15] We assume that there is some surplus left in period 2, that is, $v_2 \geq s$. When this does not hold, the principal effectively commits not to trade with the agent in period 2 which goes against the logic of the no-commitment case. Also, this possibility complicates unnecessarily the paper as one would need to consider two cases depending on whether $v_2 \geq s$ binds or not. For both reasons, we assume that $v_2 \geq s$ does not bind in the equilibrium. Since we will need a stronger version in the next Section, we will introduce this assumption formally there (Assumption 1).

[16] This is essentially the example of Sobel and Takahashi (1983), p. 420, with two small differences: they consider the private information on the side of the buyer and do not allow the valuation of the uninformed party to change from one period to another.

The final comment concerns the efficiency in this model. The welfare, only affected by which types work and when, is

$$\int_0^s (v_1 - \theta)\, d\theta + \delta \int_s^{p_2} (v_2 - \theta)\, d\theta.$$

In the first best all the types up to v_1 work in period 1 and no type works in period 2. The inefficiency is associated with the facts that types in $(s, p_2]$ work in period 2 and types in $(p_2, v_1]$ do not work at all.

3 Analysis

We solve the model in the following way. In period 1, agent j, $j = A, B$, is indifferent about working or not working when his type is the cut-off type s^j. If his type is below, he strictly prefers to work in period 1, while he strictly prefers not to work if his type is above. The information revealed by hierarchy j after period 1 is a partition, whether the type is below or above s^j. Thus, s^j summarizes all the information revealed by hierarchy j after period 1 and only s^j is payoff-relevant for the other principal. In an equilibrium, any strategy $\left\{p_1^j, \left(p_2^{yj}, p_2^{nj}\right)\right\}$ maps into cut-off type s^j of agent j. The strategies of the two principals can then be characterized by reaction functions $s^A\left(s^B\right)$ and $s^B\left(s^A\right)$ and their intersections are equilibria. Since the two hierarchies are (ex ante) identical, $s^A\left(s^B\right)$ and $s^B\left(s^A\right)$ are symmetric about the diagonal.

3.1 Optimal behavior of hierarchy A given s^B

In what follows, we focus on hierarchy A and derive the reaction function $s^A\left(s^B\right)$. There are two cases to consider, depending on whether $s^A > s^B$ (case a for *above*) or $s^A \leq s^B$ (case b for *below*). Start with the case $s^A > s^B$. The information generated by hierarchy B is useless for hierarchy A since, following a rejection of an offer in period 1, principal A learns that θ is higher than s^A which is more precise than the fact that θ is higher than s^B. Then, the bargaining in hierarchy A proceeds as if it were in the autarky. Proposition 1 applies provided $2\frac{v_1 - \delta v_2}{4 - 3\delta} > s^B$.

Consider now case b, $s^A \leq s^B$.[17] The information generated by hierarchy B is

[17]When $s^A = s^B$ the principal still uses the information from the other firm to detect deviations of

useful since, following a rejection of a first-period offer, principal A learns if the cost of the agent is in the interval $(s^A, s^B]$ or $(s^B, 1]$. She then solves a problem similar to (2) on the relevant interval. In the former case, she offers the second-period price $p_2^y = \min\left\{\frac{s^A+v_2}{2}, s^B\right\}$ and in the latter one she offers $p_2^n = \frac{s^B+v_2}{2}$.[18] Her problem at the onset of the relationship is:

$$\begin{cases} \max\limits_{p_1, p_2^y, p_2^n, s^A} (v_1 - p_1) s^A + \delta\left[(v_2 - p_2^y)\left(p_2^y - s^A\right) + (v_2 - p_2^n)\left(p_2^n - s^B\right)\right] \\ \text{s.t.} \quad s^A \leq s^B,\ s^A = \frac{p_1 - \delta p_2^y}{1-\delta},\ p_2^y = \min\left\{\frac{v_2+s^A}{2}, s^B\right\} \text{ and } p_2^n = \frac{v_2+s^B}{2}, \end{cases} \tag{5}$$

where the second constraint is (1) taking into account that the cut-off type s^A, being lower than s^B, will face p_2^y in period 2 if he rejects p_1.

Similarly to the autarky case, we have assumed above that in the equilibrium $v_2 \geq s^A$ for any s^B, that is, there is some surplus left in period 2 (see fn. 15). This requires assuming $\frac{v_2}{v_1} \geq \frac{1}{2-\delta}$ since the highest cut-off is $\frac{v_1-\delta v_2}{2(1-\delta)}$ (Lemma 1 below). However, we need a slightly stronger assumption to simplify the characterization of the function $s^A\left(s^B\right)$ there. In the proof we give the full characterization.

Assumption 1 $\frac{v_2}{v_1} \geq \frac{(2-\delta)^2 - \sqrt{(4-3\delta)(1-\delta)^3}}{4-3\delta-\delta^2(1-\delta)}$.[19]

The next lemma characterizes optimal cut-off s^A for any s^B.

Lemma 1 *The optimal cut-off s^A is*

$$s^A = \begin{cases} 2\frac{v_1-\delta v_2}{4-3\delta} & \text{if } s^B < \theta^* \\ s^B & \text{if } \theta^* \leq s^B < \frac{v_1-\delta v_2}{2(1-\delta)} \\ \frac{v_1-\delta v_2}{2(1-\delta)} & \text{if } \frac{v_1-\delta v_2}{2(1-\delta)} \leq s^B < \theta^b \\ 2\frac{v_1-\delta v_2}{4-3\delta} & \text{if } s^B \geq \theta^b \end{cases}$$

where $\theta^* < \frac{v_1}{2}$ and $\theta^b = \frac{1}{2}\left(v_2 + \frac{v_1-\delta v_2}{\sqrt{(1-\delta)(4-3\delta)}}\right)$.[20]

Proof. See Appendix. ■

her agent.

[18] The hierarchy indices in prices are omitted as this does not create any confusion.

[19] This expression is monotonically increasing in δ, equals to $\frac{1}{2-\delta}$ at $\delta = 0$ and $\delta = 1$ and is higher than $\frac{1}{2-\delta}$ otherwise but by no more than 8,6%.

[20] If $v_1 = (1+\delta) v_2$, then $\theta^* = \frac{2+\delta-\sqrt{\delta\frac{8-8\delta-3\delta^2}{4-3\delta}}}{4-\delta} v_2$.

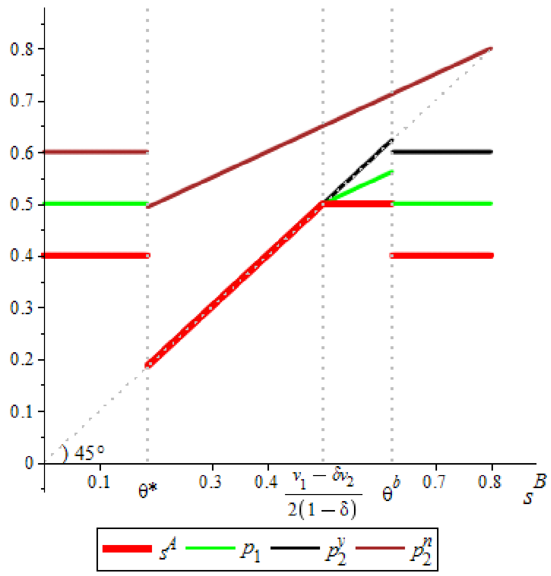

Figure 1: Prices and cut-off s^A as a function of s^B for $v_1 = 0.9$, $v_2 = 0.8$, $\delta = 0.5$. For $s^B \in \left[\theta^*, \frac{v_1-\delta v_2}{2(1-\delta)}\right]$ $s^A = p_1 = p_2^y = s^B$. For $s^B \leq \theta^*$ the information is not used and the autarky solution is obtained.

See Figure 1 for a numerical example. When $s^B < \theta^*$, the information from hierarchy B is not used. Using it would imply decreasing the first-period contracting a lot. The principal then prefers to implement the autarky solution (case a). At $s^B = \theta^*$, the principal finds it worthwhile to use the information (the solution switches to case b). However, since s^B is still quite low, she does it decreasing the first-period contracting as little as possible, that is, setting $s^A = s^B$. The most important feature of this arrangement is that the information from hierarchy B is not used to learn anything new but only to discipline the agent. In other words, it is used off the equilibrium path to detect the agent's deviations.

At $s^B = \frac{v_1-\delta v_2}{2(1-\delta)}$, s^A becomes lower than s^B. The information from hierarchy B is used in order to learn upon rejection of the first-period offer if the agent's cost is in the interval $(\frac{v_1-\delta v_2}{2(1-\delta)}, s^B]$ or $(s^B, 1]$. In the former case the principal sets $p_2^y = \min\left\{\frac{s^A+v_2}{2}, s^B\right\} = s^B$ since s^B is still close to s^A. Thus, information from hierarchy B provides some partial commitment to the principal not to increase the second-period price to its otherwise ex post optimal level $\frac{s^A+v_2}{2}$. Because of this, the principal can set s^A at a higher level than in the autarky, $\frac{v_1-\delta v_2}{2(1-\delta)} > 2\frac{v_1-\delta v_2}{4-3\delta}$.

Finally, at $s^B = \theta^b$ the constraint $p_2^y = \frac{s^A+v_2}{2} \leq s^B$ does not bind any longer. The principal uses the information from hierarchy B to contract with the agent when his cost is above s^B but it does not intervene with the contracting when the cost is below s^B. Optimal s^A is then as in the autarky case, $2\frac{v_1-\delta v_2}{4-3\delta}$. The information from hierarchy B is used to contract with the types with which the principal would not have contracted without it.

