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Centre for Economic Policy Research
25-28 Old Burlington Street
London W1X 1LB
Tel: (44 71) 734 9110

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ABSTRACT

A Model of the ERM Crisis*

Existing models of exchange rate crises do not provide a good explanation for the breakdown of the ERM in 1992-3. This paper presents an alternative model which captures some of the important features of that period. The switch from a fixed to a floating rate is triggered by an optimizing government that wants to loosen monetary policy and boost aggregate demand. Agents in the foreign exchange market know the government's objective function and therefore build expectations of a regime switch into interest differentials. It is shown that this interaction between private sector expectations and government preferences can imply a breakdown of the fixed rate sooner than the government would like.

JEL classification: F31, F32

Keywords: speculative attacks, balance of payments crises, EMS, ERM

F Gulcin Ozkan and Alan Sutherland
Department of Economics and Related Studies
University of York
Heslington
York
YO1 5DD
UK
Tel: (44 904) 433780

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NON-TECHNICAL SUMMARY

This paper presents a theoretical model that captures the main features of the crises of 1992 and 1993 in the Exchange Rate Mechanism (ERM) of the European Monetary System. The virtual collapse of the ERM raises a number of problems for existing theoretical models of speculative attacks and balance of payments crises. The existing literature emphasizes the role that limited reserves play in precipitating a crisis. In the Krugman (1979) model, a balance of payments crisis is generated by a government that operates a policy of domestic credit expansion while simultaneously fixing the exchange rate. The government inevitably runs out of foreign exchange reserves and the fixed rate has to be abandoned. Krugman shows that, with forward looking exchange markets, the final stage of the crisis involves a sudden discrete loss of reserves in a 'speculative attack'.

The crisis which preceded the exit of the pound and the lira from the ERM in September 1992 certainly involved a large and sudden loss of reserves for the UK and Italian authorities, but it is not clear that the other features of the Krugman model are relevant to this period. The central cause of a regime collapse in the Krugman model is the unsustainability of a fixed exchange rate when the authorities are allowing domestic credit to expand. Policy inconsistencies of this type do not appear to have been a major factor in generating the ERM crisis.

Obstfeld (1986) has provided an alternative model of speculative attacks which differs from the Krugman model in that it does not depend on a government following an unsustainable policy stance. Instead the possibility of a speculative attack is generated by private sector expectations of a loosening of monetary policy after a collapse of the fixed rate regime. The expectation of a devaluation triggers an attack which exhausts reserves and forces the authorities to abandon the fixed rate.

In CEPR Discussion Paper no. 817 Eichengreen and Wyplosz argue that the Obstfeld model does provide a good explanation for the ERM crisis in Autumn 1992. They suggest that, up to that point, ERM members were against the principle of realignments because of the requirement in the Maastricht Treaty that participants in EMU should not have devalued in the two years prior to Stage 3. Eichengreen and Wyplosz point out that if a speculative attack exhausted a country's reserves it would be forced to accept a realignment, however. Having been forced to accept the principle of a realignment it would no longer be eligible for Stage 3 of EMU and would therefore have no incentive to resist a large

devaluation. Thus Eichengreen and Wyplosz argue that the Maastricht Treaty created the very conditions needed for self-fulfilling speculative attacks.

We believe, however, that there are a number of important features of the ERM crisis which the Obstfeld model does not capture. In particular, the model gives no role to the pressures put on the ERM by German reunification. In both the September 1992 and the July 1993 crises there was a widespread belief in financial markets that high German interest rates were placing an intolerable burden on ERM members such as France, Italy and the United Kingdom. Speculators were clearly concerned that these governments would at some point find it acceptable to abandon the ERM in order to loosen monetary policy. This lack of credibility resulted in even higher interest rates in the countries concerned, which, in turn, increased the pressure. Interaction of this type between credibility and interest rates appeared to be an important element in the crises.

It is also not clear what role the loss of foreign exchange reserves played in the decision of each government to abandon its fixed rate. In theory the governments concerned could have defended their currencies by raising interest rates to whatever level necessary. Their decision not to do so reveals something about their preferences. It appears that the decision to leave the ERM was to some extent an optimizing decision rather than a policy action forced upon the authorities by a loss of reserves (as is true in both the Krugman and Obstfeld models).

In this paper we propose a model of the collapse of a fixed rate which incorporates a number of the features which we have emphasized above. In our model the fixed rate system is dominated by a centre country that sets its interest rate in order to achieve its own policy objectives. This implies that, from the point of view of the non-centre countries, foreign interest rates are exogenous and subject to stochastic shocks. We believe that this is a good representation of the processes at work in the ERM. The Bundesbank set its own interest rates to maintain price stability in Germany. Shocks to the German economy (such as reunification) were therefore transmitted to other ERM countries via movements in interest rates.

Our model concentrates on the behaviour of the government of a non-centre country. We assume that the government is setting policy so as to maximize a welfare function which depends on domestic output. In the fixed rate system the domestic interest rate must be set so as to maintain the currency parity. The domestic government therefore has no power to influence domestic monetary policy. Increases in foreign interest rates are therefore transmitted directly into reductions in output and government welfare. If, on the other hand, the domestic government allows the exchange rate to float, it finds that it can regain control

over domestic monetary policy and can use the exchange rate to offset the adverse effects of shocks from the centre country.

A country which is currently a member of the fixed rate system is assumed to have a once-and-for-all option to switch to the floating rate system. It turns out that the government's optimal strategy is to choose a trigger level of the interest rate in the centre country. While the centre country interest rate is below the trigger level it is optimal for the domestic country to remain in the fixed rate system. If the centre country interest rate hits the trigger level, however, it is optimal for the domestic country to leave the fixed rate system and allow its currency to float.

The optimizing nature of the regime switch is the first main feature of our model of the collapse of the ERM. The second main feature is the interaction between the government's optimizing decision and private sector expectations. We assume that the private sector knows the government's preferences and policy options. The private sector is therefore able to calculate the trigger level of foreign interest rates at which the collapse of the fixed rate regime will occur. This implies that, as foreign interest rates rise towards the trigger level, the expectation of a regime switch (and associated devaluation) drives up domestic interest rates so that the interest differential between domestic and foreign rates widens. In turn, domestic interest rates affect output, so private sector expectations impinge on the government's optimizing problem.

We show that this interaction between private sector expectations and the government's objective function gives rise to a credibility problem in that the government will be induced to abandon the fixed rate at a time which is *ex ante* sub-optimal. In particular, we show that in certain circumstances the government would like to set a very high exit trigger level of foreign interest rates. We suggest that this can be thought of as a policy announcement that a regime switch is very unlikely. The incentive for the government to make this announcement is the beneficial effect it has on the differential between domestic and foreign interest rates. The private sector understands the government's incentive to manipulate expectations, however, and thus, in the absence of any mechanism which allows the government to pre-commit to a trigger point, disbelieves the government's announcements. We show that in such circumstances it is possible that the regime switch will take place much sooner than the government would have liked.

Unlike the Krugman and Obstfeld models, foreign exchange reserves play no explicit part in determining the timing of the crisis. The switch of exchange rate regime is the result of an optimizing decision by the government (which may nevertheless give rise to a sub-optimal result).

Introduction

The virtual collapse of the Exchange Rate Mechanism raises a number of problems for existing models of speculative attacks and balance of payments crises. The existing literature¹ emphasises the role that limited reserves play in precipitating a crisis. In the Krugman (1979) model a balance of payments crisis is generated by a government which operates a policy of domestic credit expansion while simultaneously fixing the exchange rate. The government inevitably runs out of foreign exchange reserves and the fixed rate has to be abandoned. Krugman shows that, with forward looking exchange markets, the final stage of the crisis involves a sudden discrete loss of reserves in a "speculative attack".

