No. 907

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ISSN 0265-8003

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Discussion Paper No. 907
January 1994

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CEPR Discussion Paper No. 907

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ABSTRACT

Choosing the Width of Exchange Rate Bands – Credibility vs. Flexibility*

The purpose of this paper is to provide a framework for the analysis of policy-makers' choices regarding unilateral exchange rate bands. Exchange rate bands are viewed as the outcome of an optimization problem by a policy-maker whose objective function weighs the level of the real exchange rate against the level and variability of the real exchange rate. The analysis endogenizes policy decisions about realignments and about the width of the band. Conditions are developed for the case in which the optimal band width widens in response to a decrease in policy-makers' commitment reputation, an increase in the cost of reneging on the existing band, and an increase in the variance of fundamental shocks.

JEL classification: F31, F33
Keywords: exchange rates, realignments, commitment, credibility

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*This paper is produced as part of a CEPR research programme on *Market Integration, Regionalism, and the Global Economy*, supported by a grant from the Ford Foundation (no. 920-1265). The authors thank Giuseppe Bertola and Lars Svensson for their detailed comments on a previous draft and Gil Bufman for his valuable help on various aspects of the paper. Earlier versions of the paper were presented at the CEPR/Pinhas Sapir/Bank of Israel conference on 'Capital Mobility: New Perspectives', Tel-Aviv, December 1992; at the Research Department Seminar of the IMF; at the Macroeconomics Workshop of Tel-Aviv University; at the Summer Meetings of the Society for Economic Dynamics and Control, Nafplio, Greece, June 1993, and at the Latin American Econometric Society Meeting, Tucuman, Argentina, August 1993.

Submitted 4 October 1993

NON-TECHNICAL SUMMARY

An increasing number of countries have adopted in recent years unilateral exchange rate bands. Under this regime, there is a policy commitment to intervene so that exchange rates fall within a zone of known width around an announced reference rate. Some of the Nordic countries as well as Chile, Israel and Mexico fall into this category. Other countries, especially LDCs and former socialist economies are presently considering the adoption of exchange rate bands. Adopting such a band forces the policy-maker to take a stand on key choices regarding the width of the band, the frequency and form of realignments, the method of intervention for supporting the band, and the specific exchange rate to be used as a central parity.

Most of the existing literature on currency bands offers little guideance on the policy trade-offs involved in these choices. While some of the earlier work on exchange rate target zones partially dealt with these important policy choices, the recent voluminous literature on target zones has taken the existence of bands and their width to be exogenous. Thus recent work can not easily account for policy changes such as the recent widening of exchange rate bands in Europe.

The purpose of this work is to provide a framework for analysing policy-makers' choices regarding unilateral exchange rate bands. We view exchange rate bands as the outcome of an optimization problem of a policy-maker whose objective function weighs the level of the real exchange rate against the level and variability of the nominal exchange rate. We believe this formulation captures an important real world aspect of exchange rate policy determination in countries such as Chile, Israel and Mexico, where the authorities have shown their concern to preserve and improve the competitiveness of exports and the current account position, while at the same time avoiding the possible inflationary consequences of nominal exchange rate depreciation. Exchange rate bands are seen in this context as a simple and verifiable system for the policy-maker to make a credible inflation-target commitment while allowing for some degree of exchange rate flexibility needed to shield exports and the current account from the impact of adverse shocks.

Our analysis endogenizes policy decisions about band width and about realignments. In particular, we consider conditions under which the shocks are such that reneging on the existing commitment and setting a new set of parameters for the band is optimal from the policy-maker standpoint. Imperfect band credibility arises from the public's uncertainty about the size of the shocks and about the strength of commitment of the policy-maker in office. We view the determination of band width as a choice between credibility and flexibility.

We show conditions under which optimal band width widens in response to an increase in the policy-makers' commitment reputation, an increase in the cost of reneging on the existing band, and an increase in the variance of fundamentals. In general, the existence of a band has a moderating impact on the variability of expected currency depreciation. This moderating effect is stronger the greater is the reputation of policy-makers and the higher is the cost of reneging on the band. The model is also used to discuss the credibility of the band and its dependence on policy-makers' reputation, on the cost of reneging on the band, and on the position of the exchange rate within the band. An important empirical implication of the model is that the contribution of expected realignments to expected depreciation increases as the exchange rate increases toward the upper bound of the band.

1. INTRODUCTION

An increasing number of countries have, in recent years, adopted unilateral exchange rate bands. Under this regime, there is a policy commitment to intervene so that exchange rates fall within a zone of known width around an announced reference rate. Some of the Nordic countries as well as Chile, Israel and Mexico fall into this category. Other countries, especially LDCs and former socialist economies are presently considering the adoption of exchange rate bands. Adopting an exchange rate band forces the policymaker to take a stand on key choices regarding the width of the band, the frequency and form of realignments,[1] the method of intervention for supporting the band, and the specific exchange rate to be used as a central parity.

It turns out that most of the existing literature on currency bands offers little guide on the policy tradeoffs involved in these choices. While some of the earlier work on exchange rate target zones partially dealt with these important policy choices,[2] the recent voluminous literature on target zones has taken the existence of bands and their width to be exogenous. That is, the recent work,[3] although elegant and useful, has not addressed the considerations involved in real world decisions whether or not to adopt an exchange rate band and it cannot easily account for policy changes such as the recent widening of

[1] For example, the bands in Chile, Israel, and Mexico feature crawling central parity rates, where the daily rate of crawl (i.e., exchange rate depreciation) is announced in advance. In addition, the width of Mexico's band increases automatically as time progresses. For a description, see Helpman and Leiderman (1992) and Cukierman, Kiguel, and Leiderman (1993a).

[2] See, for example, Williamson (1985), Frenkel and Goldstein (1986), and Williamson and Miller (1987).

[3] For comprehensive surveys of recent work see chapters 1 and 2 in Krugman and Miller (1992a), and Svensson (1992a).

currency bands in Europe to ± 15 percent around a central parity. In Krugman's own words:

> "... the target zone literature has given us a wonderfully lucid account of how a currency band might work, but it has shed little light on why such an exchange regime might be desirable." (Krugman, in Krugman & Miller (1992a), p.14)).

Similarly, Krugman and Miller (1992b) have argued that the academic target zone literature has not addressed the case for target zones made by policymakers themselves.

The purpose of this paper is to provide a framework for analyzing policymakers' choices regarding unilateral exchange rate bands. We view exchange rate bands as the outcome of an optimization problem of a policymaker whose objective function weighs the level of the real exchange rate against the level and variability of the nominal exchange rate. We believe this formulation captures an important real world aspect of exchange rate policy determination in countries such as Chile, Israel, or Mexico where the authorities have shown their concern to preserve and improve the competitiveness of exports and the current account position, while at the same time avoiding the possible inflationary consequences of nominal exchange rate depreciation.[4] Exchange rate bands are seen in this context as a simple and verifiable system for the policymaker to make a credible anti–inflation commitment while allowing for some degree of exchange rate flexibility needed to shield exports and the current account from the impact of adverse shocks.[5] Our

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[4] That this is the case for Chile is transparent from a recent statement by the president of the Banco Central de Chile; see Zahler (1992), who focuses his discussion on policy dilemmas that arise under high capital mobility.

[5] Different rationales for the existence of exchange rate bands are provided in recent work by Krugman and Miller (1992b) and Svensson (1992b). The former argue that the real world motivation for target zones is, to a large extent, the concern about irrational and unstable market behavior. The latter stresses the role of exchange rate bands in increasing the independence and flexibility of monetary policy compared to fixed exchange rates.

model differs from most existing models of exchange rate bands in two main aspects. First, the width of the band is determined endogenously as the result of maximization of policymakers' objectives. Second, we endogenize policymakers' decisions about realignments. That is, we consider conditions under which the realization of shocks is such that reneging on the existing commitment and setting a new set of parameters for the band is optimal from policymakers' perspective. In this context, imperfect band credibility arises from the public's uncertainty about the size of the shocks and about the strength of commitment of the policymaker in office. We view the determination of the band width as a choice between credibility and flexibility. This policy tradeoff has recently been investigated by Flood and Isard (1989) and Lohmann (1992) for closed economies and by Cukierman, Kiguel and Liviatan (1992) for open economies.[6] Whether an existing band is maintained or not depends on the realization of shocks and on the political cost of abandoning the band. These costs imply the existence of a range of effective commitment in which the preannouncement of the band prevents policymakers from adjusting the exchange rate. Since these costs apply only when the band is violated, the range of effective commitment is outside the band.

After describing the optimal determination of band width by the policymaker, we use the model to characterize the relation between underlying parameters and band width. In particular, we show conditions under which optimal band width narrows in response to

[6] The models in the last paper and in this one complement each other. In Cukierman et.al. the exchange rate is pegged by assumption, but policymakerrs can set the degree of commitment to the peg by choosing the cost of reneging on it. Here the cost of reneging on the band is taken as given, but policymakers can influence the exante tradeoff between credibility and flexibility by choosing the width of the band. Also relevant in this context is the work on exchange–rate escape clauses, which are a specific form of limited commitment, by Persson and Tabellini (1990) and Obstfeld (1991).

an increase of the policymaker's commitment reputation, a decrease in the cost of reneging on the existing band, and a decrease in the variance of fundamental shocks. In general the existence of a band has a moderating impact on the variability of expected currency depreciation.[7] This moderating effect is stronger the higher are the reputation of policymakers and the cost of reneging on the band. The model is also used to discuss the credibility of the band and to characterize its dependence on the reputation of policymakers, the cost of reneging on the band, and the position of the exchange rate within the band. We also characterize conditions that give rise to a "Peso problem". An important empirical implication of the model is that the contribution of expected realignments to expected depreciation increases as the exchange rate increases toward the upper bound of the band. We briefly discuss available empirical evidence in support of this implication.