3.2 Equilibria

Equilibria are characterized in the next proposition.

Proposition 2 *i) There is a continuum of symmetric equilibria with the same cut-off in both hierarchies* $s^A = s^B \in \left[\theta^*, \frac{v_1-\delta v_2}{2(1-\delta)}\right]$. *There are no other equilibria.*

ii) The maximum welfare and the agents' rent are obtained in the "highest" equilibrium $s^A = s^B = \frac{v_1-\delta v_2}{2(1-\delta)}$. *The maximum profits are obtained in the equilibrium* $s^A = s^B = \frac{2v_1-\delta v_2}{4-\delta}$.

iii) Both hierarchies setting the autarky cut-off $s^A = s^B = 2\frac{v_1-\delta v_2}{4-3\delta}$ *is an equilibrium.*

Proof. See Appendix. ■

The first part of Proposition 2 says there are multiple symmetric equilibria and no asymmetric ones. The two agents always work simultaneously, either in period 1 or in period 2. They generate the same partition of the type space, i.e., the same information, and the principals use it to commit not to improve their offer, $p_2^y = p_1$. Thus, in an equilibrium the principals do not use the information from the other hierarchy to learn anything they do not already know. They use it off the equilibrium path, that is, if the agent deviates.

The second part of Proposition 2 characterizes the "best" equilibrium from different perspectives; see also Figure 2. A higher cut-off $s^A = s^B$ means more types work in period 1 and also higher overall contracting as types up to $p_2^n = \frac{v_2+s}{2}$ work either in period 1 or 2. Thus, the welfare increases and the highest welfare is reached in the "highest" equilibrium. A higher cut-off $s^A = s^B$ also means higher prices obtained by the agents and, therefore, they get the highest rent in the "highest" equilibrium, too. The principals, however, obtain the highest profits in some lower equilibrium. By definition, at the "highest" equilibrium the cut-off $s^A = s^B$ is so high that the benefits

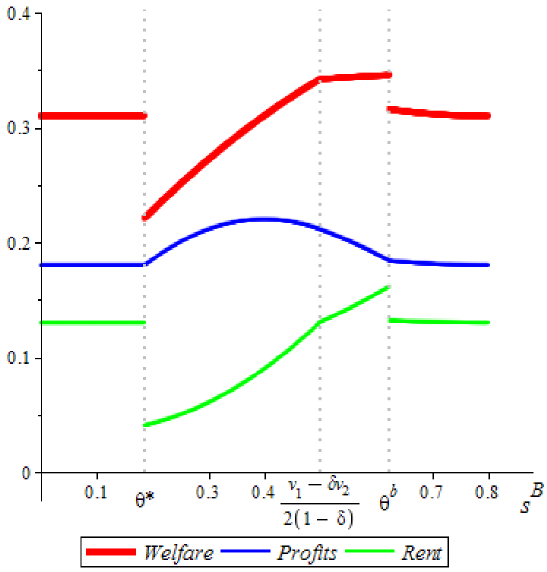

Figure 2: Welfare, profits and rent as a function of s^B for $v_1 = 0.9$, $v_2 = 0.8$, $\delta = 0.5$. For $s^B \leq \theta^*$ the information is not used and the autarky solution is obtained.

of benchmarking the agents equal the costs of paying the agents high prices. At a lower cut-off the principals earn then higher profits as the benefits of benchmarking the agents are still there, but the agents are paid less.

The welfare in the "highest" equilibrium is higher than in the benchmark case of autarky since the cut-off there, $\frac{v_1 - \delta v_2}{2(1-\delta)}$, is higher than in the autarky, $2\frac{v_1 - \delta v_2}{4-3\delta}$. On the other hand, since $\theta^* < 2\frac{v_1 - \delta v_2}{4-3\delta}$, in the "low" equilibria the welfare is lower. Comparison with the profit-maximizing equilibrium is ambiguous as $\frac{2v_1 - \delta v_2}{4-\delta}$ may be higher or lower than $2\frac{v_1 - \delta v_2}{4-3\delta}$. However, if $v_1 = (1+\delta)\, v_2$, then the cut-off in the profit-maximizing equilibrium is lower than in the autarky, that is, the welfare is lower.

3.3 Imperfect correlation of types

Here, we relax the assumption that the costs of the two agents are exactly the same. Suppose that with probability λ the costs of the two agents are the same, while with probability $1 - \lambda$ they are independent.[21] For example, the agents use different technologies with probability $1-\lambda$ in which case the information revealed by each hierarchy is irrelevant for the other one. Importantly, in period 1 it is not known whether the

[21] We use the term "correlation" even though the correct one is "dependence" since λ does not measure a linear relationship between the types of the two agents. When we say "higher (lower) correlation" it refers to a higher (lower) λ.

agents are the same or not, it becomes known in between the two periods. A higher λ means more, or better, information being revealed by each hierarchy in expectation.

When $s^A > s^B$ (case a), hierarchy A does not use information it receives from hierarchy B. Therefore, case a is not affected by λ. In the case b, problem (5) becomes:

$$\begin{cases} \max\limits_{p_1, p_2^y, p_2^n, p_2^i, s^A} (v_1 - p_1) s^A + \delta\lambda \left[(v_2 - p_2^y)\left(p_2^y - s^A\right) + (v_2 - p_2^n)\left(p_2^n - s^B\right)\right] \\ \qquad\qquad +\delta(1-\lambda)\left(v_2 - p_2^i\right)\left(p_2^i - s^A\right) \\ \text{s.t.} \quad s^A \leq s^B,\ s^A = \frac{p_1 - \delta\left[\lambda p_2^y + (1-\lambda) p_2^i\right]}{1-\delta},\ p_2^y = \min\left\{\frac{v_2+s^A}{2}, s^B\right\},\ p_2^n = \frac{v_2+s^B}{2},\ p_2^i = \frac{v_2+s^A}{2} \end{cases} \tag{6}$$

where p_2^y or p_2^n are prices offered in period 2 (depending on whether the other agent worked in period 1 or not) when the agents' types turned out to be the same and p_2^i is the price offered in period 2 when the agents' types are independent.

Next lemma characterizes reaction function $s^A\left(s^B\right)$. The proof is omitted as it is very similar to the one of Lemma 1.

Lemma 2 *The optimal cut-off s^A is*

$$s^A = \begin{bmatrix} 2\frac{v_1-\delta v_2}{4-3\delta} & \text{if } s^B < \theta_\lambda^* \\ s^B & \text{if } \theta_\lambda^* \leq s^B < 2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta} \\ 2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta} & \text{if } 2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta} \leq s^B < \theta_\lambda^b \\ 2\frac{v_1-\delta v_2}{4-3\delta} & \text{if } s^B \geq \theta_\lambda^b \end{bmatrix}$$

where $\theta_\lambda^* < \frac{v_1}{2}$ and decreasing in λ; $\theta_\lambda^b = \frac{1}{2}\left(v_2 + 2\frac{v_1-\delta v_2}{\sqrt{(4-3\delta)(4-3\delta-\lambda\delta)}}\right)$.[22]

In the case of perfect correlation, $\lambda = 1$, we are back to Lemma 1. If the agents are independent, $\lambda = 0$, we obtain the autarky solution $s = 2\frac{v_1-\delta v_2}{4-3\delta}$ (see Proposition 1). Indeed, both θ_λ^* and $2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta}$ become $2\frac{v_1-\delta v_2}{4-3\delta}$ at $\lambda = 0$.

The most interesting feature of Lemma 2 is that s^A increases with λ when it is equal to $2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta}$. In other words, better information in period 2 increases contracting

[22]Assumption 1 can be somewhat relaxed in the case of imperfect correlation as the necessary threshold for $\frac{v_2}{v_1}$ is increasing in λ. When $\lambda = 0$, it becomes $\frac{v_2}{v_1} \geq \frac{1}{2-\frac{\delta}{2}}$ which ensures that the autarky cut-off $2\frac{v_1-\delta v_2}{4-3\delta} \leq v_2$.

in period 1, that is, before this information becomes known. The reason is that the agent becomes more accommodating when λ increases since, upon rejecting p_1, he is more likely to face a relatively low p_2^y. This can be traced to $s^A = \frac{p_1 - \delta\left[\lambda p_2^y + (1-\lambda) p_2^i\right]}{1-\delta}$ in (6) which is decreasing in λ when $p_2^y = s^B$ since it is then smaller than $p_2^i = \frac{v_2 + s^A}{2}$. On the other hand, when s^B is at (slightly below) θ_λ^*, a higher λ decreases s^A since it changes from $2\frac{v_1 - \delta v_2}{4 - 3\delta}$ to s^B. Thus, we get a result similar to Drugov (2010) (cf. Proposition 5(i)) that the delay is not monotonic in information.

The equilibria and their comparative statics of equilibria with respect to λ are summarized in the next proposition.

Proposition 3 *Under imperfect correlation: i) There is a continuum of symmetric equilibria with the same cut-off in both hierarchies $s^A = s^B \in \left[\theta_\lambda^*, 2\frac{v_1 - \delta v_2}{4 - 3\delta - \lambda\delta}\right]$. This interval expands in λ. There are no other equilibria.*

ii) If $s^A = s^B$ is an equilibrium for λ' and $\lambda'' > \lambda'$, the agents' rent is higher for λ' while the welfare is the same.

iii) The maximum welfare and the agents' rent are obtained in the "highest" equilibrium $s^A = s^B = 2\frac{v_1 - \delta v_2}{4 - 3\delta - \lambda\delta}$. The welfare increases with λ while the effect on rent is ambiguous. The maximum profits are obtained in the equilibrium $s^A = s^B = \frac{2v_1 - 2\delta v_2 + \lambda\delta v_2}{4 - 3\delta + 2\lambda\delta}$ and increase with λ.

iv) Both hierarchies setting the autarky cut-off $s^A = s^B = 2\frac{v_1 - \delta v_2}{4 - 3\delta}$ is an equilibrium.