The crisis which preceded the exit of the pound and the lira from the ERM in September 1992 certainly involved a large and sudden loss of reserves for the UK and Italian authorities, but it is not clear that the other features of the Krugman model are relevant to this period. The central cause of a regime collapse in the Krugman model is the unsustainability of a fixed exchange rate when the authorities are allowing domestic credit to expand. Policy inconsistencies of this type do not appear to have been a major factor in generating the ERM crisis.

Obstfeld (1986) has provided an alternative model of speculative attacks which differs from the Krugman model in that it does not depend on a government adopting an unsustainable policy stance. Instead the possibility of a speculative attack is generated by private sector expectations of a loosening of monetary policy after a collapse of the fixed rate regime. The expectation of a devaluation triggers an attack which exhausts reserves and forces the authorities to abandon the fixed rate. If the authorities do in fact then loosen monetary policy the exchange rate will depreciate and the expectations of speculators will be fulfilled. Thus a rational expectations equilibrium can exist with a speculative attack even when the initial policy stance of the government is sustainable.

¹ See Agenor, Bhandari and Flood (1992), Blackburn and Sola (1993) and Willman (1992) for recent surveys of the literature.

Eichengreen and Wyplosz (1993) argue that the Obstfeld model does provide a good explanation for the ERM crisis in Autumn 1992. They suggest that up to that point ERM members were against the principle of realignments because of the requirement in the Maastricht Treaty that participants in EMU should not have devalued in the two years prior to Stage 3. However, Eichengreen and Wyplosz point out that if a speculative attack exhausted a country's reserves it would be forced to accept a realignment. Having been forced to accept the principle of a realignment it would no longer be eligible for Stage 3 of EMU and would therefore have no incentive to resist a large devaluation. Thus Eichengreen and Wyplosz argue that the Maastricht Treaty created the very conditions needed for self-fulfilling speculative attacks.

We believe however that there are a number of important features of the ERM crisis which the Obstfeld model does not capture. In particular the model gives no role to the pressures put on the ERM by German reunification. In both the September 1992 and the July 1993 crises there was a widespread belief in financial markets that high German interest rates were placing an intolerable burden on ERM members such as Italy, the UK and France. Speculators were clearly concerned that these governments would at some point find it acceptable to abandon the ERM in order to loosen monetary policy. This lack of credibility resulted in even higher interest rates in the countries concerned, which, in turn, increased the pressure. Interaction of this type between credibility and interest rates appeared to be an important element in the crises.

It is also not clear what role the loss of foreign exchange reserves played in the decision of each government to abandon its fixed rate. In theory the governments concerned could have defended their currencies by raising interest rates to whatever level necessary. Their decision not to do so reveals something about their preferences. It appears that the decision to leave the ERM was to some extent an optimising decision rather than a policy action forced upon the authorities by a loss of reserves (as is true in both the Krugman and Obstfeld models).

In this paper we propose a model of the collapse of a fixed rate which incorporates a number of the features which we have emphasised above. In our model the economy is subjected to stochastic shocks which are interpreted as movements in foreign interest rates. The government is assumed to be setting policy so as to maximise a welfare function which depends on domestic output. Increases in foreign interest rates reduce output and therefore reduce government welfare relative to that which could be obtained if the exchange rate were floating. The government's optimising problem is to choose the trigger level of foreign interest rates at which it will leave the fixed rate system.²

The private sector knows the government's preferences and its policy options and is therefore able to forecast the collapse of the fixed rate. This is built into domestic interest rates so that the interest differential between domestic and foreign rates widens as foreign interest rates rise.³ This reduces output further and hastens the onset of the crisis. We show that this interaction between private sector expectations and the government's objective function gives rise to a credibility problem in that the government will be induced to abandon the fixed rate at a time which is *ex ante* suboptimal. Unlike the Krugman and Obstfeld models, foreign exchange reserves play no explicit part in determining the timing of the crisis. The switch of exchange rate regime is the result of an optimising decision by the government (which may nevertheless give rise to a suboptimal result).

In sections 1, 2 and 3 of the paper we describe our basic framework. In section 4 we derive the government's optimal regime switching strategy and show that it is "state inconsistent" in the sense that the government will choose a different policy stance in each state of the world. The "state-consistent" but non-optimal policy is derived in section 5.

² Flood, *et al* (1989) also model exchange rate regime switches as being generated by an optimal state-contingent policy strategy. Their model differs from ours in two important respects. Firstly in their model a switch of regime is triggered by changes in the variability of shocks, whereas in our model it is the level of the shock which is important. And secondly, the Flood *et al* model does not incorporate forward looking behaviour on the part of agents in financial markets.

Another example of an optimal state-contingent regime switch is analysed in De Kock and Grilli (1993). In that model a regime switch follows adverse shocks to the fiscal deficit. The government abandons the fixed rate so that it may exploit the inflation tax.

³ Gros (1992) analyses a model where private sector expectations of devaluation increase as interest rates rise. Gros, however, does not explicitly model the linkage in terms of the government's optimising problem.

Stansfield and Sutherland (1993) consider a similar linkage between realignment expectations and economic activity in a model of overlapping wage contracts. Again, in that paper, the government's optimising problem is not modelled explicitly.

Section 6 considers the determinants of these two alternative solutions to the model and shows how the state-consistent policy may involve a breakdown of the fixed exchange rate much sooner than the government would have liked. In section 7 we use the model to consider a number of policy measures which may preserve the fixed rate regime.

1. The Basic Structure of the Model

In order to avoid obscuring the important features of our analysis we model the underlying structure of the macro-economy in the most basic way. We assume that goods prices are fixed and that output, y , is determined by aggregate demand. Aggregate demand depends on the interest rate, i , and the exchange rate, s , as follows

$$y = -\gamma i + \eta s \quad (1)$$

where s is the log of the exchange rate (defined as the domestic price of foreign currency). Given the assumption of fixed prices there is no distinction between nominal and real variables.

Above we asserted that high German interest rates caused by reunification were a major part of the crisis. We therefore model the foreign interest rate as a stochastic variable, ε , i.e. $i^* = \varepsilon$ and

$$d\varepsilon = \sigma dz \quad (2)$$

where z is a standard brownian motion process with unit variance. This is the only source of noise in the model. Uncovered interest parity is assumed to hold so the domestic interest rate can be written as the sum of the foreign interest rate, i^* , and the expected rate of change of the exchange rate, denoted F , thus $i = i^* + F$. The shocks to the foreign interest rate are therefore transmitted to domestic demand via the domestic interest rate. The determination of F is explained in detail later.

We restrict the government's choice of policy regime to either a completely fixed rate or a completely floating rate. We assume that initially a fixed rate regime is in force (with a parity of $s=0$ for convenience). In order to fix the exchange rate the government varies the money supply and hence the interest rate. The interest rate effectively becomes determined by the private sector's expectation of devaluation. (The determinants of devaluation expectations are explained more fully later.) The level of output can be written as follows

$$y_x = -\gamma(F + \epsilon) \quad (3)$$

The subscript x indicates that this relationship is for the fixed rate regime. (The exchange rate drops out of equation (3) given the assumption that $s=0$ in the fixed rate regime.)