The paper is organized as follows. Section 2 lays out the basic model and develops the notion that an exchange rate band can be seen as a partially credible commitment device. Section 3 shows that each band is associated with a range of effective commitment, that is located outside the band, and analyzes the determination of the rate of change of the exchange rate, including the possibility of realignment. Section 4 examines how the width of the band affects exchange rate expectations. The choice of band width is discussed in section 5. In section 6 we characterize how band width is affected by the reputation of policymakers and the political costs of realignments. The relation between the currency band and the variability of expected depreciation is discussed in section 7. Section 8 defines the credibility of the band and examines how it depends on various

[7] This is somewhat similar to the "honeymoon" effect discussed in the recent target zone literature; see Krugman and Miller (1992a) and Svensson (1992a).

underlying parameters. The section also derives the relation between the contribution of expected realignments to expected depreciation, interest rate differentials, and the position of the exchange rate within the band. Section 9 concludes.

2. A STRATEGIC MODEL OF EXCHANGE RATE BANDS

a. Policy tradeoffs

In the basic model, policymakers face a fundamental short–run tradeoff between the level of the real exchange rate and the level and variability of the nominal exchange rate. This tradeoff arises under either current account shocks and capital account shocks. Accordingly, the policymaker is assumed to choose π to maximize:

$$V(\pi) \equiv x[\pi - \pi^e] - \frac{\pi^2}{2}. \tag{1}$$

Here π and π^e are the actual and the (previously) expected rates of nominal exchange rate depreciation and x is a stochastic variable. Assuming that there exists partial stickiness in nominal wages and prices, expected depreciation affects the predetermined component of these variables, which is assumed to be set in the previous period. Given π^e, the policymaker can then produce a real depreciation by making π larger than π^e, and a real appreciation by making it smaller than π^e. If real trade shocks, or other disturbances, are such that the real exchange rate desired by policymakers differs from its historically given level, they can improve the value of their objectives by appropriately adjusting the nominal exchange rate.

Part 1 of the appendix derives the correspondence between the first part of the objective function in equation (1) and the divergence between the desired and the existing real exchange rates explicitly. This analysis shows that x is directly related to the existing divergence between the actual and the desired real exchange rates at the historically given value of the nominal rate of exchange. This divergence depends, in turn, on various stochastic shocks to fundamentals affecting the current account or the capital account of the balance–of–payments. A positive value of x, in equation (1), means that the desired real exchange rate is larger than the existing one. This could be the result of a fall in the country's terms of trade, e.g. in this case, policymakers derive positive utility from a positive value of unanticipated depreciation. The converse is true when x is negative. When $x = 0$ the actual and the desired real exchange rates are equal.

The second term in equation (1) reflects the costs of nominal exchange rate depreciation and its variability when π is positive,[8] and only the latter when π is negative. We use the same functional form for both positive and negative values of π for simplicity. In countries with persistent exchange depreciation the mean value of x is positive, yet particular realizations may be negative. The mean value of x is a measure of the average degree of dissatisfaction of policymakers with the existing real exchange rate relative to exchange rate variability. Thus x can be considered as a measure of real exchange rate misalignment, as perceived by policymakers.

b. Exchange rate bands as a partially credible commitment device

We consider exchange rate bands as a partial commitment device. It is partial because it commits policymakers to maintain the (nominal) exchange rate within the band only for some realizations of the fundamental shocks. It will be shown that by undertaking

[8] The presumption is that depreciations are inflationary.

this commitment, policymakers are able to increase their credibility, and consequently lower exchange rate depreciation expectations, and thus increase the value of their objective function when the support of x is predominantly positive.[9] The cost associated with this gain in credibility is a loss in flexibility. The politically optimal width of the band is determined by balancing the benefits of credibility against the cost of reduced flexibility.[10]

The timing of moves is modeled as follows. In the first stage, policymakers announce a central parity rate and a (symmetric) band around it. In the second stage the value of x realizes. In the third stage, expectations about exchange rate changes are formed (and wages and prices are set in accordance with those expectations). In the last and fourth stage, the actual rate of nominal exchange rate depreciation (or appreciation) is chosen by policymakers. This timing of moves is summarized in Figure 1.

Figure 1 to be inserted here

Since typically there is considerable uncertainty about the commitment ability of policymakers, we assume that there are two possible types of policymakers. One, to which we refer as dependable (D), incurs a fixed cost — c — whenever he allows the exchange rate to be set outside the band after having preannounced the existence of such a band. The other type, referred to as weak (W) incurs no such cost.[11] We assume that when a

[9] The support of x is the range of values of x for which the probability density of x is positive.

[10] This tradeoff has recently been investigated in Flood and Isard (1989), Lohmann (1992), Cukierman, Kiguel and Liviatan (1992) and Cukierman (1992). As far as we know the present work is the first to apply this approach to the choice of exchange rate bands.

[11] This difference in costs may be thought of as reflecting a difference in rates of time preference between the two types in a framework in which the cost of reneging arises from a reduction in future credibility.

given band is announced, the public holds a probability α that the policymaker in office is of type D and a probability $1-\alpha$ that he or she is of type W.[12]

The equilibrium strategies of policymakers and the equilibrium value of expectations can be obtained by using the principle of dynamic programming or dynamic consistency. Given the choice of band, the realization of x and the level of expectations, the policymaker picks π in stage 4 so as to maximize the value of the objective function in equation (1). If the policymaker is of type D he also takes into consideration the cost of reneging on the band. In stage 3, expectations are formed on the basis of the public's knowledge of the objective function (equation (1)), the previously realized value of x, and the probability, α, that the band has been preannounced by a dependable policymaker. In stage 1 the width of the band is chosen so as to maximize the expected value of the objective function in equation (1) taking into consideration the way expectations are formed and the fact that the policymaker knows, already in stage 1, his decision rule for stage 4 as a function of x. If type D is in office he also takes into consideration the cost of reneging on the band which will be incurred for some realizations of x. Policymakers of type W are not subject to this cost. Hence they do not take it into consideration when announcing the band. However, it will be shown later on that type W always finds it advantageous to mimic the announcement of D. Hence the band chosen by both policymaker types is determined by the solution to the decision problem of the dependable policymaker in the first stage.

[12] This way of modeling uncertainty about the ability to commit has been used in Barro (1986), Cukierman and Liviatan (1991) and Cukierman, Kiguel and Liviatan (1992). In Cukierman, Kiguel and Leiderman (1993a) the present model is extended to account for various degrees of transparency of policymakers' announcements about a band.

3. EQUILIBRIUM EXCHANGE RATE CHANGES AND THE RANGE OF EFFECTIVE COMMITMENT

Suppose type W is in office. Since this policymaker faces no cost of abandoning the band, the value of π is chosen so as to maximize the value of the objective function in (1) for all values of x. This maximization problem has the solution:

$$\pi = x \qquad \text{for all } x . \tag{2}$$

This is analogous to the well–known Barro–Gordon (1983) discretionary solution for inflation. They assume x is positive and therefore obtain an inflationary bias. However, in the present context the realization of x may be either positive or negative, and thus π may be of either sign.

When D is in office he also behaves according to the decision rule in equation (2), but only as long as the rate of exchange falls within the band. When the decision rule in equation (2) calls for abandoning the band, D may or may not abandon the band, depending on whether the value of his objectives net of the cost of reneging on the band is larger or smaller than that value when the band is maintained. Which of these two magnitudes is larger depends on the realization of x, on the width of the band, and on the initial position of the exchange rate within the band.

Let B be the (one–sided) width of the band, e_c the band's central parity rate, e_{-1} the exchange rate inherited from the previous period, $\bar{e}$ the upper limit for e under the band, and $\underline{e}$ its lower limit. All exchange rates are defined as domestic currency units per unit of foreign currency. Hence, an increase in e denotes a depreciation of the domestic currency. By definition for a symmetric band,

$$\bar{e} \equiv (1 + B)e_c \ , \ \underline{e} \equiv (1 - B)e_c \ . \tag{3}$$

Let π_m be the maximum permissible depreciation that would leave the exchange rate within the band. By definition

$$\pi_m \equiv \frac{\bar{e}}{e_{-1}} - 1 = (1 + B)\frac{e_c}{e_{-1}} - 1 \ . \tag{4}$$

Suppose that the realization of x is such that $x > \pi_m$. If D follows the strategy of his weak counterpart he has to incur the cost, c, for reneging on the band. He abandons the commitment to the band if and only if[13]

$$V(\pi_m) < V(x) - c \ . \tag{5}$$

Using (1) in (5) and rearranging, condition (5) becomes equivalent to

$$\frac{x^2}{2} - \pi_m x + \frac{\pi_m^2}{2} - c > 0 \ . \tag{5a}$$

This is a quadratic equation in x which has a (negative) minimum and two roots that are given by

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[13] When $x > \pi_m$, $V(\pi_m) > V(x^1)$ for all $x^1 < \pi_m$. Hence, if he decides to respect the band, D is always better off setting π equal to π_m rather than to any other lower value of x.

$$x_{1,2} = \pi_m \pm \sqrt{2c}\,. \tag{6}$$

Since we started from the premise that $x > \pi_m$ the negative root is irrelevant.

<u>Lemma 1</u>: D devalues and adjusts the band upward if and only if

$$x > \pi_m + \sqrt{2c} \equiv \pi_m + d\,. \tag{7}$$

In the particular case in which the initial exchange rate is at the central parity rate ($e_{-1} = e_c$) this condition reduces to

$$x > B + \sqrt{2c}\,. \tag{7a}$$

In summary, when x is positive D's equilibrium policy is

$$\pi = \begin{cases} x & \text{for } 0 \leq x \leq \pi_m \text{ and } x > \pi_m + d \\ \pi_m & \text{for } \pi_m < x \leq \pi_m + d \end{cases}\,. \tag{8}$$

Thus, the existence of a band produces a <u>range of effective commitment</u> that is <u>outside</u> the upper limit of the band. The range of exchange rates over which this commitment is binding is illustrated in Figure 2. It is given by $\bar{e} < e \leq \bar{e} + e_{-1}\sqrt{2c}$. The specific commitment is that when fundamental shocks tempt policymakers to move the exchange rate into that range, the dependable policymaker resists the temptation and effectively maintains the exchange rate at $\bar{e}$. Equivalently, in terms of the realizations of x, the range of effective commitment is

Figure 2 to be inserted here

$$\pi_m < x \leq \pi_m + \sqrt{2c}\ .$$

What happens when the realization of x is negative? The analysis follows along similar lines. For negative x's discretion leads to an appreciation of the currency (see equation 2). Let π_r be the maximum rate of appreciation that would leave the exchange rate within the band. By definition[14]

$$\pi_r \equiv 1 - \frac{\underline{e}}{e_{-1}} = 1 - (1-B)\frac{e_c}{e_{-1}}\ .$$

If $x \geq -\pi_r$ D lets the currency appreciate at rate x. If $x < -\pi_r$ D maintains the band if and only if

$$V(\pi_r) > V(x) - c\ .$$

In such a case the currency appreciates up to the lower limit of the band $\underline{e}$. Using arguments similar to those used for the case of a positive x it can be shown that

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[14] When $e_{-1} = e_c$, $\pi_r = B$.