Proof. See Appendix. ■

As in the perfect correlation case, there are only symmetric equilibria $s^A = s^B \in \left[\theta_\lambda^*, 2\frac{v_1 - \delta v_2}{4 - 3\delta - \lambda\delta}\right]$ (cf. Proposition 2). Intuitively, the set of these equilibria becomes smaller as correlation becomes lower. Setting the same cut-off enables the principal not to improve upon the first-period offer if the other agent has accepted that offer. But this comes at a cost of distorting the timing of contracting. The principal is less willing to tolerate this distortion when the future information is less beneficial.

If a certain cut-off s is an equilibrium for two levels of correlation, the welfare is the same in the two cases. Indeed, in period 1, by definition, there is no difference with respect to which types work. In period 2 there is no difference either since principals offer the same prices whether the agents turned out to have the same cost or not, $p_2^n = p_2^i = \frac{v_2+s}{2}$. The principals do not learn anything from each other on the equilibrium path since the information is used only to detect agents' deviations. The first-period price, however, is lower as the agent's incentive compatibility becomes easier to satisfy as we discussed above (see expression for s^A in (6)). Thus, the agents' rent goes down.

A higher correlation leads to a higher cut-off being possible in the equilibrium and, since a higher cut-off increases the first-period and the total work, the welfare in the "highest" equilibrium increases. There are two opposite effects on the agents' rent: the cut-off in the "highest" equilibrium rises but the first-period price falls for a given cut-off, as we explained above. The total effect is ambiguous.

The comparison of the welfare with the autarky is the same as the one of the perfect correlation case of Section 3. The welfare in the "highest" equilibrium is higher since the cut-off there, $2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta}$, is higher than in the autarky, $2\frac{v_1-\delta v_2}{4-3\delta}$, for $\lambda > 0$. On the other hand, since $\theta_\lambda^* < 2\frac{v_1-\delta v_2}{4-3\delta}$, in the "low" equilibria the welfare is lower. Comparison with the profit-maximizing equilibrium is ambiguous as $\frac{2v_1-2\delta v_2+\lambda\delta v_2}{4-3\delta+2\lambda\delta}$ may be higher or lower than $2\frac{v_1-\delta v_2}{4-3\delta}$. However, if $v_1 = (1+\delta)\, v_2$, then the cut-off in the profit-maximizing equilibrium is lower than in the autarky, that is, the welfare is lower.

4 Competition

In this Section, we introduce competition between the two hierarchies. They compete in a market every period. When a principal successfully contracts with her agent, the agent (when retailer) engages in a demand-increasing activity or (when supplier) exerts the effort to reduce the costs of the hierarchy or agrees to sell the new technology. Unlike the model studied before, the revenues of each principal directly depend on what the other hierarchy does. Also, even if the agent does not work, the principal may still earn some revenues.

We thus need to adjust the model of Section 2. Denote by v_{yy} the one-period profit of each principal when both have agreed with their agents (in the current or in the previous period), v_{nn} the one-period profit of each principal when none has agreed with the agents, v_{yn} the one-period profit of the principal that has agreed with her agent while the other principal has not and, finally, v_{ny} the one-period profit of the principal that has not agreed with the agent while the other principal has agreed. For some results and interpretation we will use the following example.

Example 1 *The system of (inverse) market demands is* $p_A = 1 - q_A + \rho q_B$ *and* $p_B = 1 - q_B + \rho q_A$*, where* $|\rho| \leq 1$.[23] *Both hierarchies initially have constant marginal cost* c. *If an agent works for his principal, the marginal cost is permanently reduced to zero.*[24] *The per-period profits in the product market equilibrium are*

$$v_{yy} = \frac{1}{(2-\rho)^2}, v_{nn} = \frac{(1-c)^2}{(2-\rho)^2}, v_{yn} = \frac{(2+\rho-c\rho)^2}{(2-\rho)^2(2+\rho)^2} \text{ and } v_{ny} = \frac{(2+\rho-2c)^2}{(2-\rho)^2(2+\rho)^2}. \tag{7}$$

This system of market demands is used in many oligopoly models with information sharing or externalities.[25] When $\rho = 0$ and $c = 1$, we are back to our original model with $v_2 = v_{yy} = v_{yn}$ and $v_1 = (1+\delta) v_2$ (note that $v_{nn} = v_{ny} = 0$).

It will be important to distinguish between two cases depending on whether the one-period equilibrium profit is supermodular or submodular in the marginal costs of the two hierarchies, that is, if $v_{yy} + v_{nn} \geq v_{yn} + v_{ny}$ or $v_{yy} + v_{nn} < v_{yn} + v_{ny}$. For our example (7) it is supermodular (submodular) if and only if hierarchies compete in strategic complements, i.e., $\rho \geq 0$ (substitutes, i.e., $\rho < 0$). In what follows we refer to the two cases as *complements* and *substitutes*, respectively, even though this is not entirely precise in general.

4.1 Continuation equilibria in period 2

When no principal has contracted with her agent in period 1, the period-2 game is more involved than in the absence of competition. In an equilibrium both principals know that the type of their agents is above $s = \max\{s^A, s^B\}$. The profits each of them obtains from any second-period price depend on what the other principal is doing. The next lemma finds the continuation equilibria in period 2.

[23] p_A and p_B are prices in the product markets and are not related to the prices offered by principals to their agents.

[24] With linear demand and constant marginal cost decreasing the cost is equivalent to increasing the demand intercept.

[25] See Gal-Or (1985), Raith (1996) and Piccolo and Pagnozzi (2013) among others.

Lemma 3 *If no agreement has been reached in period* 1 *and both principals know that the type of the agents is above* s*, the continuation equilibria are*

- $p_2^j = \frac{v_{yy}-v_{ny}+s}{2} < p_2^{-j} = \frac{v_{yn}-v_{nn}+s}{2}$, $j = A, B$, *in the case of substitutes, that is,* $v_{yy} + v_{nn} < v_{yn} + v_{ny}$*;*
- $p_2^A = p_2^B \in \left[\frac{v_{yn}-v_{nn}+s}{2}, \frac{v_{yy}-v_{ny}+s}{2}\right]$ *in the case of complements, that is,* $v_{yy} + v_{nn} \geq v_{yn} + v_{ny}$.

Proof. See Appendix. ■

When principal A sets $p_2^A < p_2^B$, the revenues from the marginal type of her agent, p_2^A, are $v_{yy} - v_{ny}$ since agent B, whose marginal type is above, p_2^B, surely works. Principal B's revenues from the marginal type of her agent are $v_{yn} - v_{nn}$ since agent A does not work. This is an equilibrium if the former marginal revenues are smaller in which case p_2^A is indeed below p_2^B. This gives the condition for the case of substitutes. When it is not satisfied, both principals set the same prices.

In what follows we assume that in the case of substitutes the two equilibria are equally likely and in the case of complements the "average" equilibrium is played. The expected price offered by both principals is then

$$p_2^n = \frac{\frac{1}{2}\left(v_{yn} - v_{nn} + v_{yy} - v_{ny}\right) + s}{2}, \tag{8}$$

in both substitutes and complements cases which simplifies the subsequent analysis.

4.2 Equilibria of the whole game

The analysis proceeds in a similar way to the one of Section 3. However, we can preview the new result already. In the case of the competition in substitutes the principals prefer to differentiate as the continuation equilibrium in Lemma 3 shows. Thus, principals face a trade-off between benchmarking the agents in period 1 to save on their rent and differentiating to increase profits in the market. When the discount factor is low, the game essentially proceeds as if it were a one-period game and the agent's rent is limited by the low value of the future. Thus, asymmetric equilibria may appear. In the case of complements, market competition pushes in the opposite direction and the trade-off disappears.

We first characterize the reaction function $s^A\left(s^B\right)$.

Lemma 4 *If principal A sets cut-off s^A*

- *Above s^B (case a), then $s^A = 2\frac{v_{yn}-v_{nn}}{4-3\delta}$.*
- *Below or equal s^B (case b), then $s^A = \begin{bmatrix} s^B & \text{if } s^B < \frac{v_{yy}-v_{ny}}{2(1-\delta)}, \\ \frac{v_{yy}-v_{ny}}{2(1-\delta)} & \text{if } \frac{v_{yy}-v_{ny}}{2(1-\delta)} \leq s^B < \theta_c^b, \\ 2\frac{v_{yy}-v_{ny}}{4-3\delta} & \text{if } s^B \geq \theta_c^b, \end{bmatrix}$*

  where $\theta_c^b = \frac{1}{2}\left(1+\frac{1}{\sqrt{(1-\delta)(4-3\delta)}}\right)(v_{yy}-v_{ny})$.