It is assumed that the government has a once-and-for-all option to switch to a floating rate regime. This option can be exercised at any time but once it is exercised the government cannot revert to the fixed rate.⁴ Hence, after a regime switch there are no expectations of further regime switches. Given the assumption of brownian motion shocks the expected change in the exchange rate becomes zero and the domestic interest rate therefore falls into line with the foreign interest rate.

In the floating rate regime we assume that monetary policy is used to offset demand shocks coming from the foreign interest rate, thus $y_f = 0$ at all times (where the subscript f indicates a floating rate regime). Any shock is immediately offset by an adjustment to monetary policy (and hence the exchange rate) which crowds-out or crowds-in the required amount of net-exports. The exchange rate therefore has the following relationship with the shock variable

$$s_f = \frac{\gamma}{\eta} \epsilon = k\epsilon \quad (4)$$

⁴ Krugman and Rotemberg (1992) model the collapse of a fixed rate as a reversible state-contingent regime switch. In their model the authorities re-enter the fixed rate system immediately stochastic shocks take the economy back to the state at which the regime collapsed. Flood et al (1989) also model exchange rate regime switches as reversible. But in their case lump-sum costs of adjustment imply that entry and exit take place at different trigger points. In this paper we rule out the possibility of a reversible regime switch in order to keep the analysis simple. But the techniques we use can be extended to the reversible case.

where again the subscript l indicates that a floating rate regime is in force.

It is important to note that these assumptions about monetary policy will typically imply that the government implements a step-change in the money supply at the time of a regime switch in order to bring output to its target level (assumed to be zero). This implies that a regime switch is accompanied by a step-change in the exchange rate.

The framework we have laid out here is obviously highly restricted and stylised but we feel that it captures some of the main features of the EMS crisis. In the EMS countries such as France, Italy and the UK all suffered from high interest rates which were necessary to defend their currencies in the face of high German interest rates. These countries all faced a basic choice between remaining in the fixed rate system or opting out and regaining control over domestic monetary policy. The main incentive to switch regime was the opportunity a floating rate provides to loosen monetary policy and to boost output.

2. The Determination of Interest Rates

Having set up this basic framework it is now possible to describe in more detail the determination of the interest rate. The interest rates i and i^* are defined as composite rates which are formed as averages over all maturities. We assume that there is a continuous distribution of debt contracts with different maturities. Consider a debt contract starting at time t and terminating at time τ . Uncovered interest parity implies that the following must hold

$$i_{(\tau,t)} = i^*_{(\tau,t)} + \frac{E_t[s(\tau)] - s(t)}{(\tau - t)} \quad (5a)$$

where $i_{(\tau,t)}$ indicates an interest rate over the period $\tau-t$ and E_t is the expectations operator conditional on information available at time t .

We assume that debt contracts signed at time t are distributed according to a Gamma distribution of the form $r^2(\tau - t)e^{-r(\tau-t)}$. (It will be shown below that the parameter

r has a useful interpretation.) Applying this distribution to equation (5a) yields the following relationship between i and i^* (the composite interest rates)

$$i = i^* + E_t \int_t^{\infty} [s(\tau) - s(t)] r^2 e^{-r(\tau-t)} d\tau \quad (5b)$$

The composite expected rate of depreciation of the exchange rate, F , is therefore defined as

$$F = E_t \int_t^{\infty} [s(\tau) - s(t)] r^2 e^{-r(\tau-t)} d\tau \quad (6)$$

In effect the structure of F implies that the private sector discounts the future at rate r . Given the assumed distribution of maturities, the average maturity of a new debt contract is given by $(2/r)$. If r is very large (i.e. the average debt contract is very short) then the private sector discounts the future very heavily. If on the other hand r is small then the private sector discounts the future only lightly.

It is now possible to determine the behaviour of the rate of interest in the floating rate regime. Given the assumption of driftless brownian motion shocks it follows that $E_t s(\tau) = s(t)$, hence $F(t) = 0$. Uncovered interest parity therefore implies that the nominal interest differential is zero.

The behaviour of the interest differential during the fixed rate phase requires that account be taken of the possible switch of regime. The structure of the optimising problem facing the government (which will be described below) is such that the government chooses a trigger value of ϵ at which the switch of regime will take place. This trigger value will be denoted $\bar{\epsilon}$. It is assumed that the current value of ϵ is below $\bar{\epsilon}$ and that the regime switch is anticipated by the private sector to take place when ϵ hits $\bar{\epsilon}$ for the first time. At this stage we will distinguish between the private sector's expectation of $\bar{\epsilon}$, denoted $\bar{\epsilon}^e$, and the true value of $\bar{\epsilon}$.

It is shown in Appendix 1 that the solution for F in the fixed rate regime is given by

$$F = f(\varepsilon) = rk\bar{\varepsilon}^* e^{\lambda(\varepsilon - \bar{\varepsilon}^*)} \quad (7)$$

where $\lambda = \sqrt{2r/\sigma^2}$. An example of the relationship between F and the demand shock variable is illustrated as the solid curve in the upper panel of Figure 1.

Intuitively it is clear that the effect of the anticipated regime switch will be to drive a wedge between the domestic and foreign interest rates. In the case illustrated in Figure 1 the regime switch at $\bar{\varepsilon}_1^*$ brings about a devaluation of the currency so the anticipation of the switch implies $F > 0$ for all ε below $\bar{\varepsilon}_1^*$. When the shock variable is far away from the expected trigger point the private sector regards the probability of a regime switch in the near future to be low so there is a relatively small interest differential. Hence the curve in Figure 1 asymptotes to the horizontal axis as ε tends to minus infinity. But as the shock variable moves closer to $\bar{\varepsilon}_1^*$ the probability that the private sector attaches to a regime switch rises and the interest differential therefore also rises. We show in Appendix 1 that in the limit, as the shock variable approaches $\bar{\varepsilon}_1^*$, and the regime switch becomes almost certain, the interest differential approaches the value $rk\bar{\varepsilon}$. Hence the solid curve in Figure 1 passes through the point A.

In this respect the model captures an important feature of the EMS crisis. High German interest rates, represented by high values of ε , made it more likely that countries such as the UK would leave the EMS and allow their currencies to float down. This increased the interest differential between UK and German interest rates.⁵ Had German interest rates fallen (represented by lower values of ε) it seems likely that the interest differentials would also have declined in the same way as depicted in Figure 1 and equation (7).

⁵ In principle, it is possible to test this aspect of the model against the data by, for instance, regressing some measure of realignment expectations on macroeconomic variables such as output and the trade balance. Measuring realignment expectations is, however, not straightforward. In target zone systems such as the EMS the observed interest differential between two currencies is influenced both by expectations of realignment and by expected movement of the exchange rate within the band. Measuring the former requires some assumption to be made about the latter. Rose and Svensson (1993) use one technique for disaggregating expectations and find no clear relationship between realignment expectations and most macroeconomic variables. However, Chen and Giovannini (1993), using a different technique, do find some weak evidence of a link between realignment expectations and macroeconomic variables.

Note that the interest differential for any given level of the shock variable depends on the expected regime switch point. The dashed curved schedule in Figure 1 illustrates this. Here the expected regime switch point has been raised to $\bar{\epsilon}_2^*$. This causes the interest differential to fall for all levels of the shock variable. It is important to note, however, that increasing $\bar{\epsilon}^*$ only has a negative effect on the interest differential when $\bar{\epsilon}^* > 1/\lambda$.