LEMMA 2: D revalues and adjusts the band downward if and only if[15]

$$x < -(\pi_r + \sqrt{2c}) . \tag{9}$$

From now on, we shall use the term realignment to indicate situations in which the new exchange rate ends up outside the existing band, thus implying that a new band is set. The terms depreciation and appreciation will refer mainly to exchange rate fluctuations within an existing band.

Lemma 2 implies that when x is negative

$$\pi = \begin{cases} x & \text{for } -\pi_r \leq x \leq 0 \text{ and } x < (\pi_r + d) \\ -\pi_r & \text{for } -(\pi_r + d) \leq x < -\pi_r \end{cases} . \tag{10}$$

Figure 2 to be inserted here

Again the range of effective commitment is outside the band. But since now x is negative, it is below the lower limit of the band rather than above its upper limit as was the case for positive x.

4. EXCHANGE RATE EXPECTATIONS AND THE WIDTH OF THE BAND

Expectations are formed after the realization of x. Hence the only remaining uncertainty concerns the policymaker's type. Based on equations (8) and (10) the range of x can be usefully segmented into a range in which $\pi = x$ independently of the identity of the policymaker in office and a (two–piece) range of effective commitment in which the

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[15] In the particular case $e_{-1} = e_c$ condition (9) reduces to $x < -(B + \sqrt{2c})$.

weak type sets $\pi = x$ but the dependable type sets $\pi = \pi_m$ or $\pi = -\pi_r$ depending on whether the realization of x is positive or negative. Figure 3 summarizes the strategies of the two policymakers in the different ranges of x. π^D and π^W denote the strategies of the dependable and of the weak policymaker respectively. It follows that expectations of depreciation (or of appreciation) are

$$\pi^e(x) = \begin{cases} x, & x < -(\pi_r + d) \\ -(\alpha\pi_r + (1-\alpha)x), & -(\pi_r + d) \leq x < -\pi_r \\ x, & -\pi_r \leq x \leq \pi_m \\ \alpha\pi_m + (1-\alpha)x, & \pi_m < x \leq \pi_m + d \\ x, & \pi_m + d < x \end{cases} \tag{11}$$

It can be seen from equation (11) that the width of the band affects expectations only in the range of effective commitment. More precisely, by using equation (4) in equation (11) we obtain that for

$$(1+B)\frac{e_c}{e_{-1}} - 1 < x < (1+B)\frac{e_c}{e_{-1}} - 1 + d, \tag{12a}$$

$$\pi^e(x) = \alpha\left[(1+B)\frac{e_c}{e_{-1}} - 1\right] + (1+d)x. \tag{12b}$$

An increase in the width of the band has two effects. First it shifts the range of effective commitment to higher values of x without changing the width of this range.[16] Second, for a given value of x that is in the range of effective commitment both before and after the change, expected depreciation increases. The reason for this is that, with a wider band, the commitment becomes effective at a higher value of the shock x. Expected depreciation also depends on α. It can be seen from equation (12b) that a stronger reputation of the policymaker (a higher α) reduces expectations of depreciation in the range of effective commitment. Similarly, it can be shown that when x is negative (and within the range of effective commitment) an increase in B raises expected appreciation and an increase in α reduces it. Thus a stronger reputation for dependability may reduce expected depreciation when x is positive and expected appreciation when x is negative.

From the perspective of policymakers, an important feature of equation (12b) is that the choice of band's width B can affect expectations. However, given the timing of events (Figure 1), even after the announcement of B, policymakers are uncertain about the value of exchange rate expectations. The expected value, as of stage 1, of expectations can be calculated from equation (11) yielding (after some algebra),

$$E_x\pi^e(x) = Ex - \alpha\left[\int_{\pi_m}^{\pi_m+d}(x-\pi_m)dF(x) + \int_{-(\pi_r+d)}^{-\pi_r}(x+\pi_r)dF(x)\right] \tag{13}$$

Figure 3 to be inserted here

[16] The width of the range of effective commitment depends only on the cost, c, of adjusting the band.

where $F(x)$ is the cumulative distribution function of x. $E_x\pi^e(x)$ is the mean expected change in the exchange rate. Ex is the average depreciation or appreciation bias under discretion. The presence of a band mitigates the effect of this bias on expectations. To see this, consider, for example, the case in which the support of x is predominantly positive. In this case there is an average depreciation bias under discretion and the last term on the right–hand–side of equation (13) is small, in absolute value, in comparison to the term that precedes it. Hence, the entire term in brackets is positive. It follows that the existence of a band reduces the magnitude of the expected depreciation bias. Similarly when Ex is negative and there is an appreciation bias, the existence of a band mitigates the effect of this bias on the average value of expected appreciation. Note that in both cases the moderating effect of a preannounced band on expectations is lower the lower is reputation, α. It is virtually non existent when α tends to zero.

5. CHOOSING THE WIDTH OF THE BAND

As indicated earlier, from the perspective of the dependable policymaker, the choice of band width involves a tradeoff between credibility and flexibility.[17] Concretely D picks the width of the band, B, so as to maximize the expected value of his objective function, taking into consideration the way expectations are formed and the fact that he will follow the contingent policy in equations (8) and (10) in stage 4. Substituting these policies into equation (1) we obtain the following values of objectives for the different ranges of x

[17] A similar, but not identical tradeoff between reduced expectations and flexibility arises in Lohmann (1992) and in Cukierman, Kiguel, and Liviatan (1992). In the second paper the exchange rate is fixed (zero band width) by assumption, but the degree of commitment as measured by the cost of reneging on the exchange rate is a choice variable.

$$-W(x,\pi_m) \equiv V[\pi(x,\pi_m)] = \begin{cases} -[x^2/2 + c] & x < -(\pi_r + d) \\ -[x(1-\alpha)(\pi_r + x) + \pi_r^2/2] & -(\pi_r + d) \leq x < -\pi_r \\ -x^2/2 & \pi_r \leq x \leq \pi_m \\ -[x(1-\alpha)(x-\pi_m) + \pi_m^2/2] & \pi_m < x \leq \pi_m + d \\ -[x^2/2 + c] & \pi_m + d < x \end{cases} \tag{11}$$

We shall assume, without loss of generality, that when policymaker D chooses the band width, the central parity rate is always set equal to the then prevailing actual rate, i.e., $e_c = e_{-1}$. Hence, from equations (4) and (9),

$$\pi_m = \pi_r = B\,. \tag{15}$$

D's problem in stage 1 is

$$\underset{B}{\text{Max}}\ E\ V[\pi(x),B] = \underset{B}{\text{Min}}\ E\ W(x,B) \equiv \underset{B}{\text{Min}}\ L(B)\,. \tag{16}$$

Inserting (15) into (14) and the resulting expression into (16) D's problem may be expressed as

$$\underset{B}{\text{Min}}\ L(B) = \underset{B}{\text{Min}} \int_{\underline{x}}^{-(B+d)} (\frac{x^2}{2} + c)dF(x) + \int_{-(B+d)}^{-B} \left[x(1-\alpha)(B+x) + \frac{B^2}{2}\right] dF(x) \tag{17}$$

$$+ \int_{-B}^{B} \frac{x^2}{2}\, dF(x) + \int_{B}^{B+d} \left[x(1-\alpha)(x-B) + \frac{B^2}{2}\right] dF(x) + \int_{B+d}^{\overline{x}} \left[\frac{x^2}{2} + c\right] dF(x).$$

Here $\underline{x}$ and $\bar{x}$ are the minimal and the maximal possible values of x.

To focus and simplify the analysis we shall, from now on, concentrate on the case of a depreciation bias by assuming that $\underline{x} \geq 0$. This is the case that applies mostly to the countries mentioned in the Introduction. Thus, the realizations of x are restricted to the non–negative range.[18] This assumption eliminates the first two terms in equation (17). The first–order condition for an internal minimum is (after some algebra),

$$L'(\cdot) \equiv \frac{\partial EL(\cdot)}{\partial B^2} = \int_B^{B+d} (B - (1-\alpha)x)dF(x) - \alpha d(B+d)f(B+d) = 0. \tag{18}$$

The second–order condition for an internal minimum is

$$L''(\cdot) \equiv \frac{\partial^2 EL(\cdot)}{\partial B^2} = \int_B^{B+d} dF(x) - \alpha Bf(B) + (\alpha B - d)f(B+d)$$
$$-\alpha d(B+d)f'(B+d) > 0. \tag{19}$$

Here $f(\cdot)$ denotes the density function of x and $f'(\cdot)$ denotes its first partial derivative.

To gain insight into how the width of the band depends on reputation, consider the two extreme cases in which policymakers have extremely weak reputation $(\alpha=0)$ and extremely strong reputation $(\alpha=1)$. In the first case,

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[18] By similarly restricting x to the non–positive range $(\bar{x} \leq 0)$ the case of an appreciation bias can be analyzed too.

$$L'(b) = \int_{B}^{B+d} (B-x)dF(x) < 0$$

for all values of B for which this expression has a non–degenerate probability. Hence, it pays to increase B to the point at which there is no effective commitment of any kind (B $> \bar{x}$). The intuition is that when reputation is extremely weak, a preannounced band has no effect on expectations. Since such an announcement does carry a cost and has no benefit, policymakers opt then for a fully flexible exchange rate system.