Proof. See Appendix.[26] ■

The characterization of $s^A\left(s^B\right)$ in Lemma 4 is similar to the ones in Lemmas 1 and 2. It is interesting to note the following differences. First, Lemma 4 does not feature threshold θ_c^* at which principal A starts using the information from hierarchy B, that is, when she first sets $s^A \leq s^B$. The reason is that the exact location of this threshold depends on the direction and the strength of the competition and it is too cumbersome to characterize it in general. We will do it for our Example (7) when $\rho = -1$ (see Proposition 4 below), that is, for the strongest competition in substitutes. Second, in the case a the gain from contracting with the marginal type of the agent is proportional to $v_{yn} - v_{nn}$ since agent B does not work while in the case b it is proportional to $v_{yy} - v_{ny}$ since agent B works. In Lemmas 1 and 2 the competition is absent and that gain is $v_1 - \delta v_2$ in both cases. Otherwise, the characterization is remarkably similar.

The equilibria are characterized in the next proposition.

Proposition 4 *Take Example (7) with $\rho = -1$. There exists threshold $\widehat{\delta}$ such that for $\delta \geq \widehat{\delta}$ there exist symmetric equilibria with the same cut-off in both hierarchies $s^A = s^B \in \left[\theta_c^*, \frac{v_{yy}-v_{ny}}{2(1-\delta)}\right]$. For $\delta < \widehat{\delta}$ there are two asymmetric equilibria in which one*

[26] When $v_1 = (1+\delta)\, v_2$, Assumption 1 writes as $\delta < 0.4548$. Also assume that in period 2 the principals are willing to contract with the agent which can be written as $\frac{2}{4-3\delta} \leq \frac{v_{yy}-v_{ny}}{v_{yn}-v_{nn}} \leq 2(1-\delta)$.

hierarchy sets its cut-off at $2\frac{v_{yn}-v_{nn}}{4-3\delta}$ *and the other one at* $\frac{v_{yy}-v_{ny}}{2(1-\delta)}$ *or* $2\frac{v_{yy}-v_{ny}}{4-3\delta}$.[27] *In both cases, there are no other equilibria. Moreover,* $\widehat{\delta}$ *is increasing in* c.[28]

Proof. See Appendix. ■

Proposition 4 confirms our intuition that we outlined above. When the competition is in substitutes, one-period game does not have symmetric equilibria as the two principals try to differentiate. When the discount factor is small this is the dominant effect. In particular, it dominates the desire to discipline the agents by setting the cut-offs at the same level. A higher cost c makes it more important to contract with the agent (both $v_{yn} - v_{nn}$ and $v_{yy} - v_{ny}$ increase) but it of course also makes it more important for hierarchies to differentiate ($(v_{yn} + v_{ny}) - (v_{yy} + v_{nn})$ increases) and this effect is larger.

To see better the effect of the degree of competition ρ, we plot the equilibria in Figure 3 for different ρ. The range of symmetric equilibria is increasing in ρ. When competition is in substitutes, there is a tension between two forces: principals want to differentiate but they also want to limit their agents' rent. In Proposition 4 we considered the extreme case of $\rho = -1$ when the first effect is dominant. As ρ increases the first force weakens and at $\rho = 0$ the principals do not compete with each other at all. As ρ becomes positive, the principals want to contract simultaneously also for competition reasons, that is, the two forces become aligned. Doing the same as the other principal is beneficial in the product market and also in bargaining with the agent.

In Figure 3 we also plot $s^{\Pi\max} = \frac{2+\delta}{4-\delta}(v_{yy} - v_{nn})$, the cut-off at which the principals' profits are maximized among all the points $s^A = s^B$.[29] Interestingly and unlike the previous cases, this point is not an equilibrium for high or low values of ρ. In other words, the principals would do better if they could agree to set their cut-offs at $s^{\Pi\max}$ than in any equilibrium. When the competition is in substitutes, the joint profit

[27] It depends on whether $2\frac{v_{yn}-v_{nn}}{4-3\delta} \lesseqgtr \theta_c^b$. For this example it can be written as $c \lesseqgtr 1 - \frac{4}{4-3\delta}\frac{1}{1+\frac{1}{\sqrt{(1-\delta)(4-3\delta)}}}$ which is decreasing in δ.

[28] As ρ increases, the form of equilibria and the expressions for the thresholds stay the same; but at some point, when the competition is in substitutes but weak enough, $\widehat{\delta}$ disappears as asymmetric equilibria do not exist. See Figure 3 for an illustration.

[29] The maximum welfare and the agents' rent are still obtained in the "highest" equilibrium as before (see Propositions 2(ii) and 3(iii)).

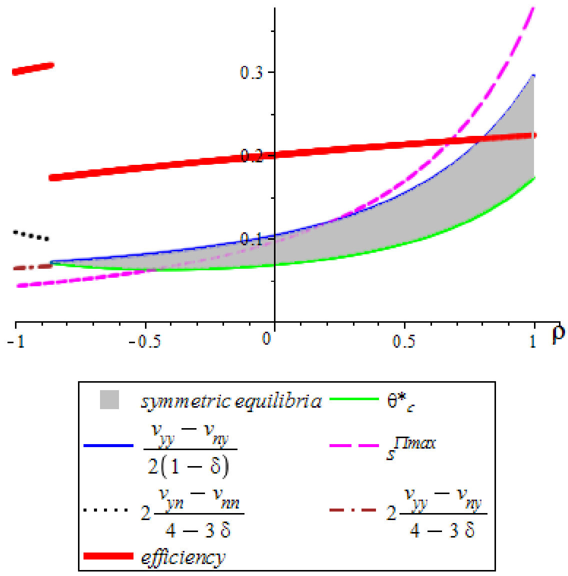

Figure 3: Equilibria for different ρ and $\delta = 0.2, c = 0.41$. Symmetric equilibria are $s^A = s^B \in \left[\theta_c^*, \frac{v_{yy}-v_{ny}}{2(1-\delta)}\right]$ (grey area). To the left of the grey area equilibria are asymmetric with $s^j = 2\frac{v_{yn}-v_{nn}}{4-3\delta}$ and $s^{-j} = 2\frac{v_{yy}-v_{ny}}{4-3\delta}$. At $s^{\Pi\max}$ principals get the highest profits among all the points $s^A = s^B$. The scale for efficiency is adjusted.

maximization calls for a relatively low level of contracting in period 1. Each principal, however, prefers to deviate to a higher cut-off and the relative gain $\frac{v_{yn}-v_{yy}}{v_{yy}}$ is decreasing in ρ. When the competition is in complements, the joint profit maximization means a high level of contracting in period 1. If a principal deviates to a lower cut-off, she looses in the product market but saves on her agent's rent and the relative loss $\frac{v_{ny}-v_{yy}}{v_{yy}}$ is decreasing in absolute terms in ρ. Thus, when $|\rho|$ is close to 1 the incentives to deviate are the highest and the joint profit-maximizing contracting profile $s^A = s^B = s^{\Pi\max}$ is not an equilibrium.

4.3 Comparison with the benchmarks

In the autarky a principal does not obtain any information from the other hierarchy but does not compete with it either, that is, $\rho = 0$. The autarky cut-off is then equal to $2\frac{v_{yy}-v_{ny}}{4-3\delta}$, see Proposition 1 for $v_2 = v_{yy} - v_{ny} = v_{yn} - v_{nn}$ and $v_1 = (1+\delta)\, v_2$. We also compare to the standard benchmark in the literature on competing hierarchies and, more generally, in the contract theory literature, namely, the benchmark of complete information, that is, when the principals observe the type of their agents. Without competition, as we noted in the end of Section 2.1, under complete information all the types up to v_1 should work in period 1 and no type should work in period 2. With competition, there are product market externalities and we characterize the equilibria in Proposition 5 in the Appendix. In the case of substitutes there are two asymmetric equilibria in which the cut-offs are $(v_{yy} - v_{ny})(1 + \delta)$ and $(v_{yn} - v_{nn})(1 + \delta)$ and no contracting in period 2. In the case of complements there are multiple symmetric equilibria with any cut-off between $(v_{yn} - v_{nn})(1 + \delta)$ and $(v_{yy} - v_{ny})(1 + \delta)$ and in some cases there are equilibria with contracting in period 2. Let us focus on the ones without contracting in period 2 (which always exist) in order to simplify the comparison and make it more homogenous for the substitutes and complements. Let us measure then the welfare under complete information by the average cut-off

$$s^{ci} = (1 + \delta) \frac{v_{yy} - v_{ny} + v_{yn} - v_{nn}}{2}. \tag{9}$$

In the case of substitutes, this is the average cut-off. In the case of complements, the "average" equilibrium is played.[30]

There are two distortions brought about by the asymmetric information: contracting with some types occurs in period 2 instead of period 1 and contracting with some types never occurs while it should occur in period 1. We measure welfare by the average discounted agreement time. In the case of multiple symmetric equilibria, focus on the "average" equilibrium, $s^{sym} = \frac{1}{2}\left(\theta_c^* + \frac{v_{yy}-v_{ny}}{2(1-\delta)}\right)$. In the case of asymmetric equilibria, cut-offs are given in Proposition 4. The (relative) efficiency is then

$$E = \left[\begin{array}{l} \frac{1}{s^{ci}} \left[s^{sym} + \delta \left(p_2^n - s^{sym}\right)\right], \text{when equilibria are symmetric} \\ \frac{1}{s^{ci}} \left[\frac{1}{2}\left(s^A + s^B\right) + \frac{1}{2}\delta\left(p_2^{yA} - s^A\right) + \delta\left(p_2^n - s^B\right)\right], \text{otherwise and } s^A < s^B \end{array} \right. \tag{10}$$

In the case of a symmetric equilibrium with cut-off s^{sym}, both agents of types below s^{sym} work in period 1 and agents with types between s^{sym} and p_2^n work in period 2. In the case of an asymmetric equilibrium with cut-offs $s^A < s^B$ agent A (B) works in period 1 if his type is below s^A $\left(s^B\right)$ and in period 2 if his type is between s^A and p_2^{yA} or between s^B and p_2^n (between s^A and p_2^n); where $p_2^{yA} = \min[\frac{v_{yy}-v_{ny}+s^A}{2}, s^B]$ and p_2^n

[30] We made a similar assumption for p_2^n in (8).

is given by (8) for $s = s^B$ (note that p_2^n is based on the higher of the two first-period cut-offs s^B).