3. The Government's Objective Function

The next stage in describing the model is to show how the government chooses the trigger point, $\bar{\epsilon}$. We assume that the government is attempting to maximise a welfare function of the form

$$W(t) = E_t \int_0^{\infty} y(\tau) e^{-\delta(\tau-t)} d\tau \quad (8)$$

where δ is the government's discount rate. This form of objective function implies that the government dislikes recessions and likes booms. In the fixed rate regime the government's flow utility falls as interest rates rise. In the floating rate regime all shocks are offset by changes in the exchange rate so the government's utility is constant (at zero).

It must be pointed out that neither the fixed rate nor the floating rate regime described above are optimal given this objective function. In this paper we are not concerned with the optimal design of an exchange rate regime, we are concerned with optimal switching between a prescribed pair of regimes. In this respect we believe that model captures the essential short term choice that EMS members faced during the crisis. And we believe that modelling the government's objective function in the form given above captures the main motivating factors that were driving government behaviour in that period.

In each regime the value of the government's objective function can be expressed as a function of the current value of the shock variable. Thus in the fixed rate regime we write $W=V_x(\epsilon)$ and in the floating rate regime we write $W=V_f(\epsilon)$. Explicit solutions for these two

functions are now obtained conditional on an arbitrary (i.e. not necessarily optimal) value of $\bar{\epsilon}$. The solution for the value function in the floating rate regime is easily obtained given the assumption that once in the floating rate regime it is not possible to switch again. It therefore follows that the government's flow payoff is zero for ever and that the value function must also be zero, i.e. $V_1(\epsilon)=0$.

The solution for the government's value function in the fixed rate regime is derived in Appendix 2. The expression obtained is as follows

$$V_x(\epsilon) = -\frac{\gamma}{\delta}\epsilon - \frac{\gamma rk\bar{\epsilon}^*}{\delta - r}e^{\lambda(r-r^*)} + \gamma \left[\frac{\bar{\epsilon}}{\delta} + \frac{rk\bar{\epsilon}^*}{\delta - r}e^{\lambda(r-r^*)} \right] e^{\theta(r-r)} \quad (9)$$

where $\theta = \sqrt{2\delta/\sigma^2}$, and λ is defined in equation (7). An example of this relationship is illustrated in the lower panel of Figure 1. Here the straight line XX is the relationship between the shock variable and the value function which would arise if no regime switch ever takes place. (The schedule XX corresponds to the first term in equation (9).) The function $V_x(\epsilon)$ is illustrated as the solid curve (where it is assumed that $\bar{\epsilon}=\bar{\epsilon}^*=\bar{\epsilon}_1$). For values of ϵ far below $\bar{\epsilon}_1$ the value of the government's welfare is very close to XX. This reflects the fact that a regime switch is very unlikely to happen in the near future. As ϵ approaches $\bar{\epsilon}_1$, $V_x(\epsilon)$ curves away from XX and approaches zero at $\bar{\epsilon}_1$. This reflects the increasing likelihood that a regime switch will be triggered. At $\bar{\epsilon}_1$, $V_x(\epsilon)$ takes the value zero, which is the value in the floating rate regime. The dashed curve illustrates the effect of choosing a different $\bar{\epsilon}$. The new choice of $\bar{\epsilon}$ raises the government's welfare at some levels of ϵ and reduces it at others.

4. The Optimal Trigger Point

The objective of the government is to choose the value of $\bar{\epsilon}$ so as to maximise V_x as given in equation (9). The structure of equation (9) however suggests that the equilibrium value of $\bar{\epsilon}$ is not obtained by simply maximising V_x with respect to $\bar{\epsilon}$. It can be seen that V_x is a function of $\bar{\epsilon}$, the current value of ϵ and also the private sector's expectation of $\bar{\epsilon}$. The fact that V_x depends on $\bar{\epsilon}^*$ arises from the effect of an anticipated regime switch on the

interest differential. Changes in $\bar{\epsilon}^e$ cause changes in the rate of interest and changes in the level of output and therefore changes to V_x . This dependence of the government's objective function on private sector expectations immediately raises the issue of credibility. The government's choice of $\bar{\epsilon}$ will depend on the extent to which the government can influence private sector expectations. In turn private sector expectations will only be influenced by the government's announced choice of $\bar{\epsilon}$ if the private sector has good reason to believe that the government will implement a regime switch at the announced value of $\bar{\epsilon}$.

We shall now proceed by first deriving the government's optimal choice of $\bar{\epsilon}$ on the assumption that policy announcements are fully credible. We will then show that as circumstances change the government has an incentive to alter its choice of $\bar{\epsilon}$. The implication of this will be that government announcements about $\bar{\epsilon}$ will not in fact be credible. We will therefore then derive the government's choice of $\bar{\epsilon}$ in the case where its announcements are not believed.

The optimal regime switch point when announcements are credible, denoted $\bar{\epsilon}_0$, satisfies the following first order condition⁶

$$(1 - \lambda \bar{\epsilon}_0) \frac{rk}{\delta - r} e^{\lambda(\epsilon - \bar{\epsilon}_0)} - (1 - \theta \bar{\epsilon}_0) \left[\frac{1}{\delta} + \frac{rk}{\delta - r} \right] e^{\theta(\epsilon - \bar{\epsilon}_0)} = 0 \quad (10)$$

where in taking the derivative of V_x it has been assumed that $\bar{\epsilon}^e$ is identically equal to $\bar{\epsilon}$, i.e. the private sector believes the government's announcements. It is not possible to solve explicitly for $\bar{\epsilon}_0$ but equation (10) shows clearly that the optimal choice of $\bar{\epsilon}$ depends on the current value of ϵ (we can therefore write $\bar{\epsilon}_0(\epsilon)$). This dependence of $\bar{\epsilon}_0$ on ϵ is the essence of the credibility problem that arises in this model.

The implications of equation (10) can be more easily explained with reference to Figure 2 where numerical solutions for $\bar{\epsilon}_0$ are plotted against ϵ for a particular set of parameter values. Figure 2 shows that when $\epsilon=0.0$ the government would announce its

⁶ Usually in an optimal regime switching problem the optimal trigger point is obtained by applying the "smooth pasting" condition. However, in this model the dependence of V on the expected regime switch point implies that the smooth pasting condition does not apply.

intention to switch to a floating rate when $\varepsilon=6.85$ (i.e. $\bar{\varepsilon}_0=6.85$). But if there is a series of adverse shocks so that ε rises to 4.0 the government finds it can increase its welfare by announcing a new regime switch point at 6.27. (In this case there is a negative relationship between $\bar{\varepsilon}_0$ and ε . For other parameter sets it will be found that the relationship is positive.)

We shall use the term "state inconsistency" to describe the property that the government's choice of $\bar{\varepsilon}$ depends on ε . There is an obvious parallel between this feature of our model and the well known property of time inconsistency (originating with Kydland and Prescott, 1979) which has been extensively analysed in rational expectations models. However, it is important to recognise the differences between the two concepts. A policy plan is said to be time inconsistent if the government has an incentive to change its plans purely because of the passage of time and not because of any change in the state of the world. The problem of time inconsistency does not arise in our model, i.e. the government's choice of $\bar{\varepsilon}$ is time consistent. This can be checked by noting that equation (10) is independent of time. Thus the value of $\bar{\varepsilon}$ obtained by solving (10) will be identical at time t_0 and time $t_1 > t_0$ provided $\varepsilon(t_0)=\varepsilon(t_1)$. The government's choice of $\bar{\varepsilon}$ can however be said to be state inconsistent because the solution to (10) is not invariant to changes in the state of the world, i.e. changes in ε .