At the other extreme, when $\alpha = 1$,

$$L'(B) = [F(B+d) - F(B) - df(B+d)]B - 2cf(B+d) .$$

Now a value of B that is larger than $\bar{x}$ is no longer optimal. To see that let $B = \bar{x} - \epsilon$ where ϵ is an arbitrarily small positive number. In particular $\epsilon < d$. Then

$$L'(B) = L'(x-\epsilon) = [F[\bar{x}-\epsilon+d] - F[\bar{x}-\epsilon][\bar{x}-\epsilon) > 0$$

since $f(B - \epsilon + d) = 0$. Hence, losses can be reduced by reducing B even further. It follows that in the presence of perfect reputation an exchange rate band will be announced.

Taking a linear approximation of f(B) around f(B+d), substituting it into equation (19), and rearranging yields

$$L''(B) = \int_{B}^{B+d} dF(x) - df(B+d) - 2\alpha cf'(B+d) . \qquad (19a)$$

Equation (19a) implies that an internal optimal value of B cannot occur in a range of values of B for which f(x) is predominantly increasing in the range [B,B+d]. For had that been the case $L''(B)$ would be negative and the second–order condition for an internal minimum would be violated. A (somewhat overly strong) sufficient condition for the fulfillment of the second–order condition in (19a) is $f'(x) < 0$ for x in the range [B,B + d].[19] We shall focus for simplicity on internal maxima in which this is the case and on unimodal distributions of x.

<u>ASSUMPTION 1</u>: The distribution of $\tilde{x}$ is unimodal.

<u>ASSUMPTION 2</u>: When the objective function in equation (17) has an internal minimum, $f'(x) < 0$ for all values of x in the effective commitment range, [B, B + d].

Figure 4 to be inserted here

Stated somewhat loosely, the second–order condition for an internal minimum implies that an optimal value of B must occur in a range of values of x for which the probability density function, f(x), decreases in x. Figure 4 illustrates such a situation

The intuition underlying this result can be understood by noting that the expected cost of having a band has two components. One derives from the fact that the policymaker is prevented from following the discretionary policy in the effective commitment range [B, B + d]. The other is that for $x > B + d$ he has to pay a cost c for implementing the preferred ex–post policy. When the segment [B, B + d) is in the increasing range of $f(\cdot)$ the sum of those expected costs exceeds the benefits in terms of lower expected costs of

[19] In this case the difference between the first two terms in (19a) is positive and so is the last term.

expectations of depreciation for two reasons. First, the probability that D will have to abandon the band and incur the cost of reneging is high. Second, by moving the range of effective commitment up sufficiently far to the right, the expected costs of inflexibility are also reduced. The reason is that the probability that x will fall in the effective commitment range decreases by more than the increase in the values of x.[20]

We close this section by noting that the discussion was conducted from the perspective of the dependable policymaker, who incurs a cost if he reneges on the band. In contrast, the weak policymaker always reneges on the band since he does not incur a similar cost. Yet, when in office, such a policymaker always announces the <u>same band</u> as a dependable policymaker would have. This strategy is always preferable to the alternative of being revealed as weak already at the outset since it moderates the expected rate of depreciation.

6. THE EFFECT OF REPUTATION AND THE COST OF RENEGING ON THE WIDTH OF THE BAND

This section considers the impact of changes in the parameters α and c on the optimal choice of B. We do so by performing two comparative–statics experiments. Application of the implicit function theorem to the first–order condition in equation (18) yields (after some algebra)

[20] Obviously it is possible that the optimal B will fall a bit below the mode of x. However, even in that case most of the range $[B, B + d]$ <u>must be</u> to the right of the mode. In particular $f'(B + d)$ is always negative at an optimum. Hence, by making assumption 2 we only ruled out the cases in which a small part of $[B, B + d]$ is to the left of the mode at an internal optimum.

$$\frac{dB}{d\alpha} = \frac{1}{L''}\left[d(B+d)f(B+d) - \int_{B}^{B+d} xf(x)dx\right] \qquad (a)$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (20)$$

$$\frac{dB}{dc} = \frac{1}{L''}[(1+d)f(B+d)+d(B+d)f'(B+d)] . \qquad (b)$$

Since L'' is positive by the second–order condition, the signs of these expressions are determined by the signs of the bracketed expressions in equation (20). Since $f'(B+d)$ is negative, the signs of both expressions are generally ambiguous. But if the probability density $f(\cdot)$ decreases sufficiently fast in the range of effective commitment, $dB/d\alpha$ is negative. The precise condition follows.

<u>PROPOSITION 1</u>: If the elasticity of $f(x)$ with respect to x in the range of effective commitment is not smaller than 1.0 in absolute value, and is strictly larger than 1.0 for at least some value of x, <u>a larger value of α is associated with a narrower band</u>.

<u>PROOF</u>: The first term in the bracketed expression on the right–hand–side of equation (20a) is the product $xf(x)$ at the point $x=B+d$ multiplied by the range, d, of the effective commitment. The second term also sums up products of the type $xf(x)$ over the same range, but lets x vary over the range of effective commitment. When the elasticity of $f(\cdot)$ is uniformly 1 in this range, the first and the second term are equal and $dB/d\alpha = 0$. When the elasticity is larger than 1 for at least some x, $(B+d)f(B+d) < xf(x)$ for these values of x and $dB/d\alpha$ is positive.

□

In order to obtain an intuitive understanding of the proposition as well as of the origin of the generally ambiguous effect of α on B it is useful to focus on the first–order condition that determines B. The economic content of this condition is that the optimal band width is determined by equating the expected marginal costs of inflexibility plus the expected marginal political cost of reneging to the expected marginal gain of lower expectations of depreciation. However, the first marginal quantity is really a cost only if $f(\cdot)$ decreases sufficiently slowly in the range of commitment. The reason is that an increase in B has two conflicting effects on the expected cost of inflexibility. By shifting the range of commitment to higher values of x it raises the expected cost of inflexibility. But it also reduces the probability of x falling in this range and this lowers the expected cost of inflexibility. When $f(\cdot)$ decreases sufficiently fast in the commitment range the second effect dominates so that, at the optimum, the marginal expected costs of inflexibility are negative.

A similar ambiguity arises with respect to the impact of the band on the expected cost of high expectations. More generally it can be shown (see part 2 of the appendix) that the first stage objective function in equation (17) may be decomposed as follows:

$$L \equiv EW(x,B) = \frac{1}{2}\,ECI + ECR + ECE - \frac{S}{2} \qquad (21)$$

where

$$ECI \equiv \int_{B}^{B+d} (x-B)^2 dF(x) \text{ -- Expected cost of inflexibility} \qquad (a)$$

PROPOSITION 3: Under the condition of Proposition 2, an increase in the cost of reneging leads to a reduction in the probability of reneging.

PROOF: When c increases B increases too by Proposition 2. Hence B+d increases and so does F[B+d]. Hence the probability that the policymaker will have to renege — (1–F[b+d]) — is reduced.

□

A priori, it is plausible to argue that the width of the band should rise with the cost of reneging and fall when there is a stronger reputation of policymakers' dependability. The following proposition lays down conditions that make this intuition correct.

PROPOSITION 4: If condition (25) is satisfied and if the absolute value of the elasticity of f(x) is not smaller than 1 in the range of effective commitment, then the width of the band is narrower the stronger the reputation of policymakers (α) and the lower the cost of reneging (c).

PROOF: By combining Propositions 1 and 2.

□

Note that the sign of $dB/d\alpha$ depends on the size of $\eta(x)$ over the entire commitment range whereas the sign of dB/dc depends only on the magnitude of $\eta(b+d)$.

We turn now to the effect of a change in the distribution of shocks on the width of the band. Note, from the first–order condition in (18), that only the characteristics of the distribution within the range of effective commitment affect the equilibrium value of B. Let k be a shift parameter that determines the position of the distribution in this range so that f(x) is rewritten as f(x,k). We shall consider the case in which the effect of k on B(·) in the commitment range is positive. This corresponds roughly to a flattening of the tails of the distribution if B is sufficiently far out from the center of the distribution. Using the implicit function theorem on equation (18)

$$\frac{dB}{dk} = \frac{1}{L^{??}}\left[ad(b+d)\,\frac{\partial f(B+d),k)}{\partial k} - \int_{B}^{B+d} (B-(1-\alpha)x)\,\frac{\partial f(x,k)}{\partial k}\,dx\right]. \tag{26}$$

The sign of this expression depends on the relative rates of increase of f(·) at different values of x in the commitment range. A useful benchmark is the case of an equiproportional increase in all the densities. That is

$$\frac{\partial f(x,k)}{\partial k} = qf(x,k)\ \forall\ x \in [B,B+d\} \tag{27}$$

where $q > 0$. Substituting (27) into (26) we obtain an expression that is proportional to the first–order condition in (18). Hence an equiproportional increase in f(·) for all $x \in [B, B+d)$ does not alter the equilibrium value of B. Suppose alternatively that

$$\frac{\partial f(x,k)}{\partial k} = q > 0\,. \tag{28}$$

In this case smaller values of f(·) increase proportionally more than larger values. This case describes a gradual fattening of tails. Substituting equation (28) into (26) we obtain

$$ECR \equiv c\int_{B+d}^{\overline{x}} df(x) \text{ -- Expected cost of reneging} \qquad (b) \quad (22)$$

$$ECE \equiv Ex\pi^{e}(x) \equiv S - \alpha\int_{B}^{B+d} (x-B)x dF(x)$$
$$\text{-- Expected cost of expectations} \qquad (c)$$

$$\text{and } S \equiv \int_{\underline{x}}^{\overline{x}} \frac{x^2}{2} dF(x) . \qquad (23)$$

From (21c) the expected marginal cost of an increase in B through its impact on expectations is

$$\frac{\partial ECE}{\partial B} = \alpha\left[\int_{B}^{B+d} x\, dF(x) - d(B+d)f(B+d)\right] . \qquad (24)$$

If $f(\cdot)$ decreases sufficiently fast in the commitment range the impact of B on the expected cost of expectations is positive so that an increase in B induces a cost through a higher expected value of expected depreciation. But if $f(\cdot)$ decreases sufficiently slowly, the impact of B on this cost is negative so that an increase in B reduces the cost from expectations.