In the case of symmetric equilibria, efficiency can be rewritten as $E = \frac{\delta}{2(1+\delta)} + \left(1 - \frac{\delta}{2}\right) \frac{s^{sym}}{s^{ci}}$ and, thus, it depends on the ratio of the cut-offs in the "average" equilibrium.[31] In numerical examples that we tried E increases in ρ (see Figure 3). Thus, the relative efficiency in the autarky is higher than in the case of substitutes but lower than in the case complements. However, we were unable to show it generally since the expression for θ_c^* is complicated (see 21). The efficiency in the profit-maximizing equilibrium $s^{\Pi\max} = \frac{2+\delta}{4-\delta}(v_{yy} - v_{nn})$ is increasing in ρ since the value of joint contracting $v_{yy} - v_{nn}$ increases faster than the average value of contracting $v_{yy} - v_{ny} + v_{yn} - v_{nn}$ in s^{ci} (9).

In the case of asymmetric equilibria, there are two cases depending on whether the lower cut-off is $s^A = 2\frac{v_{yy}-v_{ny}}{4-3\delta}$ (in which case $p_2^y = \frac{v_{yy}-v_{ny}+s^A}{2}$) or $s^A = \frac{v_{yy}-v_{ny}}{2(1-\delta)}$ (in which case $p_2^y = s^B$), see Proposition 4 and fn. 27. In both cases efficiency E can be explicitly rewritten as a function of ratio $\frac{v_{yy}-v_{ny}}{v_{yn}-v_{nn}}$ that measures the substitutability/complementarity and is monotonically increasing in ρ. In the first case E increases in this ratio while in the second case it decreases.

Finally, as can be noted in Figure 3, the efficiency in asymmetric equilibria is higher than in the symmetric ones. The reason is that in asymmetric equilibria the two principals learn more since the higher cut-off is larger than in the symmetric equilibria. While the comparison of the average cut-offs in period 1 is ambiguous, the second-period cut-off p_2^n is then higher since it depends on the highest of the first-period cut-offs.

5 Discussion

Other contractual settings. In the previous version of the paper Drugov (2011) we considered two other contractual settings. Under full commitment, the principals can commit to two-period offers. In order to make that setting comparable, we restricted

[31] In the absence of competition $p_2^n = \frac{v_2+s}{2}$ and $s + \delta(p_2^n - s) = s\left(1 - \frac{\delta}{2}\right) + \delta\frac{v_2}{2}$. That is why we could focus on the first-period cut-off only when discussing the efficiency in Propositions 2 and 3.

the first-period offer to a single price only, that is, each principal offers $\left\{p_1^j, \left(p_2^{yj}, p_2^{nj}\right)\right\}$ in the beginning of period 1.[32] As is well known (see Sobel and Takahashi (1983), Hart and Tirole (1988) and others), in the autarky the principal makes a take-it-or-leave-it offer $p_1 = p_2 \left(= \frac{v_1}{2}\right)$.

When the other hierarchy is present, the principal offers $p_1 = p_2^y = s^A$ and $p_2^n = \frac{v_2+s^B}{2}$ (in the case b). In other words, the principal makes a take-it-or-leave-it offer, as in the autarky, but improves it to the ex post optimal level if he obtains the evidence that it was below the agent's cost. The case of "partial commitment" not to increase p_2^y a lot ex post becomes irrelevant since the principal can directly commit to any p_2^y. Thus, the principal either sets s^A at the autarky level or follows the other principal, $s^A = s^B$ (see Lemma 3 in Drugov (2011)) which results in a set of symmetric equilibria $s^A = s^B \in \left[\theta^{FC}, \frac{v_1}{2}\right]$. Actually, when the agents' types are perfectly correlated following the other principal yields the same profits in both the full and no commitment settings since the prices are as stated above. Obviously, as soon as $s^A \neq s^B$ each principal does strictly worse under no commitment than under full commitment. Therefore, the set of symmetric equilibria in Proposition 2 is larger than the set of equilibria under full commitment (see Proposition 9 in Drugov (2011)).

We also considered the case of the agents who are not strategic in the sense that they accept any offer above their cost in period 1, i.e., $s^A = p_1$. The principals do not need to detect the deviations of their agents and, therefore, never set $s^A = s^B$. Then, even without competition, there are only asymmetric equilibria in which one principal, the *leader*, acts as if it were in the autarky while the other, the *follower*, plays "wait-and-see" strategy by making a low offer in period 1 and using the information from the leader to better contract in period 2 (see Proposition 1 in Drugov (2011)). Under competition, symmetric equilibria may appear in this setting or disappear in the full and no-commitment ones but the comparison between the three settings does not change: the incentives to match the other hierarchy are always the highest under no commitment and the lowest with non-strategic agents.[33]

[32] Otherwise, each principal could ask her agent his type, observe the communication in the other hierarchy and make the payment to the agent in period 2 achieving thus the first best.

[33] Another interesting possibility is the so-called renting case, that is, when the principal deals with the agent every period; for example, a firm licenses a technology from its supplier every period instead of buying it once. In the autarky the selling (studied in this paper) and renting are equivalent since there are only two periods (see Bolton and Dewatripont (2005), ch. 9.1.1.3 for a two-type model and it is easy to verify for the Sobel and Takahashi (1983) model). With informational externalities the

A single principal. In many cases a single principal contracts or bargains with the two agents as analyzed by Crémer and McLean (1985), Shleifer (1985) and others. What would a single principal do in our model? Clearly, she would always pick the "best" equilibrium, characterized in Propositions 2(ii) and 3(iii)), among many symmetric equilibria. In the case of competition with relatively high $|\rho|$ she would pick a profile $s^A = s^B$ which is even not an equilibrium as we discussed at the end of Section 4.[34]

More importantly, however, our informational and contractual assumptions will not be satisfied with a single principal. First, a single principal will use the information she obtains from the agents already in period 1. Second, she will do better with more complicated mechanisms of Crémer-McLean type. She will ask both agents to report their types and contract with them on the terms that depend on both reports. While agents' limited liability will prevent the principal from reaching the first best as shown first by Robert (1991), she will still do better than with price posting.

More than two hierarchies. Take the baseline model of Section 3 and suppose that there are more than two hierarchies. There is always a symmetric equilibrium as in Proposition 2(i). It can be shown that there is another equilibrium with two cut-offs and with at least two hierarchies at each cut-off but there are no other equilibria. Indeed, in an equilibrium with multiple cut-offs being at any of them has to yield the same profits which implies that the different cut-offs cannot be too close to each other.[35] This limits the maximum number of possible equilibrium cut-offs and in this model this number is two. Two implications directly follow. First, when there are three hierarchies, there are only symmetric equilibria as with two hierarchies. Second, the information revealed after period 1 does not increase with the number of hierarchies when there are at least four of them.

new effect is that the principal can adjust the price he is paying to the types with which he already contracted in period 1. However, benchmarking the agents is still the most effective way to discipline them and there is still an interval of symmetric equilibria around the autarky cut-off.

[34] However, in the case of competition the analysis of a single principal becomes very different even absent information externalities since there are externalities in competition that she internalizes.

[35] Another candidate equilibrium is to have one firm at the autarky cut-off while others are at some lower cut-off(s). The incentives compatibility constraints are then slightly different but it turns out that they cannot be satisfied simultaneously.

6 Conclusion

In this paper we studied two parallel dynamic bargaining games in which two principals bargain with their respective agents and the private information of agents is correlated. We showed that the principals want to benchmark their agent against each other which may delay or advance the agreement compared to the autarky. When principals compete in the product market, the product market externalities interact with the informational ones. Under competition in substitutes, the principals trade off the benefits of differentiation in the product market against the cost of the agents' rent and the first effect may dominate if the competition is particularly strong. In the case of complements, both the product market competition and the agency problem call for symmetric behaviour and the benchmarking pattern is reinforced.

Appendix

Proof of Lemma 1. Since case a is the autarky case characterized in Proposition 1, we need to solve case b, that is, problem (5), and then find where switching between the two cases occurs.