5. The State-Consistent Trigger Point

State inconsistency gives rise to a credibility problem in the same way as time inconsistency. It is clear that rational private sector agents will be able to predict the government's incentive to deviate from its original choice of $\bar{\varepsilon}$ and will therefore not believe the government's announcements. Rational agents will use their knowledge of the government's optimising problem to determine where the actual regime switch point will be (without reference to any announcements by the government). We call this point the state-consistent regime switch point (denoted $\bar{\varepsilon}_c$). This regime switch point is analogous to the concept of a time-consistent (or discretionary) policy plan in a dynamic rational expectations model. The time consistent policy plan is the one which the government has an incentive to implement at each point in time given that the private sector believe that it will be implemented. In exactly the same way, the state-consistent regime switch point is

the point at which the government has an incentive to implement the regime switch given that the private sector believe that it will be implemented at that point.

The state-consistent regime switch point can be found in a number of different ways. The most intuitive method can be illustrated with reference to Figure 2. The curved schedule shows the regime switch point that the government would announce at each level of ϵ given that its announcements are believed. At the point where this schedule intersects the 45° line (where $\epsilon=5.70$) the government's optimal regime switch point coincides with the current value of ϵ . In other words at $\epsilon=5.70$ it is optimal for the government to implement the regime switch immediately. The private sector can forecast this and will therefore expect the regime switch to be implemented at that point. Thus, if the government announces its intention to switch regimes at $\epsilon=5.70$ the private sector will believe the announcement and the government will find it optimal to switch regimes at that point.⁷ An explicit expression for $\bar{\epsilon}_c$ can be obtained by setting $\bar{\epsilon}_c = \epsilon = \bar{\epsilon}_c$ in equation (10) and solving for $\bar{\epsilon}_c$ as follows⁸

$$\bar{\epsilon}_c = \frac{1}{\left[\frac{\delta r k(\theta - \lambda)}{\delta - r} + \theta \right]} \quad (11)$$

It is important to emphasise that even though this point is optimal at the time the regime switch is implemented, it is not optimal in other states of the world. This is clear from Figure 2. If the current value of ϵ is zero the government maximises its welfare by choosing $\bar{\epsilon}=6.85$. The option to choose an alternative regime switch point is however only available to the government if it can credibly pre-commit itself to switching regimes at that point; otherwise the private sector will not believe the announcement. The implication of

⁷ The choice of the regime switch point and private sector expectations of the regime switch point can be thought of as a nash game between the government and the private sector. If the government is a nash player it will choose the value of $\bar{\epsilon}$ to maximise (9) given some value of $\bar{\epsilon}^e$. This defines the government's reaction function. The private sector's reaction function gives the choice of $\bar{\epsilon}^e$ for a given value of $\bar{\epsilon}$. The private sector simply want to minimise their forecast error so they set $\bar{\epsilon}^e = \bar{\epsilon}$. The nash equilibrium of this game is given by the intersection of the government's and the private sector's reaction functions. Solving these two equations yields the value for $\bar{\epsilon}$ given in equation (11).

⁸ It can be shown that at $\bar{\epsilon}_c$ $V'(\epsilon)=0$, i.e. the "smooth pasting" condition is satisfied. Thus in our model the smooth pasting condition yields the state-consistent trigger point rather than the optimal trigger point.

this result (given these parameter values) is that a government which can pre-commit can make a fixed rate last longer than a government which cannot pre-commit.⁹

Before carrying out a more detailed analysis of the model it is important to consider the real world relevance of the two trigger points. In describing the optimal trigger point it was emphasised that the government had to announce and pre-commit to the chosen trigger. It is obvious that no EMS member has ever made an announcement of a trigger point for leaving the EMS (let alone attempted to pre-commit to such a policy stance). This seems to undermine the relevance of the optimal trigger point. However, it is the case that EMS governments frequently announced their intention never to leave the system. It can be argued that such announcements are approximately equivalent to setting a trigger point which is so high that it is very unlikely to be reached. In this sense the optimal regime switch point may be relevant as an explanation for the announcements of EMS governments.

The optimal trigger point is, however, unlikely to be relevant for explaining the actual breakdown of the regime itself. The EMS did not provide a strong mechanism for governments to pre-commit to their announced policy so the state-consistent trigger point is more relevant in this context. It was shown above that, with appropriate parameter values, the state-consistent trigger point is less than the optimal trigger point. It was also explained that at the state-consistent trigger point it becomes optimal for the government to switch regimes immediately despite any previous announcement. This may explain why at the height of the crisis in September 1992 the UK government chose to break its promise never to leave the EMS.

6. Determinants of the Optimal and State-Consistent Trigger Points

We now describe and discuss the effects of different parameter values on the solutions to the model. We conduct the analysis in two stages. In the first stage we

⁹ Our assumption of driftless brownian motion implies that the expected survival time of the fixed rate regime is always infinite (except when the shock variable hits a trigger point). However, for any given finite trigger point the median survival time is finite and is positively related to the distance between the current value of ϵ and the trigger point.

consider the behaviour of the absolute values of the two trigger points. In the second stage we consider how the optimal and state-consistent trigger points change relative to each other. In all cases the optimal trigger point is evaluated at $\epsilon=0$. Our base parameter values are $\eta=0.5$, $\gamma=5$, $\sigma=1$, $\delta=0.01$ and $r=0.1$. The important features of the model are entirely determined by the values of k , and the two discount rates δ and r .¹⁰

(a) The absolute values of the trigger points

Figure 3 plots the optimal and state-consistent trigger points against values of k . It can be seen that the two trigger points decline as k rises. Higher values of k imply a larger devaluation in the event of a regime switch, and hence higher interest rates in the period before a switch. A regime switch therefore becomes more attractive and the trigger points are reduced.

The effect of changing the parameter r is illustrated in Figure 4. The average length of debt contracts is given by $2/r$ so the larger is r the shorter are debt contracts. Figure 4 shows that as r increases the optimal trigger point is not greatly affected while the state-consistent trigger point decreases. For low values of r the average debt contract is very long so the devaluation associated with a regime switch is spread over a long horizon. Interest rates are therefore little affected by the prospect of a regime switch. Increasing r increases the effect of a possible regime switch on interest rates and therefore increases the government's incentive to switch to a float. Hence the decline in the state-consistent trigger point. Increasing r also increases the government's incentive to manipulate expectations so there is also an increasing divergence between the optimal and state-consistent trigger points.

In Figure 5 the two trigger points are plotted against the government's discount rate, δ . As δ is increased both the optimal and state-consistent trigger points decline. When the government is deciding on a possible regime switch it has to weigh the immediate gain in output it will obtain by floating against the loss of possible high levels of output it may

¹⁰ The two trigger points also depend on the noise parameter σ . However, the median survival time is independent of σ so we do not consider this parameter in our analysis.

gain in the future in the fixed rate system if the shock turns favourable. The more the government discounts the future the more likely it is to opt for a switch in regime. Hence higher δ reduces both trigger points.

(b) The optimal trigger point relative to the state-consistent trigger point.

We now consider how varying the parameter values affects the optimal trigger point relative to the state-consistent trigger point. First of all notice from Figure 3 that for $k=0.0$ the two trigger points are equal. This reveals the crucial role that γ plays in generating the government's credibility problem. If γ is zero (and hence $k=0$) the rate of interest has no effect on the level of output. Private sector expectations of a regime switch therefore have no effect on the government's optimising problem and the government has no incentive to manipulate expectations through its choice of trigger point.