As a matter of fact (comparing expressions (20a) and (24)) it can be seen that $dB/d\alpha$ and $\partial ECE/\partial B$ have opposite signs. This is not accidental. When an increase in B raises the cost of expectations to the policymaker, a larger reputation reduces the width of the band. In this case an increase in reputation raises the moderating impact of the original band on expected depreciation and raises the marginal benefit of a tighter band through lower expectations. This happens without impact effects on the marginal costs of inflexibility and of reneging. Hence, at the original value of B the total marginal loss becomes positive. Since, by the second–order condition the marginal loss is increasing in B, the new optimal band width must be lower.

Thus the ambiguity about the effect of reputation on the width of the band originates from the ambiguous effect of B on the expected value of the costs of high expectations to the policymaker.

The ambiguity about the effect of an increase in the cost of reneging on the width of the band is due to the ambiguous effect that an increase in c has on $\partial ECR/\partial B$ and on $\partial ECE/\partial B$ at the original level of B. If $f'(B+d)$ is not too negative, both of these marginal costs decrease making the total marginal cost of an increase in B negative at the original equilibrium. To restore equilibrium, B must increase. The following proposition states the precise condition.

<u>PROPOSITION 2</u>: A larger cost of reneging is associated with a wider optimal band if and only if the absolute value of the elasticity of $f(\cdot)$ at $x = B+d$ is smaller than $1 + 1/\alpha$.

<u>PROOF</u>: From equation (20b) dB/dc is positive if and only if

$$|\eta| \equiv -\frac{B+d}{f(B+d)} f'(B+d) < 1 + \frac{1}{\alpha}. \tag{25}$$

□

A corollary to Proposition 2 is

$$\frac{dB}{dk} = \frac{q}{L''} \left[\alpha d(B+d) - \int_{B}^{B+d} (B-(1-\alpha)x)dx \right] . \qquad (26a)$$

The term in brackets in this expression is identical to the term in brackets in the first–order condition (equation (18)) except that it is not weighted by probability densities. By assumption 2 these densities decrease in x. Hence in equation (18), which equals zero, larger values of $B-(1-\alpha)x$ get relatively higher weights. In equation (26a) they all get equal weights. In comparison to equation (18) this increases the relative size of $\alpha d(B+d)$ and of the small (and possibly negative) values of $B-(1-\alpha)x$. Hence $dB/dk > 0$. This result is summarized in the following proposition.

PROPOSITION 5: When smaller values of $f(\cdot)$ in the range of effective commitment increase proportionally more than larger values of $f(\cdot)$ in this range (as specified in equation (28)) the equilibrium width of the band increases.

7. THE VARIABILITY OF EXPECTATIONS, THE BAND AND THE COST OF RENEGING

This section briefly explores the effect of the band and of the political cost of abandoning it on the variability of expected depreciation. It is convenient to perform the analysis on the assumption that the actual exchange rate is at the center rate.[21] From equation (11) specialized to the case $\underline{x} \geq 0$ and $\pi_m = B$ we obtain

[21] This is obviously a partial experiment since the exchange rate need not be equal to the center rate and since the width of the band is itself a function of more basic parameters as seen in the previous section.

$$\pi^e(x) = \begin{cases} x & x \le \pi_m \text{ or } x > \pi_m + d \\ \alpha B + (1-\alpha)x & B < x \le B + d \end{cases} .$$

The solid line in Figure 5 represents expected depreciation as a function of the realization of x for a given band width. In the absence of a band, the relation between $\pi^e(x)$ and x is given by the 45–degree line. The presence of a band flattens this relationship, but only within the range of effective commitment $[B, B+d]$. In all other ranges the relation between $\pi^e(x)$ and x remains along the 45–degree line. Hence, the existence of a band reduces the variability of expected depreciation by reducing it in the range of effective commitment. This reduction, which is similar to the "honeymoon effect" stressed in the recent target zone literature, is stronger the larger is the range of effective commitment, which is the case in turn, the higher is the political cost of reneging on the band. Given this cost, the moderating impact of the band on the variability of expectations is larger the stronger is reputation, α. When reputation is perfect, $\alpha = 1$ and the relation between $\pi^e(x)$ and x in the range of commitment is given by the horizontal line DE. Thus, variability vanishes completely in the effective commitment range when $\alpha = 1$. At the other extreme, when reputation is very low $(\alpha \to 0)$ the relation tends to the segment DF along the 45 degree line. Thus with poor reputation a band has very little impact on the variability of expectations even if the cost of reneging on it is substantial.

Figure 5 can be used to present a taxonomy of different types of exchange rate arrangements. In each case the type of arrangement is determined by the width of the band, B, the political cost of reneging, c(or $d = \sqrt{2}c$), and in some cases by reputation, α. The cases are:

1)	$B \to \infty$	Flexible Exchange rate
2)	$\infty > B > 0$, $d < \infty$	Adjustable band
3)	$\infty > B > 0$, $d \to \infty$, $\alpha = 1$	Irrevocable band
4)	$B = 0$, $d < \infty$, $\alpha = 1$	Adjustable peg
5)	$B = 0$, $d \to \infty$, $\alpha = 1$	Irrevocably fixed exchange rate.

These cases can be understood by reference to Figure 5.

Figure 5 to be inserted here

8. THE INTEREST RATE DIFFERENTIAL, REALIGNMENT EXPECTATIONS AND THE CREDIBILITY OF THE BAND

Recent research has focused on the extent to which movements in the interest rate differential provide useful information about expected realignments. This section illustrates the implications of a modified version of our framework for the interest rate differential, the variability of interest rates, expected realignments (when the center rate changes), and the credibility of the band. One refutable implication of the theory is that the contribution of expected realignments to expected depreciation is larger the nearer is the exchange rate to the upper limit of the band.

Under the maintained hypothesis of uncovered interest parity and a negligible foreign exchange risk premium, the behavior of the interest rate differential reflects the expected rate of depreciation of the currency. This expectation is affected in turn by the characteristics of the distribution of the shock x and the reputation of policymakers. To capture the effect of uncertainty about both the realization of x and the type of policymaker in office on interest rate differentials we consider a modified framework in which the basic sequence of events (Figure 1) repeats twice.[22] We think of each such sequences as occurring within a period. Figure 6 illustrates the extended two–periods framework. The sequence of shock realizations and of decisions in the first period is as described in the previous sections. To prevent the public from discovering the identity of the policymaker after the realization of π, we assume that the policymaker type in office is drawn anew at the beginning of each period from an i.i.d. with stable probability α of being type D. Since the cost c is related to how much policymakers value the future, a change in their evaluation of future political survival may change their dependability even without a formal change in government. If the realizations of x_1 and t_1 (the policymaker type in period 1) are such that the band is maintained $e_1 \leq \bar{e}_1$, $\bar{e}_2 = \bar{e}_1$, and $e_{c1} = e_{c2}$. In such a case the actual exchange rate at the beginning of the second period usually differs from the center rate so that $\pi_m \neq B$. Except for this difference in the position of the opening exchange rate, events and decisions in the second period occur in a manner that is similar to the first.

If x_1 and t_1 are such that there is a realignment, $e_1 > \bar{e}_1$ and a new band is instituted at the beginning of period 2, with the new center rate set at e_1. Since basic parameters such as α and c have not changed, the width of the band remains as in period 1. But now $\bar{e}_2 = e_1\,(1+B)$ since the center rate has been shifted upward. Except for the fact that they occur within the framework of a different band, events and decisions in period 2 then proceed according to the single period principles discussed in previous sections.

[22] However, we maintain the assumption that the policymaker chooses the band taking into consideration only its first–period objective.

Suppose now that <u>after</u> the the choice of π_1 but before the realization of either x_2 or π_2 a one-period loan market opens up. For convenience one can think of the loan as being contracted at point A_1 and being due at point A_2 (see Figure 6). Assuming uncovered interest parity and a negligible risk premium, the interest rate differential between domestic and foreign rates reflects the expected rate of depreciation. The latter reflects the public's expectation, as of point A_1, about the realizations of x_2 and of the policymaker type in period 2. As argued by Svensson (1992a), expected depreciation takes into consideration that both realignments and depreciations within the band are possible.

Figure 6 to be inserted here

a. <u>The probability of realignment and the credibility of the band</u>

Given the parameters α and c, the equilibrium width of the band does not change from one period to the next. But if the realization of x is sufficiently large, a realignment takes place through an upward adjustment of the central parity rate. In other words, the band shifts upward, but its width remains the same. Obviously the credibility of the existing band is higher the smaller the probability that a realignment will take place. This subsection derives an analytical expression for this probability and characterizes its determinants. We focus on the credibility of the band for the second period when there is no realignment between period 1 and period 2. This formulation has two advantages. First, it makes it possible to detect if the deviation from the center rate at the end of period 1 has any effect on the credibility of the band for period 2. Second this measure of credibility refers to the same time span as the loans discussed above. It is therefore likely to be relevant for participants in the capital market. Let p be the probability of a realignment in period 2. We shall take the complementary probability, 1−p, as a measure of the band's credibility. More formally

Credibility of the band $\equiv$ CRB $= 1-p =$

$=$ Prob [No Realignment/W is in office] Prob [W is in office]

$+$ Prob [No Realignment/D is in office] Prob [D is in office]

$= (1-\alpha)F(\pi_m) + \alpha F(\pi_m + d)$.