In the case b, there are two subcases depending on whether $p_2^y = \frac{v_2+s^A}{2}$ or $p_2^y = s^B$. In the first subcase, using $s^A = \frac{p_1-\delta p_2^y}{1-\delta}$ the prices are ($p_2^n = \frac{v_2+s^B}{2}$ always)

$$\begin{aligned} p_1 &= (1-\delta)\, s^A + \delta \frac{v_2+s^A}{2}, \\ p_2^y &= \frac{v_2+s^A}{2}. \end{aligned}$$

Problem (5) can be written in terms of s^A only

$$\max_{s^A \leq 2s^B - v_2} \left(v_1 - \left((1-\delta)\, s^A + \delta \frac{v_2+s^A}{2} \right) \right) s^A + \frac{1}{4}\delta \left(v_2 - s^A\right)^2 + \frac{1}{4}\delta \left(v_2 - s^B\right)^2 \quad (11)$$

and the solution is

$$s^A = \min\left\{ 2\frac{v_1-\delta v_2}{4-3\delta}, 2s^B - v_2 \right\}.$$

Consider now the second subcase, $p_2^y = s^B$. Using $s^A = \frac{p_1 - \delta p_2^y}{1-\delta}$, the prices are

$$\begin{aligned} p_1 &= (1-\delta)\, s^A + \delta s^B, \\ p_2^y &= s^B. \end{aligned}$$

Problem (5) can be written in terms of s^A only

$$\max_{s^A \in [2s^B - v_2, s^B]} \left(v_1 - (1-\delta)\, s^A - \delta s^B\right) s^A + \delta \left(v_2 - s^B\right)\left(s^B - s^A\right) + \frac{1}{4}\delta \left(v_2 - s^B\right)^2 \quad (12)$$

and the solution is

$$s^A = \max\left\{\min\left\{\frac{v_1 - \delta v_2}{2(1-\delta)}, s^B\right\}, 2s^B - v_2\right\}. \quad (13)$$

When s^B is very low, there should be almost no trade in period 1 in order to use the information from the other hierarchy. Then, case a gives higher profits to the principal. As s^B increases, principal A starts using the information from hierarchy B by setting $s^A = s^B$ (that is, $p_2^y \le s^B$ is binding, so the second subcase is the relevant one; the optimal cut-off (13) equals to s^B for low s^B). Threshold θ^* is found from equalizing the value of (4) to the value of (12) with $s^A = s^B$. Direct comparison shows that the former is lower at $s^B = \frac{v_1 - \sqrt{\delta} v_2}{2-\sqrt{\delta}}$, thus, $\theta^* < \frac{v_1 - \sqrt{\delta} v_2}{2-\sqrt{\delta}} < \frac{v_1}{2}$ since the principal's profits in the case a do not change with s^B while they increase in s^B in the case b for low s^B.[36]

As s^B increases further, the constraint $p_2^y \le s^B$ stops binding. This happens either when s^A still equals to s^B or when s^A equals to $\frac{v_1 - \delta v_2}{2(1-\delta)}$, see (13). In the latter case, by comparing the value of (12) for $s^A = \frac{v_1-\delta v_2}{2(1-\delta)}$ and the value of (11) for $s^A = 2\frac{v_1 - \delta v_2}{4-3\delta}$ we find threshold θ^b at which the change occurs

$$\theta^b = \frac{1}{2}\left(v_2 + \frac{v_1 - \delta v_2}{\sqrt{(1-\delta)(4-3\delta)}}\right).$$

If $\frac{v_1-\delta v_2}{2(1-\delta)} \le \theta^b$, this is the relevant case and we obtain Lemma 1; otherwise, the optimal cut-off s^A is

$$s^A = \left[\begin{array}{ll} 2\frac{v_1-\delta v_2}{4-3\delta} & \text{if } s^B < \theta^* \\ s^B & \text{if } \theta^* \le s^B < \widehat{\theta} \\ 2\frac{v_1-\delta v_2}{4-3\delta} & \text{if } s^B \ge \widehat{\theta} \end{array}\right.$$

where $\widehat{\theta}$ is found from equality of the value of (12) for $s^A = s^B$ and the value of (11) for $s^A = 2\frac{v_1-\delta v_2}{4-3\delta}$.

[36] In the full-commitment case, the analogous threshold is $\theta^*_{FC} = \frac{v_1 - \sqrt{\delta} v_2}{2-\sqrt{\delta}}$. At this point, the value of (12) with $s^A = s^B$ is $\frac{v_1^2}{4}$ since in the full-commitment autarky case the principal offers $p_1 = p_2 = \frac{v_1}{2}$. The fact that $\theta^* < \theta^*_{FC}$ is a consequence of Proposition 9 in Drugov (2011) and is commented in Section 5 in "Other contractual settings".

Finally, $\frac{v_1-\delta v_2}{2(1-\delta)} \leq \theta^b$ is equivalent to Assumption 1. ■

Proof of Proposition 2. **Part i)** Since the two hierarchies are (ex ante) identical, $s^A\left(s^B\right)$ and $s^B\left(s^A\right)$ are symmetric about the diagonal. Then, every intersection of $s^A\left(s^B\right)$ with the diagonal is an equilibrium and, using Lemma 1, we obtain the interval in the statement of the Proposition.

Let us show that there are no asymmetric equilibria. If $s^B < \theta^*$, then $s^A\left(s^B < \theta^*\right) = 2\frac{v_1-\delta v_2}{4-3\delta}$ but $s^B\left(s^A = 2\frac{v_1-\delta v_2}{4-3\delta}\right) = 2\frac{v_1-\delta v_2}{4-3\delta} > \theta^*$. If $\frac{v_1-\delta v_2}{2(1-\delta)} \leq s^B < \theta^b$, $s^A\left(s^B\right) = \frac{v_1-\delta v_2}{2(1-\delta)}$ but $s^B\left(s^A = \frac{v_1-\delta v_2}{2(1-\delta)}\right) = \frac{v_1-\delta v_2}{2(1-\delta)}$. Finally, if $s^B \geq \theta^b$, $s^A\left(s^B \geq \theta^b\right) = 2\frac{v_1-\delta v_2}{4-3\delta}$ but $s^B\left(s^A = 2\frac{v_1-\delta v_2}{4-3\delta}\right) = 2\frac{v_1-\delta v_2}{4-3\delta} < \theta^b$.

Part ii) The welfare generated by each hierarchy is

$$\int_0^s (v_1-\theta)\, d\theta + \delta \int_s^{\frac{v_2+s}{2}} (v_2-\theta)\, d\theta,$$

where s is the common cut-off of the two hierarchies. A higher s implies more work in period 1 and more work overall. Thus, the highest welfare is reached for the highest possible equilibrium value of $s = \frac{v_1-\delta v_2}{2(1-\delta)}$.

The agents face prices $p_1 = s\left(= p_y^y\right)$ and $p_2^n = \frac{v_2+s}{2}$. A higher s implies that all the prices are higher and, therefore, the agents' rent increases with s.

From (5), the profit of each principal in a symmetric equilibrium with a common cut-off s is $(v_1-s)\, s + \delta\frac{(v_2-s)^2}{4}$. Maximizing this expression with respect to s yields the result that the profits are the highest at $s = \frac{2v_1-\delta v_2}{4-\delta}$. Note that this point belongs to the interval of symmetric equilibria as $\theta^* < \frac{v_1-\sqrt{\delta} v_2}{2-\sqrt{\delta}} < \frac{2v_1-\delta v_2}{4-\delta} < \frac{v_1-\delta v_2}{2(1-\delta)}$.

Part iii) True since $\theta^* < 2\frac{v_1-\delta v_2}{4-3\delta} < \frac{v_1-\delta v_2}{2(1-\delta)}$. ■

Proof of Proposition 3. **Part i)** Analogous to Part i) of Proposition 2. The interval $\left[\theta_\lambda^*, 2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta}\right]$ expands in λ since θ_λ^* decreases in λ (see Lemma 2) while $2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta}$ increases in λ.

Part ii) The welfare generated by each hierarchy is

$$\begin{aligned}&\textstyle\int_0^s (v_1-\theta)\, d\theta + \delta\lambda \int_s^{p_2^n} (v_2-\theta)\, d\theta + \delta(1-\lambda) \int_s^{p_2^i} (v_2-\theta)\, d\theta \\ &= \textstyle\int_0^s (v_1-\theta)\, d\theta + \delta \int_s^{\frac{v_2+s}{2}} (v_2-\theta)\, d\theta\end{aligned} \tag{14}$$

where s is the common cut-off of the two hierarchies, since $p_2^n = p_2^i = \frac{v_2+s}{2}$. A higher s implies more work in period 1 and more work overall. Thus, the highest welfare is reached for the highest possible equilibrium value of $s = 2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta}$. Since $2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta}$ increases with λ, the welfare in the "highest" equilibrium increase with λ.

The agents face prices $p_1 = (1-\delta)\, s + \delta\left(\lambda p_2^y + (1-\lambda)\, p_2^i\right)$, $p_2^y = s$ and $p_2^n = p_2^i = \frac{v_2+s}{2}$ (see (6)). A higher s implies that all the prices are higher and, therefore, the agents' rent increases with s. A higher λ increases $2\frac{v_1-\delta v_2}{4-3\delta-\lambda\delta}$ but it also decreases p_1 for a given s. Depending on the parameters, the agents' rent in the "highest" equilibrium can be decreasing, increasing or U-shaped with respect to λ.

From (6), the profit of each principal in a symmetric equilibrium with a common cut-off s is

$$\begin{aligned}
&(v_1 - p_1)\, s + \delta\lambda \left[(v_2 - p_2^y)(p_2^y - s) + (v_2 - p_2^n)(p_2^n - s) \right] + \delta (1-\lambda)\left(v_2 - p_2^i\right)\left(p_2^i - s\right) \\
= &\left(v_1 - \left((1-\delta)\, s + \delta \left(\lambda s + (1-\lambda)\frac{v_2 + s}{2} \right) \right) \right) s + \frac{1}{4}\delta\, (v_2 - s)^2 .
\end{aligned}$$

Maximizing this expression with respect to s gives the result. The only effect of a higher λ is to decrease p_1 for any given s. Thus, maximum profits increase with λ.

Part iii) For a given s, welfare (14) does not depend on λ; a higher λ decreases p_1 leaving p_2^y, p_2^n and p_2^i unchanged, thus, decreasing the agents' rent.