For low values of k the state-consistent trigger point is less than the optimal trigger point. When the government cares about interest rates, i.e. γ does not equal zero, the government has an incentive to attempt to manipulate private sector expectations. This effect is built into the choice of the optimal trigger point. However, if the government cannot credibly commit to a trigger point its announcements do not affect private sector expectations. Interest rates are therefore higher than when announcements are credible. This leads the government to bring forward the switch of regimes.

It can be seen, however, that for high values of k the optimal trigger point is less than the state-consistent trigger point. In other words a government that can commit to the optimal trigger point switches regimes sooner than a government that cannot commit itself. This reversal of optimal and state-consistent trigger points can be explained by considering the effect of the trigger point on the interest rate (via the interest differential). The derivative of F with respect to the (expected) trigger point is as follows

$$\frac{\partial F}{\partial \bar{\epsilon}^e} = rke^{\lambda(\epsilon - \bar{\epsilon}^e)} - \lambda r k \bar{\epsilon}^e e^{\lambda(\epsilon - \bar{\epsilon}^e)} \quad (12)$$

A delay in the expected regime switch has two effects on the interest differential. The first effect, which is given by the first term in (12), arises from the impact of an increase in the expected switch point on the size of the devaluation. A higher value of the expected trigger point raises the devaluation and therefore raises interest rates. The second effect, given by the second term in (12), represents the reduced probability of a regime switch when the expected trigger point is increased. The overall impact of these two effects may be positive or negative depending on the sign of $(1-\lambda\bar{\epsilon}^e)$. If k is small the trigger point is relatively large and the derivative in (12) is likely to be negative. In choosing its optimal trigger point the government will therefore find it has an incentive to announce a delay in the regime switch in order to reduce interest rates. If k is large the trigger point will be relatively small so the derivative in (12) is likely to be positive. In this case bringing the regime switch forward will reduce interest rates. The government's optimal trigger point will therefore be less than the state-consistent trigger point. (It can be shown that when k is less than $(1/\delta) - (1/r)$ the optimal trigger point is greater than the state-consistent trigger point and vice versa when k is greater than $(1/\delta) - (1/r)$.)

Figure 4 shows that for low values of r (to be precise when $r < \delta/(1 - \delta k)$) the optimal trigger point is less than the state-consistent trigger point. The explanation for this is the same as given in the previous paragraph. When the trigger point is low the derivative in (12) will be positive, so reducing the trigger point reduces interest rates. A government which can make credible precommitments will therefore tend to bring a regime switch forward. While for $r > \delta/(1 - \delta k)$ the derivative in (12) will be negative and the government will prefer to delay a regime switch in order to benefit from lower interest rates. The same arguments apply to the effect of δ on the relative values of the two trigger points. For small δ the optimal trigger point is greater than the state-consistent trigger point and vice versa for large δ .

Figures 3, 4 and 5 show that the divergence between the optimal and state-consistent trigger points is large if r and k are large and δ is small. In this situation the state-consistent trigger point will be small and the optimal trigger point will be large. In other words a government in this situation would like to announce a trigger point which

implies a prolonged commitment to the fixed rate. But if that government cannot credibly pre-commit itself it will find that the private sector expects a regime switch to occur at a much lower trigger point than that announced.

It is typically assumed that financial markets have a short horizon (r is large) while governments have a relatively longer horizon (δ is small). It is also plausible to assume that the interest elasticity of aggregate demand, γ , and therefore k , will be relatively large in countries (such as the UK) where floating rate mortgages on private residential property are the norm. The case where the state-consistent trigger point is lower than the optimal trigger point therefore seems to be the empirically relevant case.

7. Policy Measures to Preserve the Fixed Rate

In this section we show how our model can be used to consider two methods by which a government can preserve the fixed exchange rate. In this first case the government uses capital controls to break the link between domestic interest rates and devaluation expectations. In the second case the government binds itself to the fixed rate in such a way that it will pay a penalty if it floats.

There are a number of ways in which capital controls can be modelled. We consider the effect of a tax on foreign investment income which drives a wedge between the level of the domestic interest rate and devaluation expectations. The tax rate is denoted μ so that the domestic interest rate becomes $i = F + \epsilon - \mu$. Initially we assume that this tax is only imposed while the fixed rate is in force. When a change of regime is triggered the tax rate is set to zero. The level of output in the fixed rate regime becomes

$$y_x = -\gamma(F_x - \mu + \epsilon) \tag{13}$$

so the effect of the tax is to add a constant amount to output (corresponding to the constant reduction in the interest rate). The level of output in the floating rate regime is unaffected by the tax. The result of this modification to the model is that the state-consistent trigger point becomes

$$\bar{\epsilon}_c = \frac{1 + \mu\theta}{\left[\frac{\delta r k(\theta - \lambda)}{\delta - r} + \theta \right]} \quad (14)$$

The denominator in this expression is always positive so the effect of the tax is to increase the trigger point. Thus a tax can achieve the desired result of prolonging the life of the fixed rate.

At first sight it may appear that the tax is achieving this affect by partly breaking the link between domestic interest rates and devaluation expectations. This however is not the case. The tax puts a fixed wedge between $F+\epsilon$ and i but it does not alter the marginal effect of devaluation expectations on i and thus on the government's objective function. The true source of the increase in the trigger point is the effect of the tax on output in the fixed rate regime. Equation (13) shows that the tax increases output by a fixed amount while the fixed rate is in force. When the regime switch is triggered the tax rate is assumed to be set to zero and the beneficial effect of the tax on output is lost. The lost of output causes the government to delay the switch in regime.¹¹

An alternative way in which a government could try to prolong the life of the fixed rate is to raise the cost to itself of switching regimes. For example a government could make an explicit commitment to resign in the event of a failure of the fixed rate policy.¹² We can represent this as a lump-sum cost of transferring from the fixed rate regime to the floating rate regime. Suppose the government suffers a fixed cost of X when it triggers a

¹¹ An alternative way in which capital controls may enter the model is by generating a variable wedge between i and $F+\epsilon$. Suppose for instance that capital controls imply that $i = \mu(F+\epsilon)$ where $0 < \mu < 1$. In this case output in the fixed rate regime becomes $y_x = -\gamma\mu(F_x + \epsilon)$. In other words the impact of changes in devaluation expectations on output is reduced. The effect of this on the state-consistent trigger point is easily deduced. It is effectively the same as reducing the value of the parameter γ . From Figure 3 it can be seen that reducing γ increases the value of the state-consistent trigger point. In this case the explanation for the increased survival time of the fixed rate is that private sector expectations have, at the margin, less of an effect on the government's objective function. The government therefore has less of an incentive to switch regimes in order to benefit from lower interest rates.

¹² By making such a commitment the government will not be able to avoid political costs if the fixed rate policy collapses. Either the government will have to resign and therefore suffer being out of office. Or it will have to suffer the loss of political face that would accompany a failure to honour its promise to resign.

regime switch. After taking account of this modification we find that the state-consistent regime switch point is now given by the expression

$$\bar{\epsilon}_c = \frac{1 + \delta\theta\chi/\gamma}{\left[\frac{\delta rk(\theta - \lambda)}{\delta - r} + \theta \right]} \quad (15)$$

The denominator of this expression is positive so the state-consistent regime switch point is positively related to the size of the lump sum cost of switching. Thus the greater the cost of abandoning the fixed rate the longer the fixed rate will survive. It is immediately apparent that formally the effect of a lump sum cost of switching is identical to the effect of the investment tax considered above. This is because the investment tax works by adding a lump sum amount to output while the fixed rate is in force. Abandoning the tax effectively imposes a lump sum cost of switching.