A recurring theme in related work is whether the credibility of the band is affected by the distance of the exchange rate at the end of period 1 from the upper limit of the band. To deal with this issue we differentiate equation (29) with respect to π_m

$$\frac{\partial CRB}{\partial \pi_m} = (1-\alpha)\, f(\pi_m) + \alpha f(\pi_m+d) > 0\,. \tag{30}$$

Hence, <u>the credibility of the band is lower the nearer is the exchange rate to its upper bound</u>. It is also of interest to investigate the effects of the cost of reneging on the band and of reputation on the credibility of the band. To do so, we differentiate equation (29) with respect to c and α, and obtain,

$$\frac{dCRB}{dc} = \alpha f(\pi_m+d)\left[\frac{1}{d} + \frac{1+\pi_m}{1+B}\,\frac{dB}{dc}\right]$$

$$\frac{dCRB}{d\alpha} = F[\pi_m+d] - F[\pi_m] + [(1-\alpha)\, f(\pi_m) + \alpha f(\pi_m+d)]\,\frac{1+\pi_m}{1+B}\,\frac{dB}{d\alpha}\,.$$

An increase in the cost of reneging has a direct and an indirect effect on the credibility of the band. The first effect operates through the change in d for a given band width. The second effect operates through the effect that c has on B and through it on π_m and credibility. The direct effect is positive. If the condition of Proposition 2 is satisfied the second effect is positive too. Hence the conditions of Proposition 2 are sufficient (but not necessary) for the conclusion that an increase in the cost of reneging raises the credibility of the (wider) band.

An increase in reputation also triggers two effects on the credibility of the band. The direct effect, which is positive, operates for a given band width. The indirect effect influences credibility by changing the band width. If the condition of Proposition 1 is satisfied, an increase in reputation reduces B. This effect tends to reduce the credibility of the band. If the first effect dominates or if $dB/d\alpha \geq 0$ a better reputation is associated with a more credible band. The results of this subsection are summarized in the following proposition.

PROPOSITION 6: The credibility of the band is larger

(i) the further away is the exchange rate from the band's upper limit,

(ii) the higher the cost of reneging on the band if the condition of Proposition 2 is satisfied,

(iii) the higher is reputation, provided $dB/d\alpha$ is non–negative or not "too negative."

b. The expected rate of depreciation and the interest rate differential

Here we develop an expression for expected depreciation in period 2 as of the end–of–period 1 in terms of the fundamental parameters of the model and of the position of the exchange rate in period 1. This is done for the case in which there is no band realignment in period 1. The expected rate of depreciation can be written

$$E\pi = E\pi_R + E\pi_{NR} \tag{31}$$

where R stands for "realignment" and NR stands for "no realignment". $E\pi_R$ is the average contribution of depreciations within the range of realignment to expected depreciation and $E\pi_{NR}$ is the average contribution of depreciations within the band to expected depreciation. The first term is given by the summation of possible rates of depreciation in the realignment range weighted by their respective probability densities

$$E\pi_R = \int_{\pi_m+d}^{\bar{x}} x \, dF(x) + (1-\alpha) \int_{\pi_m}^{\pi_m+d} x \, dF(x) \, . \tag{32a}$$

Similarly

$$E\pi_{NR} = \int_{\underline{x}}^{\pi_m} x \, dF(x) + \alpha\pi_m \int_{\pi_m}^{\pi_m+d} x \, dF(x) \tag{32b}$$

is the (density weighted) summation of rates of depreciation within the band. We refer to them as the contributions of expected realignments and of expected depreciations within the band to (overall) expected depreciation respectively.[23] Since under uncovered interest rate arbitrage $E\pi$ is equal to the differential between the domestic and the foreign interest rate, it is possible to obtain a measure of expected depreciation. Using this measure we focus on the implication of our approach for the effect of the distance of the exchange rate from the upper limit of the band on the interest rate differential. Partial differentiation of $E\pi$ with respect to π_m (taking 32a) and (32b) into consideration) yields

$$\frac{\partial E\pi}{\partial \pi_m} = \alpha\left[\int_{\pi_m}^{\pi_m+d} dF(x) + (1-\pi_m)\{\pi_m f(\pi_m) - (\pi_m+d)f(\pi_m+d)\}\right]. \tag{33}$$

This expression is generally ambiguous. But when the exchange rate is sufficiently close to the upper limit of the band $\pi_m \to 0$ it reduces to

$$\frac{\partial E\pi}{\partial \pi_m} = \alpha[F(d) - df(d)]. \tag{34}$$

When the distribution of x is unimodal the expression in equation (34) is negative. This is summarized in the following proposition.

[23] Note that the first concept is not identical to Svensson's "expected realignment" which refers to the expected value of the rate of change of the center rate rather than to that of the actual rate. A discussion of expected realignment appears in subsection d below.

PROPOSITION 7: When (i) the cost of reneging on the band is not too large, (ii) the distribution of x unimodal, and (iii) the exchange rate is sufficiently close to the upper limit of the band, an increase in the exchange rate without a realignment increases expected depreciation and the interest rate differential.

In a multiperiod framework extention of our framework there will be a "peso problem" once the exchange rate reaches the upper limit of the band. The reason is that expected depreciation is positiv as can be seen from equations (31) and (32). However, once the upper limit of the band is reached, the exchange rate is fixed at $\bar{e}$ as long-as $x \leq \sqrt{2c}$. Hence there will be sequences of periods during which unexpected depreciation is persistently negative. This phenomenon will be more persistent the larger the cost, c, of reneging on the band.

c. The Effect of the Position in the Band on the Components of Expected Depreciation

The contribution of expected realignments to expected depreciation is given in equation (32a). Differentiating this expression with respect to π_m yields

$$\frac{\partial E\pi_R}{\partial \pi_m} = -[\alpha(\pi_m+d)f(\pi_m+d) + (1-\alpha)\pi_m f(\pi_m)] \tag{35}$$

which is unambiguously negative. This leads to the following proposition.

PROPOSITION 8: The contribution of expected realignments to expected depreciation is larger the nearer is the actual exchange rate at the end of period 1 to the upper limit of the band.

d. The effect of the position within the band on expected realignment

We defined a "realignment" as a change in the central parity rate. "Expected realignment" is therefore the expected value of the change in the center rate. Let π_c be the rate of change in the center rate between period 1 and period 2. Then

$$\pi_c = \frac{e_{c2}}{e_{c1}} - 1 \equiv \log e_{c2} - \log e_{c1} \equiv \epsilon_{c2} - \epsilon_{c1} . \tag{36}$$

Since (by definition of the event NR) $E[\pi_c | NR] = 0$, it follows that

$$E\pi_c = p\, E[\pi_c | R] . \tag{37}$$

When a realignment takes place in period 2 policymakers have no incentive (within the present framework) to set the new center rate at a level that is different from the actual exchange rate in that period. Hence $\epsilon_{c2} = \epsilon_2$ and

$$\pi_c | R = \epsilon_{c2} - \epsilon_{c1} = (\epsilon_{c2} - \epsilon_1) + (\epsilon_1 - \epsilon_{c1}) = \pi | (R + y_1) . \tag{38}$$

Here y_1 is the deviation of the actual exchange rate from the center rate in the first period. Taking expected values of both sides of (38) conditional on a realignment in period 2 yields

$$E[\pi_c|R] = E[\pi|R] + y_1 \qquad (39)$$

But from (32a)

$$E[\pi|R] = \int_{\pi_m+d}^{\overline{x}} x\,\frac{dF(x)}{p} + (1-\alpha)\int_{\pi_m}^{\pi_m+d} x\,\frac{dF(x)}{p}. \qquad (40)$$

Using (40) in (39) and the resulting expression in equation (37) and using the approximation $y_1 \equiv B - \pi_m$ we obtain

$$E\pi_c = \int_{\pi_m+d}^{\overline{x}} x\,dF(x) + (1-\alpha)\int_{\pi_m}^{\pi_m+d} x\,dF(x) +$$

$$\Big[1-(1-\alpha)F[\pi_m]-\alpha F[\pi_m+d]\Big][B-\pi_m]. \qquad (41)$$

Differentiating this expression with respect to π_m and rearranging yields

$$\frac{\partial E\pi_c}{\partial \pi_m} = -[\alpha(\pi_m+d)f(\pi_m+d)+(1-\alpha)\pi_m f(\pi_m)]-p-[(1-\alpha)f(\pi_m)$$

$$+ \alpha f(\pi_m+d)]y_1. \qquad (42)$$

Since p and y_1 are non–negative, this expression is unambiguously negative. This result is summarized in the following proposition.

Proposition 9: The magnitude of realignment expected for period 2 is larger the nearer is the exchange rate at the end of period 1 to the upper limit of the band.

Empirical evidence in support of this proposition has been provided in at least three recent studies which have distinguished carefully between expected realignments and expected depreciation within the band. Chen and Giovannini (1993) used a modified version of Svensson's drift–adjustment method to explore the empirical relation between expectations of parity changes and economic variables for the French Franc/DM and Italian Lira/DM exchange rates. They find high explanatory power for the deviation of exchanges from the central parity in equations for expected parity changes. Cukierman, Kiguel and Leiderman (1993b) provide empirical evidence based on the unilateral exchange rate bands of Finland, Norway, Sweden, Chile and Israel. They found that the contribution of expected realignments to expected depreciation significantly rises with increases in the level of the exchange rate within the band. Similarly, Edin and Vredin (1993) find that the lagged central parity has a significant negative influence on the estimated probability of a devaluation in the Nordic countries.

9. CONCLUDING REMARKS

This paper developed a framework for analyzing policymakers' choices regarding unilateral exchange rate bands. Such bands were shown to emerge as the outcome of optimization under a policy objective function that weighs the level of the real exchange rate against the level and variability of the nominal exchange rate. The determination of the band's key parameters was shown to involve the resolution of a tradeoff between credibility and flexibility in the presence of positive costs of reneging on the band. We used the basic model to identify conditions under which an increase in the commitment reputation of the policymaker, a decrease in the cost of reneging on the existing band, and a decrease in the variance of fundamental shocks can produce a narrowing of band width. In addition, the band regime was shown to result in a range of effective commitment and to give rise to a "peso problem." The findings of some recent empirical studies provide evidence in support of the model's implication that the contribution of expected realignments to expected depreciation increases as the exchange rate increases toward the upper limit of the band.

The analysis in this paper can be extended in at least two interesting directions. First, it would be useful to consider policy choices in a multiperiod framework in which the policymaker can make multiperiod commitments about the time path of the central parity exchange rate. This would allow determination of conditions under which regimes of preannounced crawling -- such as in Chile, Israel and Mexico[24] would emerge. Second, it would be desirable to incorporate into the analysis the existence of intra marginal (i.e., within the existing band) intervention by the policymaker. While the present model includes intervention within the range of effective commitment, empirical evidence indicates that nonnegligible intervention does exist within the band.