Part iv) True since $\theta_\lambda^* < 2\frac{v_1 - \delta v_2}{4 - 3\delta} < 2\frac{v_1 - \delta v_2}{4 - 3\delta - \lambda\delta}$. ∎

Proof of Lemma 3. The profit of principal A is[37]

$$\left(p_2^A - s\right)\left(v_{yy} - p_2^A\right) + \left(p_2^B - p_2^A\right) v_{ny} + \left(1 - p_2^B\right) v_{nn}, \text{ if } p_2^A < p_2^B \tag{15a}$$
$$\left(p_2^A - s\right)\left(v_{yy} - p_2^A\right) + \left(1 - p_2^B\right) v_{nn}, \text{ if } p_2^A = p_2^B \tag{15b}$$
$$\left(p_2^B - s\right)\left(v_{yy} - p_2^A\right) + \left(p_2^A - p_2^B\right)\left(v_{yn} - p_2^A\right) + \left(1 - p_2^A\right) v_{nn}, \text{ if } p_2^A > p_2^B \tag{15c}$$

Suppose a candidate symmetric equilibrium $p_2^A = p_2^B$. If principal A deviates to $p_2' < p_2^B$, her profits are given by (15a) and change by

$$\frac{\partial}{\partial p_2'}\left((p_2' - s)(v_{yy} - p_2') + \left(p_2^B - p_2'\right) v_{ny} + \left(1 - p_2^B\right) v_{nn} \right) = s - 2p_2' - v_{ny} + v_{yy} \tag{16}$$

while a deviation to $p_2'' > p_2^B$ changes (15c) by

$$\frac{\partial}{\partial p_2''}\left(\left(p_2^B - s\right)(v_{yy} - p_2'') + \left(p_2'' - p_2^B\right)(v_{yn} - p_2'') + (1 - p_2'')\, v_{nn} \right) = s - 2p_2'' - v_{nn} + v_{yn}. \tag{17}$$

For $p_2^A = p_2^B$ to be an equilibrium, (16) must be non-negative at $p_2' = p_2^A = p_2^B$ and (17) must be non-positive at $p_2'' = p_2^A = p_2^B$. Thus, $p_2^A = p_2^B$ is an equilibrium if and only if

$$\frac{v_{yn} - v_{nn} + s}{2} \leq p_2^A = p_2^B \leq \frac{v_{yy} - v_{ny} + s}{2}.$$

which establishes the condition for the existence of symmetric equilibria.

[37] It should be multiplied by $\frac{1}{1-s}$ to normalize the distribution of θ conditional of being above s. This term is omitted for brevity and since Π_2 will be used from the period-1 perspective later.

Suppose a candidate asymmetric equilibrium $p_2^A < p_2^B$. Optimal p_2^A is found from equalizing (16) to zero which yields $\frac{v_{yy}-v_{ny}+s}{2}$ and optimal p_2^B is found from equalizing (17) to zero which yields $p_2^B = \frac{v_{yn}-v_{nn}+s}{2}$. This is an equilibrium if and only if $\frac{v_{yy}-v_{ny}+s}{2} < \frac{v_{yn}-v_{nn}+s}{2}$.[38] ■

Proof of Lemma 4. The proof is similar to the one of Lemma 1 but with an additional preliminary step of finding the profits in continuation equilibria of period 2, $\Pi_2\left(s^A, s^B\right)$, and how they change with s^A.

Period 2. Substitutes. In the case of substitutes ($v_{yy} + v_{nn} < v_{yn} + v_{ny}$), the expected second-price p_2^n is (8) and the expected profits are the average of (15a) and (15c) with $p_2^j = \frac{v_{yy}-v_{ny}+s}{2}$ and $p_2^{-j} = \frac{v_{yn}-v_{nn}+s}{2}$.

When principal A finds her optimal s^A, she needs to know how it affects her profits in period 2. Let us take an equilibrium with some s^A and s^B. If no agent has accepted the first-period offer, both principals know that the agents' type is above $s = \max\{s^A, s^B\}$. If $s^A < s^B$, then a small deviation to $\widetilde{s}^A < s^B$ does not affect the continuation equilibrium. If $s^A = s^B$ and principal A deviates to $\widetilde{s}^A < s^B$ the continuation equilibrium is not affected either.

Consider now the case $s^A \geq s^B$ and a deviation to $\widetilde{s}^A > s^B$. Since principal B does not observe the first-period contract, she cannot detect a deviation from s^A. She, therefore, continues to act as if the agents' type were above s^A, that is, her second-period price p_2^B does not change. Principal A, however, sets a different price $\widetilde{p}_2^A$.

In the continuation equilibrium $p_2^A < p_2^B$, she maximizes (15a) with respect to her price which yields $\widetilde{p}_2^A = \frac{v_{yy}-v_{ny}+\widetilde{s}^A}{2}$. Note that this does not depend on p_2^B and s^A. The marginal effect of $\widetilde{s}^A$ is found from differentiating (15a) with respect to $\widetilde{s}^A$ for $s = \widetilde{s}^A$ and $p_2^A = \widetilde{p}_2^A$ and it is equal to $\frac{1}{2}\left(\widetilde{s}^A - v_{ny} - v_{yy}\right)$.

In the continuation equilibrium $p_2^A > p_2^B$, she maximizes (15c) with respect to her price which yields $\widetilde{p}_2^A = \frac{v_{yn}-v_{nn}+\widetilde{s}^A}{2}$. Note that this does not depend on p_2^B and s^A. The marginal effect of $\widetilde{s}^A$ is found from differentiating (15c) with respect to $\widetilde{s}^A$ for $s = \widetilde{s}^A$ and $p_2^A = \widetilde{p}_2^A$ and it is equal to $\frac{1}{2}\left(\widetilde{s}^A - v_{nn} + v_{yn} - 2v_{yy}\right)$.

The expected marginal effect of deviating at s^A on the profits in the continuation equilibrium is then

$$\frac{\partial \Pi_2\left(s^A, s^B\right)}{\partial s^A} = \left[\begin{array}{l} 0, \text{ if } s^A < s^B \text{ or } s^A = s^B \text{ and } ds^A < 0 \\ \\ \frac{1}{4}\left(2s^A - v_{nn} - v_{ny} + v_{yn} - 3v_{yy}\right), \text{ otherwise} \end{array}\right. \tag{18}$$

Period 2. Complements. In the case of complements ($v_{yy} + v_{nn} \geq v_{yn} + v_{ny}$), we assume that the "average" symmetric equilibrium played, $p_2^A = p_2^B$ are given by (8). The profits are (15b).

[38] As before, we assume that principals are willing to contract with the agent in period 2, that is, $v_{yy} - v_{ny}$ and $v_{yn} - v_{nn}$ are higher than s. See fn. 26.

Suppose that principal A deviated to some $\widetilde{s}^A$. Principal B still sets p_2^B given by (8). If she sets $\widetilde{p}_2^A > p_2^B$, then from maximizing (15c) for $s = \widetilde{s}^A$ gives $\widetilde{p}_2^A = \frac{v_{yn} - v_{nn} + \widetilde{s}^A}{2}$. For small deviations it is close to $\frac{v_{yn} - v_{nn} + s}{2}$ and, therefore, smaller than (15c). If principal A sets $\widetilde{p}_2^A < p_2^B$, then from maximizing (15a) for $s = \widetilde{s}^A$ gives $\widetilde{p}_2^A = \frac{v_{yy} - v_{ny} + \widetilde{s}^A}{2}$. For small deviations it is close to $\frac{v_{yy} - v_{ny} + s}{2}$ and, therefore, bigger than (15c). Thus, principal A sets the same price $\widetilde{p}_2^A = p_2^B$

Then, the marginal effect of s^A is found from differentiating (15b) with respect to $\widetilde{s}^A$ for for $s = \widetilde{s}^A$ and it is equal to $p_2^A - v_{yy} = \frac{1}{4}(2s - v_{nn} - v_{ny} + v_{yn} - 3v_{yy})$. The marginal effect of deviating at s^A on the profits in the continuation equilibrium is the same as in the case of substitutes, (18).

Case a. The principal solves (we again omit superscript "A" in the prices set by principal A)[39]

$$\begin{cases} \max\limits_{p_1, p_2^n, s^A} s^B v_{yy}(1+\delta) + \left(s^A - s^B\right)(v_{yn} + \delta v_{yy}) - p_1 s^A + \left(1 - s^A\right) v_{nn} + \delta \Pi_2\left(s^A, s^B\right) \\ \\ \text{s.t. } s^A \geq s^B, s^A = \frac{p_1 - \delta p_2^n}{1-\delta} \text{ and } p_2^n \text{ given by (8).} \end{cases} \tag{19}$$

Plugging $p_1 = (1-\delta) s^A + \delta p_2^n$ and p_2^n into the objective function and maximizing with respect to s^A yields

$$s^A = \max\left\{2\frac{v_{yn} - v_{nn}}{4 - 3\delta}, s^B\right\}.$$

Case b. The principal solves

$$\begin{cases} \max\limits_{p_1, p_2^y, p_2^n, s^A} (v_{yy}(1+\delta) - p_1) s^A + \left(s^B - s^A\right) v_{ny} + \left(1 - s^B\right) v_{nn} \\ \\ +\delta\left[(v_{yy} - p_2^y)\left(p_2^y - s^A\right) + \left(s^B - p_2^y\right) v_{ny} + \Pi_2\left(s^A, s^B\right)\right] \\ \\ \text{s.t. } \ s^A \leq s^B,\ s^A = \frac{p_1 - \delta p_2^y}{1-\delta},\ p_2^y = \min\left\{\frac{v_{yy} - v_{ny} + s^A}{2}, s^B\right\} \end{cases} \tag{20}$$

The expression for p_2^y comes from maximizing $(v_{yy} - p_2^y)\left(p_2^y - s^A\right) + \left(s^B - p_2^y\right) v_{ny}$ subject to the constraint $p_2^y \leq s^B$.