Conclusion

We suggest that the model presented in this paper captures many of the mechanisms at work in the ERM crises of September 1992 and July 1993. The model emphasises the role of high German interest rates in generating (or prolonging) a negative demand shock in the rest of Europe. The output loss generated by this shock gives rise to expectations in financial markets of an abandonment of the fixed exchange rate. This in turn further increases domestic nominal interest rates and therefore hastens the collapse. We demonstrate this process in a model which pays explicit attention to the policy optimising problem facing the government. The preferences of the government, and private sector perceptions of those preferences, play a crucial role in determining the timing of the regime switch.

Eichengreen and Wyplosz (1993) present evidence to suggest that those currencies subject to speculative attacks in September 1992 were not substantially over valued. They therefore claim that the speculative attacks were driven more by self-fulfilling expectations on the part of financial markets rather than more fundamental forces. At first sight this might be taken as evidence against the relevance of our model. However, we would claim that our framework is consistent with these facts. What is important in our model is that

the government and the private sector perceive that there is some advantage for the government in abandoning the fixed rate. It does not necessarily have to be the case that the exchange rate is over-valued in any objective sense.

One way in which our model could be improved is to introduce adjustment lags into output. In the model we have presented here output falls as soon as interest rates rise. But, if there are adjustment lags a rise in foreign interest rates would not have an immediate adverse effect on output. The government would therefore find it optimal to delay any regime switch in order to see whether (or how far) foreign interest rates fall in the meantime. If there are no significant favourable shocks to foreign rates the regime switch would then be triggered. Modifying our model in this way has the advantage that it predicts the sort of delay which separated the peak in German interest rates from the onset of the ERM crisis in 1992.

Our model emphasises optimal switching between regimes while pushing into the background the optimal design of those regimes. In fact in our model when the demand shock is zero the government is indifferent between regimes. It is only when there are unfavourable shocks that the government prefers the floating rate regime. We believe that this approach captures the main forces behind the ERM crises. But it obviously does not incorporate the underlying factors which induced ERM members to fix their rates in the first place. A second important extension to our work would be to consider more complex economic structures and more realistic government welfare functions. There are likely to be important interactions between the underlying motivation for the ERM and the conditions which brought about its collapse.

Appendix 1

In order to derive an explicit solution for F in the fixed rate system it is convenient to differentiate equation (6) with respect to time. This yields

$$\frac{E_t dF}{dt} = rF \quad (\text{A1})$$

(the exchange rate does not appear since $s_x=0$). Given the structure of the model we can postulate that F will be a function of ϵ , that is $F=f(\epsilon)$. Applying Ito's lemma to this function yields

$$dF = f'(\epsilon)d\epsilon + \frac{\sigma^2}{2}f''(\epsilon)dt \quad (\text{A2})$$

After substituting from equation (A1) and taking expectations the following differential equation in $f(\cdot)$ is obtained

$$rf(\epsilon) = \frac{\sigma^2}{2}f''(\epsilon) \quad (\text{A3})$$

This has a general solution of the form

$$f(\epsilon) = Ae^{\lambda\epsilon} + Be^{-\lambda\epsilon} \quad (\text{A4})$$

where $\lambda = \sqrt{2r/\sigma^2}$ and A and B are constants to be determined by boundary conditions.

The appropriate boundary conditions are implicit in the discussion in the paragraph following equation (7) in the main text. When the shock variable is far below $\bar{\epsilon}^e$ it must be the case that F is very close to zero. Formally we require that F tends to 0 as ϵ tends to minus infinity. This immediately implies that B should be set to zero. The other arbitrary constant is tied down by a boundary condition at $\bar{\epsilon}^e$. Consider a debt contract of maturity $(\tau-t)$. As ϵ approaches $\bar{\epsilon}^e$ it must be the case that the interest differential for that maturity approaches $k\bar{\epsilon}^e/(\tau-t)$, (where k is the slope of the free float relationship between s and ϵ).

Integrating over all maturities implies that F must approach $rk\bar{\epsilon}^*$ as ϵ approaches $\bar{\epsilon}^*$. Hence the second boundary condition is $f(\bar{\epsilon}^*) = rk\bar{\epsilon}^*$. The arbitrary constant A is therefore given by

$$A = rk\bar{\epsilon}^* e^{-\lambda^*} \quad (A5)$$

and the relevant solution for the forward rate is as given in equation (7).

Appendix 2

Solving explicitly for $V_x(\epsilon)$ is complicated by the need to take account of the prospective switch of regime. The first stage is to apply Ito's lemma to $V_x(\cdot)$ to obtain

$$dV_x(\epsilon) = V_x'(\epsilon)d\epsilon + V_x''(\epsilon)\frac{\sigma^2}{2}dt \quad (A6)$$

By differentiating (8) with respect to time the following expression for $E_t(dV_x)$ is obtained

$$\frac{E_t[dV_x(\epsilon)]}{dt} = \delta V_x(\epsilon) - y(t) \quad (A7)$$

Taking expectations of (A6) and substituting from (A7) and (3) yields the following differential equation in $V_x(\cdot)$

$$\frac{\sigma^2}{2} V_x''(\epsilon) = \delta V_x(\epsilon) + \gamma F_x(\epsilon) + \gamma \epsilon \quad (A8)$$

This has a solution of the form

$$V_x(\epsilon) = -\frac{\gamma}{\delta} \epsilon - \frac{\gamma rk\bar{\epsilon}^*}{\delta - r} e^{\lambda(\epsilon - \bar{\epsilon}^*)} + Ce^{\alpha\epsilon} + De^{-\alpha\epsilon} \quad (A9)$$

where C and D are arbitrary constants to be determined by boundary conditions, $\theta = \sqrt{2\delta/\sigma^2}$, and λ is defined in equation (7). (This solution is only valid for $\delta - r \neq 0$, an alternative solution can be found for the case $\delta - r = 0$.)

The first term in equation (A9) is the value of the government's objective function in the case where the exchange rate is permanently fixed. By the same argument as was applied to the solution for F it should be the case that for values of ϵ far below the regime switch point the value function should be close to the value it would take when there is no regime switch point. This implies that $D=0$.

The value of the other arbitrary constant, C, is tied down by the "value matching" condition. At $\bar{\epsilon}$ a regime switch takes place with certainty so the value function in the fixed rate regime should be exactly equal to the value function in the floating rate regime, formally $V_x(\bar{\epsilon}) = V_1(\bar{\epsilon}) = 0$. This yields the following expression for C

$$C = \gamma \left[\frac{\bar{\epsilon}}{\delta} + \frac{rk\bar{\epsilon}^*}{\delta - r} e^{\lambda(\bar{\epsilon} - \bar{\epsilon}^*)} \right] e^{-\theta\bar{\epsilon}} \quad (\text{A10})$$

and the corresponding expression for $V_x(\epsilon)$ is as given in equation (9) in the main text. In the case where there is a lump sum cost of switching regimes, denoted X, the value matching condition becomes $V_x(\bar{\epsilon}) = V_1(\bar{\epsilon}) - X$. This yields the solution for the state-consistent trigger point given in equation (15).