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[24] Cukierman, Kiguel and Leiderman (1993a) provide a first attempt in this direction.

APPENDIX

1. The real exchange rate objectives underlying equation (1)

This part of the appendix demonstrates that equation (1) in the text can be viewed as an approximation to the following objective function

$$\text{Max} \quad -\left[\frac{A}{2}(R - R^*)^2 + \frac{1}{2}\pi^2 \right] \tag{A1}$$

where

$$R \equiv \frac{e}{p} \tag{A2}$$

is the real rate of exchange, e is the nominal exchange rate (defined as domestic currency units per unit of foreign currency), p is the domestic price level and the foreign price level is normalized to 1. R^* is the level of the real exchange rate currently desired by policymakers. The objective function in equation (A1) states that policymakers incur costs that increase with the deviation of the real exchange rate from its desired level and with the degree of instability in nominal variables. The parameter A measures the relative importance attributed by policymakers to each type of cost. The larger is A, the stronger their relative aversion to missing their real exchange rate target. We will show that R^* and x are monotonically related.

Domestic prices are temporarily sticky. To reflect this fact we assume that prices for (part of) the current period are preset in the previous period on the basis of the then prevailing, exchange rate expectations and remain fixed until after the current realization of x (or R^*). They are, then, reset again in light of this realization. Hence $p = e^e_{-1}$, where e^e_{-1} is the nominal exchange rate expected in the previous period and p is the price level prevailing in the second part of the previous period and in the first part of the current one. Let R^H (H, for "historically given") be the real exchange rate in the current period when the nominal exchange rate is maintained at its previous period's value. That is

$$R^H \equiv \frac{e_{-1}}{p} = \frac{e_{-1}}{e^e_{-1}} . \tag{A3}$$

Define

$$L(R) \equiv \frac{A}{2}(R - R^*)^2 . \tag{A4}$$

Expanding $L(R)$ linearly around R^H and using the fact that $R = e/e^e$, yields

$$L(R) = -\frac{A}{2}[(R^H)^2 - (R^*)^2] + A(R^H - R^*)\frac{e}{e^e} . \tag{A5}$$

Due to trade account and capital account shocks, and other factors which are beyond the control of policymakers, R^* fluctuates randomly. Substituting (A5) into (A1) and retaining only the terms that are affected by the choice of e the problem in equation (A1) can be reformulated as

$$\underset{e}{\text{Max}} \left[x\frac{e}{e^e} - \frac{1}{2}\pi^2\right] \tag{A1a}$$

where

$$X \equiv A(R^* - R^H) = A(R^* - \frac{e_{-1}}{e^e_{-1}}) . \tag{A6}$$

Let

$$u \equiv \frac{e}{e^e} - 1 \tag{A7}$$

be the magnitude of unanticipated depreciation. Equation (A7) implies

$$\log \frac{e}{e^e} = \log(1 + u) \cong u . \tag{A8}$$

On the other hand

$$\log \frac{e}{e^e} = \log \frac{e/e_{-1}}{e^e/e_{-1}} = \log \frac{1+\pi}{1+\pi^e} = \log(1+\pi) - \log(1+\pi^e) \cong \pi - \pi^e. \tag{A9}$$

equations (A8) and (A9) imply

$$u = \pi - \pi^e . \tag{A10}$$

Combining (A7) and (A10)

$$\frac{e}{e^e} = 1 + u \cong 1 + \pi - \pi^e . \tag{A11}$$

Substituting (A11) into (A1a) and deleting terms that do not depend on e, the maximization problem in (A1a) is equivalent to

$$\underset{e}{\text{Max}} \left\{ x(\pi - \pi^e) - \frac{\pi^2}{2} \right\} . \tag{A12}$$

This is identical to the objective function in equation (1) of the text.

Examination of equation (A6) reveals that x is positive, zero or negative depending on whether R^* is larger than, equal to, or smaller than R^H. This equation also reveals some of the factors that affect the magnitude of x. A higher current desired real exchange rate, and a higher expected depreciation in the previous period raise x. In addition the average value of x is larger the larger is A. The intuition underlying these relationships is straightforward. Any shock that raises the desired real exchange rate (e.g., an unexpected reduction in the terms of trade or a sudden capital outflow) raises the value of a surprise devaluation. Hence x increases. Anything that reduces R^H also raises the value of a surprise devaluation. This fact is again reflected by a higher value of x. For example, an increase in last period's expectations, by raising current domestic prices, reduces the current real exchange rate under a fixed peg and increases the value of a surprise devaluation. More formally, it follows from (A6) that

$$x = \begin{cases} \text{positive} & \text{if } R^* > R^H \\ 0 & \text{if } R^* = R^H \\ \text{negative} & \text{if } R^* < R^H \end{cases} .$$

2. <u>Derivation of equations (21) and (22)</u>

From equation (1) the expected value of D's objectives in stage 1, given an optimal continuation in stage 4 is

$$EV(x) = E\left[x(\pi(x) - \pi^e(x)) - \frac{(\pi(x))^2}{2} - I(x)c\right] \tag{A13}$$

where $I(x)$ is an indicator function with $I(x) = 1$ for $x > B + d$ and 0 otherwise. Suppressing the dependence on x in the notation, multiplying by -1 to get expected losses and rearranging we obtain

$$L(\cdot) = E\left[\frac{\pi^2}{2} - x\pi\right] + cEI(x) + Ex\pi^e(x) \,. \tag{A14}$$

But

$$\frac{\pi^2}{2} - x\pi = \begin{Bmatrix} \frac{x^2}{2} & x < B, \quad x > B + d \\ \frac{B^2}{2} & B < x \leq B + d \end{Bmatrix}. \tag{A15}$$

Substituting (A15) into the first term in (A14) and rearranging,

$$E\left[\frac{\pi^2}{2} - x\pi\right] = \frac{1}{2}\int_B^{B+d} (x-B)^2 dF(x) - \frac{1}{2}\int_{\underline{x}}^{\overline{x}} \frac{x^2}{2}\, dF(x) \equiv \frac{1}{2}(ECI - S). \tag{A16}$$

The first term on the right–hand–side of (A16) measures the expected cost of inflexibility in the commitment range. Not surprisingly these costs are proportional to the stochastic difference x–B.

The second term on the right–hand–side of (A14) may be written

$$CEI(x) = c \int_{B+d}^{\overline{x}} dF(x) \equiv ECR \,. \tag{A17}$$

Using equation (11) for $\pi_m = B$, the last term on the right–hand–side of equation (A14) may be written

$$Ex\pi^e(x) = S - \alpha \int_{B+d}^{B} (x-B)x dF(x) \equiv ECE \,. \tag{A18}$$

Equations (21) and (22) in the text follow from equation (A14) in conjunction with (A16), (A17) and (A18).

REFERENCES

Barro, Robert J., and David B. Gordon (1983), "A Positive Theory of Monetary Policy in a Natural Rate Model," Journal of Political Economy 91, 589–610.

Barro, Robert J. (1986), "Reputation in a Model of Monetary Policy with Incomplete Information," Journal of Monetary Economics, 17, 3–20.

Chen, Zhaohui, and Alberto Giovannini (1993), "The Determinants of Realignment Expectations under the EMS: Some Empirical Regularities," Working Paper No.4291, National Bureau of Economic Research, March.

Cukierman, Alex (1992), Central Bank Strategy, Credibility and Independence: Theory and Evidence, Cambridge, MA: The MIT Press.

Cukierman, Alex, Kiguel, Miguel A., and Leonardo Leiderman (1993a), "Transparency and the Evolution of Exchange Rate Flexibility in the Aftermath of Disinflation," Unpublished manuscript, Tel–Aviv University, May.

Cukierman, Alex, Kiguel, Miguel A., and Leonardo Leiderman (1993b), "Some Evidence on a Strategic Model of Exchange Rate Bands," forthcoming in: L.Leiderman and A.Razin (eds.), Capital Mobility: New Perspectives, Cambridge: Cambridge University Press.

Cukierman, Alex, Kiguel, Miguel A., and Nissan Liviatan (1992), "How Much to Commit to an Exchange Rate Rule? Balancing Credibility and Flexibility," Revista de Analisis Economico 7, 73–90.

Edin, Per–Anders and Anders Vredin (1993), "Devaluation Risk in Target Zones: Evidence from the Nordic Countries," Economic Journal 103, 161–175, January.

Flood, Robert P., and Peter Isard (1989), "Monetary Policy Strategies," IMF Staff Papers 36, 612–632.

Frenkel, Jacob A., and Morris Goldstein (1986), "A Guide to Target Zones," IMF Staff Papers 33, 633–673.

Helpman, Elhahan, and Leonardo Leiderman (1992), Israel's Exchange Rate Band, Tel–Aviv Israel: Israeli International Institute for Applied Economic Policy Review, August.

Krugman, Paul, and Marcus Miller (1992a) eds., Exchange Rate Targets and Currency Bands, Cambridge: Cambridge University Press.

Krugman, Paul, and Marcus Miller (1992b), "Why Have a Target Zone?'" Research Paper No.394, Department of Economics, University of Warwick, August.

Lohmann, Susanne (1992), "Optimal Commitment in Monetary Policy: Credibility Versus Flexibility," American Economic Review 82, 273–286, March.

Obstfeld, Maurice (1991), "Destabilizing Effects of Exchange–Rate Escape Clauses," Working Paper No. 3603, NBER, January.

Persson, Torsten, and Guido Tabellini (1990), Macroeconomic Policy, Credibility, and Politics, London: Harwood Publishers.

Svensson, Lars, E.O. (1992a), "An Interpretation of Recent Research on Exchange Rate Target Zones," Journal of Economic Perspectives 6, 119–144, Fall.

Svensson, Lars E.O. (1992b), "Why Exchange Rate Bands? Monetary Independence in Spite of Fixed Exchange Rates," Discussion Paper No.742, Centre for Economic Policy Research, December.