As in the proof of Lemma 1, there are two subcases depending on whether $p_2^y = \frac{v_{yy} - v_{ny} + s^A}{2}$ or $p_2^y = s^B$. Following the same steps as there and noting that $\frac{\partial \Pi_2\left(s^A, s^B\right)}{\partial s^A} = 0$

[39] We formally allow $s^A = s^B$ to make the feasible set compact. However, in this case principal A does obtain information from the other firm and, therefore, she gets a strictly higher payoff in the case b.

(see (18)), we obtain that in the former subcase

$$s^A = \min\left\{2\frac{v_{yy}-v_{ny}}{4-3\delta}, 2s^B - (v_{yy}-v_{ny})\right\}$$

and in the latter one

$$s^A = \max\left\{\min\left\{\frac{v_{yy}-v_{ny}}{2(1-\delta)}, s^B\right\}, 2s^B - (v_{yy}-v_{ny})\right\}.$$

Equalizing the values of (20) in the two subcases gives θ_c^b. The condition $\frac{v_{yy}-v_{ny}}{2(1-\delta)} \leq \theta_c^b$ is equivalent to $3\delta^3 - 4\delta^2 - \delta + 1 \geq 0$ which holds for δ below ~ 0.4548.

Finally, in order for the principals to contract with the agent in period 2, $v_{yn} - v_{nn}$ and $v_{yy} - v_{ny}$ must be above $2\frac{v_{yn}-v_{nn}}{4-3\delta}$ and $\frac{v_{yy}-v_{ny}}{2(1-\delta)}$. The two of these constraints are implied by $\delta \leq 0.4548$ and the other two can be written as $\frac{2}{4-3\delta} \leq \frac{v_{yy}-v_{ny}}{v_{yn}-v_{nn}} \leq 2(1-\delta)$. ■

Proof of Proposition 4. When there are symmetric equilibria, threshold θ_c^* is found from equalizing the value of (19) to the value of (20) at $s^A = s^B$ and is given by the smaller root of the following equation

$$\begin{aligned}&\left(2-\tfrac{1}{2}\delta\right)\left(s^B\right)^2 - \left(2v_{yn} + \delta v_{yy} - (2+\delta)\, v_{nn}\right) s^B \\ &\quad + (v_{yn} - v_{nn})\, \tfrac{(2-\delta)v_{yn} - (2+\delta)v_{nn} + \delta v_{ny} + \delta v_{yy}}{4-3\delta} = 0.\end{aligned} \tag{21}$$

For Example (7) and $\rho = -1$, (21) writes as

$$\left(2-\frac{1}{2}\delta\right)\left(s^B\right)^2 - c\,(2\delta - c\delta + 8)\, s^B + 8c^2\frac{4-2\delta+c\delta}{4-3\delta} = 0.$$

Symmetric equilibria exist if the smaller root is smaller than $\frac{v_{yy}-v_{ny}}{2(1-\delta)} = 2c\frac{1-c}{1-\delta}$, that is,

$$\begin{aligned}&\frac{c}{4-\delta}\left(2\delta - c\delta + 8 - \sqrt{\delta\frac{128c + 112\delta + 3c^2\delta^2 - 48c\delta + 12\delta^2 - 12c\delta^2 - 4c^2\delta - 128}{3\delta - 4}}\right) \\ \leq\ & 2c\frac{1-c}{1-\delta}.\end{aligned}$$

Plotting this inequality reveals that it is equivalent to $\delta \geq \widehat{\delta}$, where $\widehat{\delta}$ is increasing in c.

When $\delta < \widehat{\delta}$, symmetric equilibria do not exist. One hierarchy is then necessarily in the case a and, therefore, sets its cut-off at $2\frac{v_{yn}-v_{nn}}{4-3\delta}$. The other hierarchy is in the case b and sets its cut-off at $\frac{v_{yy}-v_{ny}}{2(1-\delta)}$ if $2\frac{v_{yn}-v_{nn}}{4-3\delta} < \theta_c^b$ and at $2\frac{v_{yy}-v_{ny}}{4-3\delta}$ otherwise. ■

Proposition 5 *Under complete information and competition, there are only the following equilibria in cut-off strategies*[40]

- *In the case of substitutes* ($v_{yy} + v_{nn} < v_{yn} + v_{ny}$),
  $s^j = (v_{yy} - v_{ny})(1+\delta) < s^{-j} = (v_{yn} - v_{nn})(1+\delta)$, $j = A, B$, *in period* 1 *and no contracting in period* 2*;*

- *In the case of complements* ($v_{yy} + v_{nn} \geq v_{yn} + v_{ny}$),
  $s^A = s^B = s_1 \in [(v_{yn} - v_{nn})(1+\delta), (v_{yy} - v_{ny})(1+\delta)]$ *in period* 1*. In period* 2 *there is either no contracting or, if* $s^A = s^B < v_{yy} - v_{ny}$, $s_2^A = s_2^B \in (s_1, v_{yy} - v_{ny}]$*.*[41]

Proof. The proof consists of two steps. First, we restrict the principals to contract with the agents in period 1 or not at all and find the candidate equilibria. Second, allowing for contracting in period 2 we show that the candidate equilibria found in the first step are indeed equilibria but some other equilibria involving contracting in period 2 also obtain.

I. Suppose that the principals can contract with their agents only in period 1. If principal B contracts with her agent, the net gain for principal A of contracting with her agent is $(1+\delta)(v_{yy} - v_{ny})$. If principal B does not contract with her agent, the net gain for principal A of contracting with her agent is $(1+\delta)(v_{yn} - v_{nn})$. If $v_{yy} - v_{ny} < v_{yn} - v_{nn}$ (substitutes), one principal sets $s^j = (1+\delta)(v_{yy} - v_{ny})$ and the other principal sets $s^{-j} = (1+\delta)(v_{yn} - v_{nn})$. If $v_{yy} - v_{ny} \geq v_{yn} - v_{nn}$ (complements), the two principals set $s^A = s^B \in [(v_{yn} - v_{nn})(1+\delta), (v_{yy} - v_{ny})(1+\delta)]$.

II. Now we check when the principals find it optimal to contract in period 2. Suppose that the agents are of type θ. For principal A there are three cases depending on when principal B contracts with her agent.

i) If principal B contracts with her agent in period 1, principal A prefers to contract with her agent in period 2 rather than in period 1 or not to contract at all if and only if $v_{ny} + \delta(v_{yy} - \theta) > v_{yy}(1+\delta) - \theta$ and $v_{ny} + \delta(v_{yy} - \theta) > v_{ny}(1+\delta)$. Rewrite these inequalities as $\theta > \frac{v_{yy} - v_{ny}}{1-\delta}$ and $\theta < v_{yy} - v_{ny}$. They are incompatible.

ii) If principal B does not contract with her agent in either period, principal A prefers to contract with her agent in period 2 rather than in period 1 or not to contract at all if and only if $v_{nn} + \delta(v_{yn} - \theta) > v_{yn}(1+\delta) - \theta$ and $v_{nn} + \delta(v_{yn} - \theta) > v_{nn}(1+\delta)$. Rewrite these inequalities as $\theta > \frac{v_{yn} - v_{nn}}{1-\delta}$ and $\theta < v_{yn} - v_{nn}$. They are incompatible.

iii) If principal B contracts with her agent in period 2, principal A prefers to contract with her agent in period 2 rather than in period 1 or not to contract at all if and only if $v_{nn} + \delta(v_{yy} - \theta) > v_{yn} + \delta v_{yy} - \theta$ and $v_{nn} + \delta(v_{yy} - \theta) > v_{nn} + \delta v_{ny}$. Rewrite these

[40] By "cut-off strategies" we mean that if a principal contracts with her agent of type θ in a given period, she also contracts with her agent of all the types lower than θ in the same period. This restriction is imposed for better comparability with the asymmetric information case when all the equilibria are necessarily in cut-off strategies.

[41] Note that equilibria with contracting in period 2 are possible only if $(1+\delta)(v_{yn} - v_{nn}) \leq v_{yy} - v_{ny}$.

inequalities as $\theta > \frac{v_{yn}-v_{nn}}{1-\delta}$ and $\theta < v_{yy} - v_{ny}$. Thus, there are symmetric continuation equilibria $s_2^A = s_2^B \in [\frac{v_{yn}-v_{nn}}{1-\delta}, v_{yy} - v_{ny}]$ provided this interval exists which is possible only in the case of complements.

Thus, in the case of substitutes contracting in period 2 is never optimal and setting $s^j = (1+\delta)(v_{yy} - v_{ny})$ and $s^{-j} = (1+\delta)(v_{yn} - v_{nn})$, found in step I, is an equilibrium. In the case of complements $s^A = s^B = s_1 \in [(v_{yn} - v_{nn})(1+\delta), (v_{yy} - v_{ny})(1+\delta)]$ combined with either no contracting in period 2 or $s_2^A = s_2^B \in [s_1, v_{yy} - v_{ny}]$ are equilibria.

Finally, note that without assuming cut-off strategies there are many other equilibria. For example, in the case of complements, in period 1 the two principals can contract with the types in any subset of $[(v_{yn} - v_{nn})(1+\delta), (v_{yy} - v_{ny})(1+\delta)]$. ■

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