Svensson, Lars E.O. (1993), "Assessing Target Zone Credibility: Mean Reversion and Devaluation Expectations in the ERM, 1979–1992," European Economic Review 37, 763–793, May.

Figure 1: Timing of Events

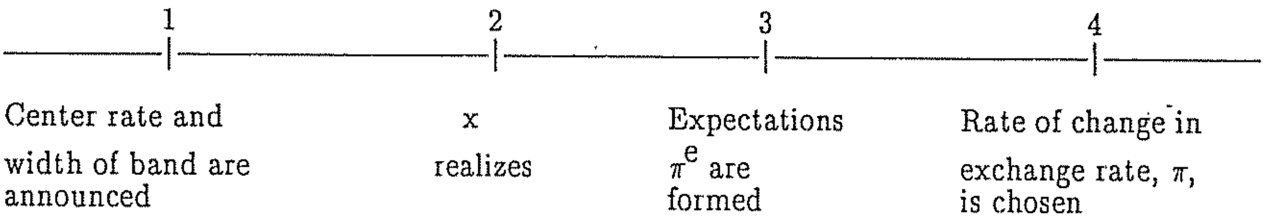

Figure 2: The Range of Effective Commitment of the Dependable Policymaker

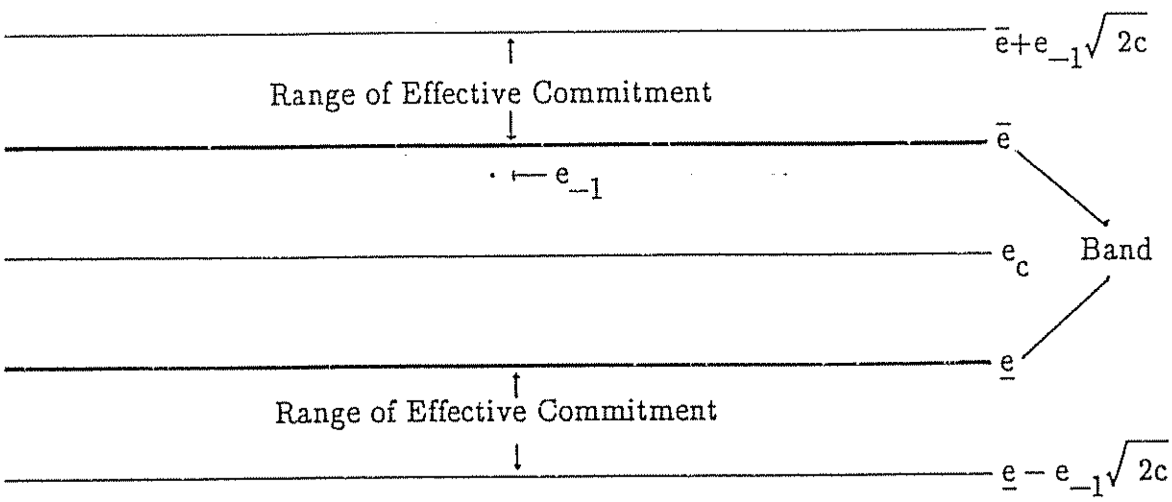

Figure 3: Last Stage Strategies of the Two Policymakers' Types for Different Values of x

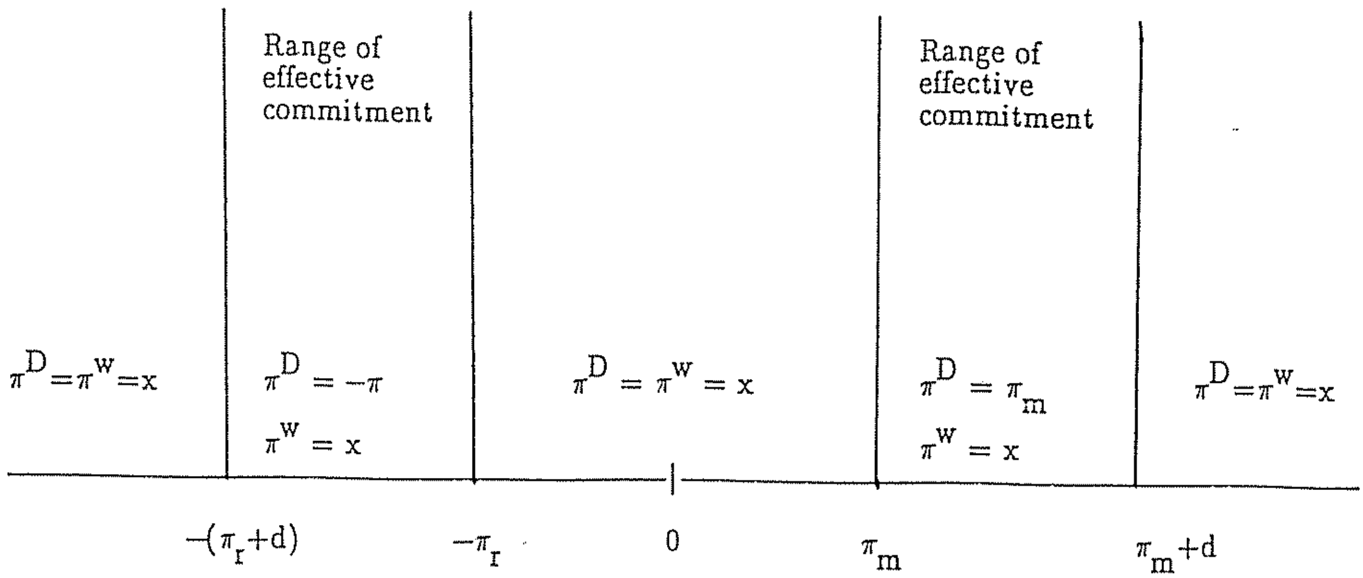

Figure 4: The Position of the Band Width Relatively to the Mode of x

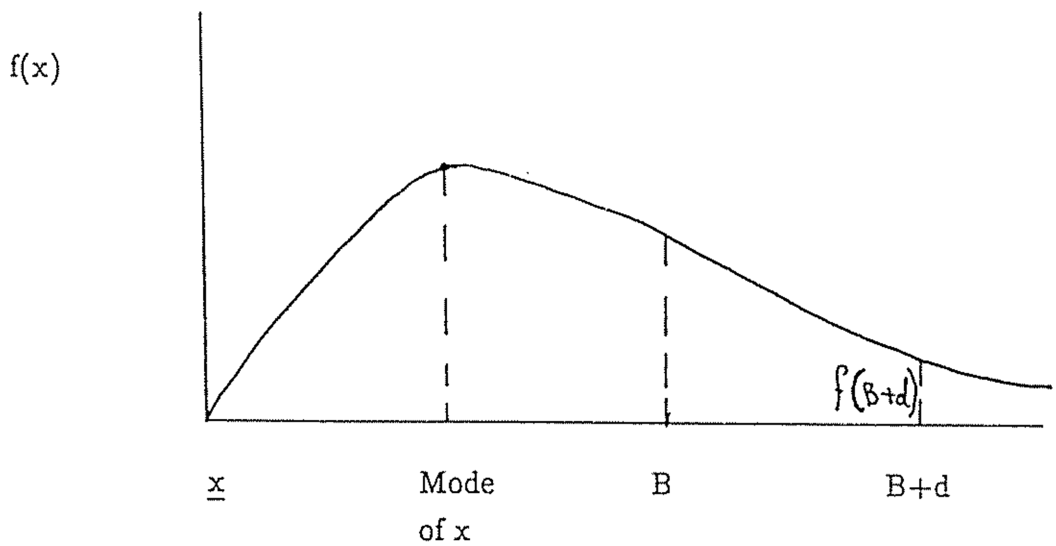

FIGURE 5: EXPECTED DEPRECIATION
AFTER THE REALIZATION OF x AS A FUNCTION OF x

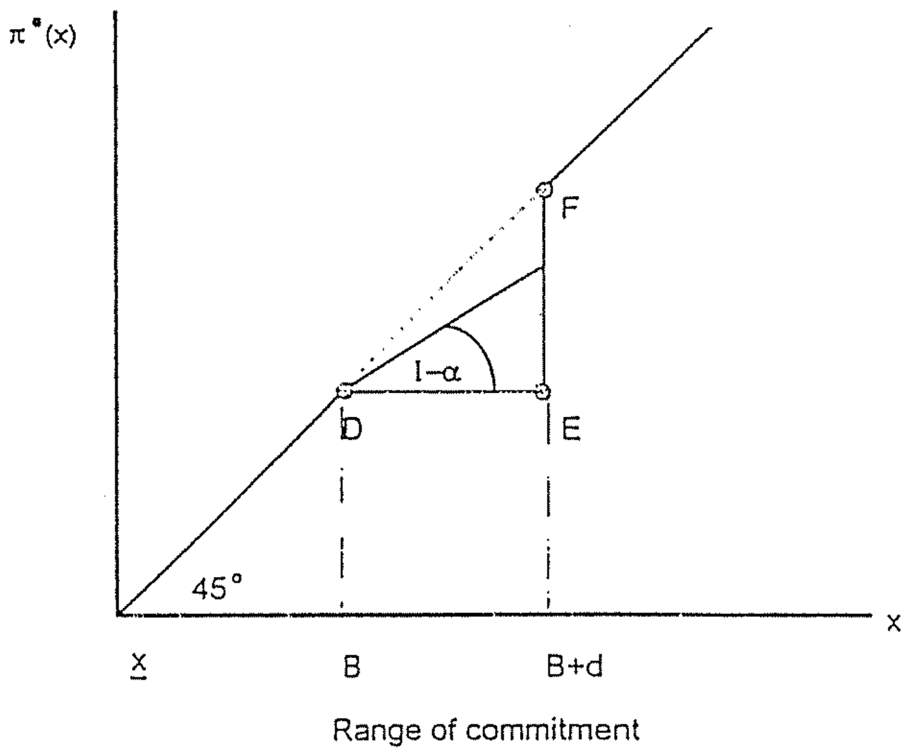

Figure 6: A Capital Market in a Two–Period Framework