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Figure 1a: The Interest Differential

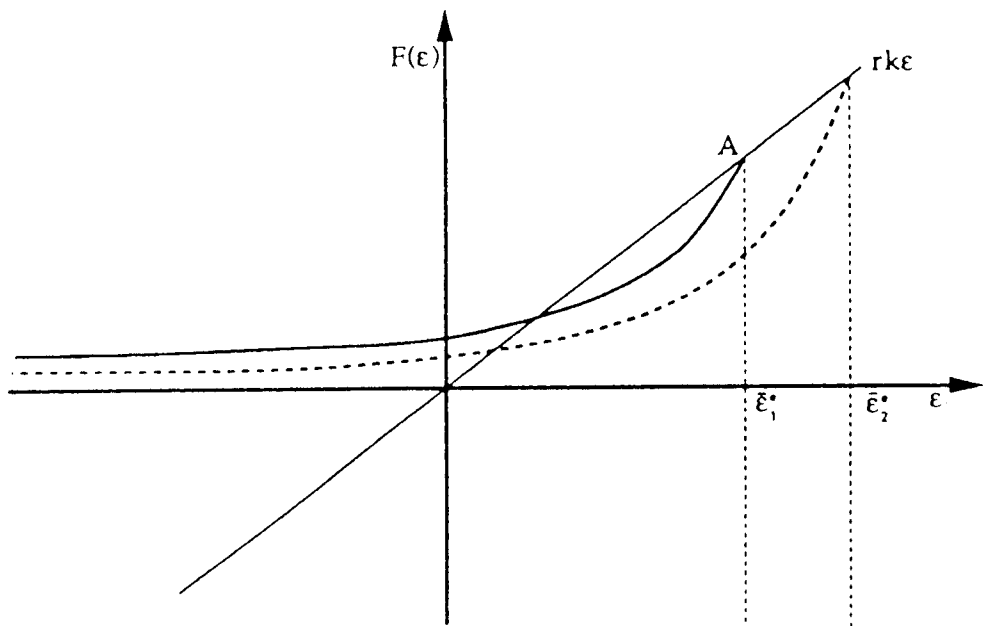


Figure 1b: The Government's Value Function

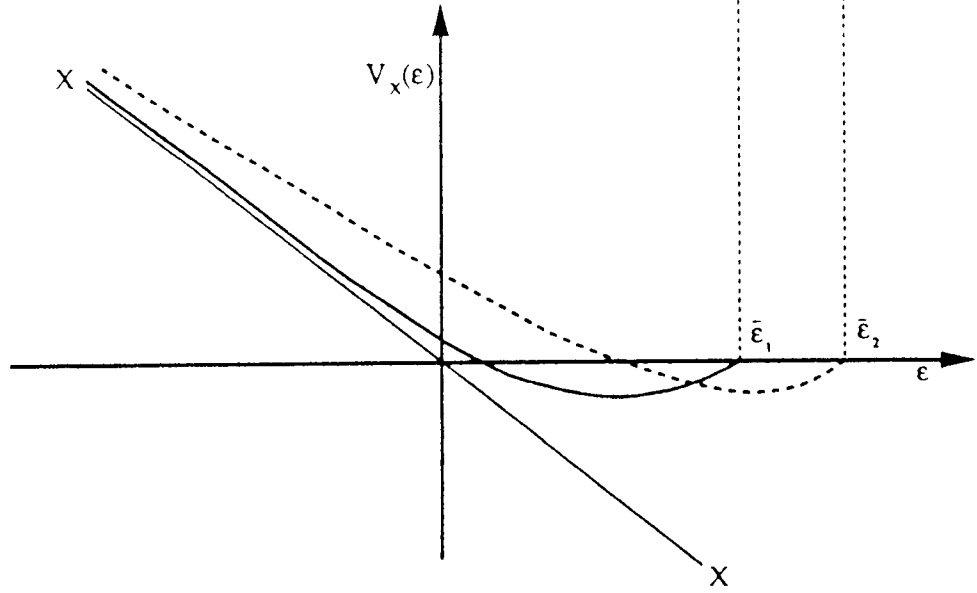


Figure 2: Optimal and State-Consistent Trigger Points.

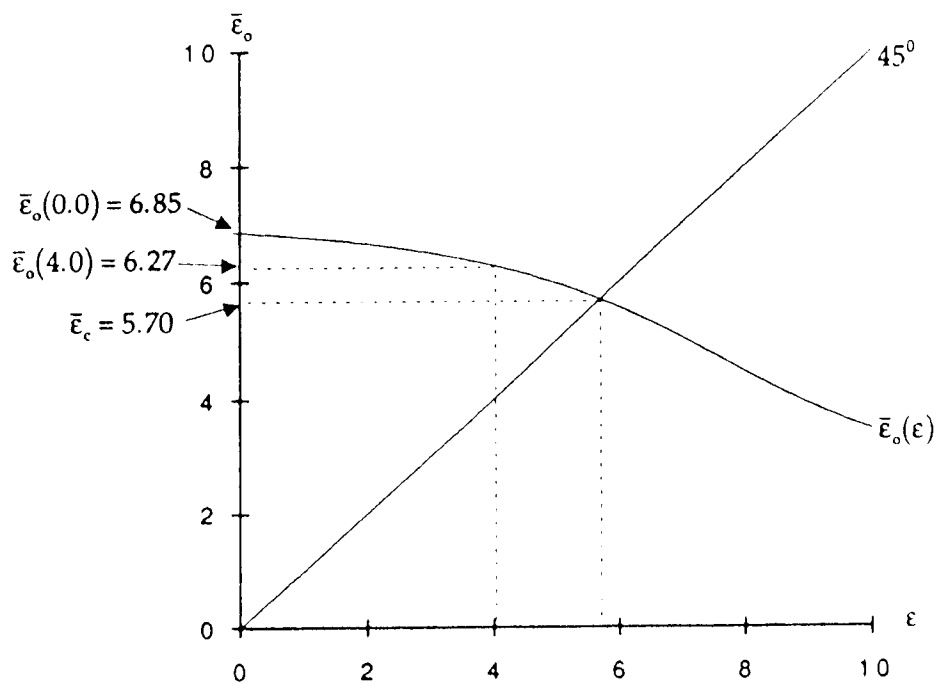


Figure 3: k and the Optimal and State-Consistent Trigger Points

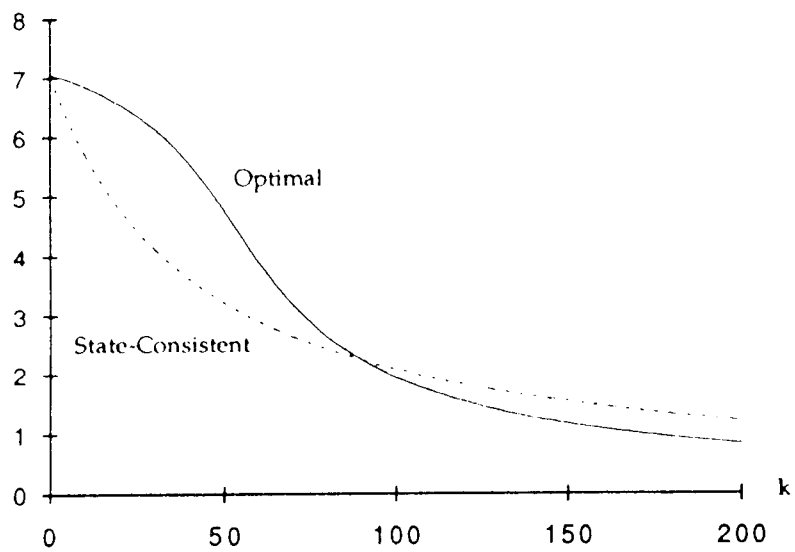


Figure 4: r and the Optimal and State-Consistent Trigger Points

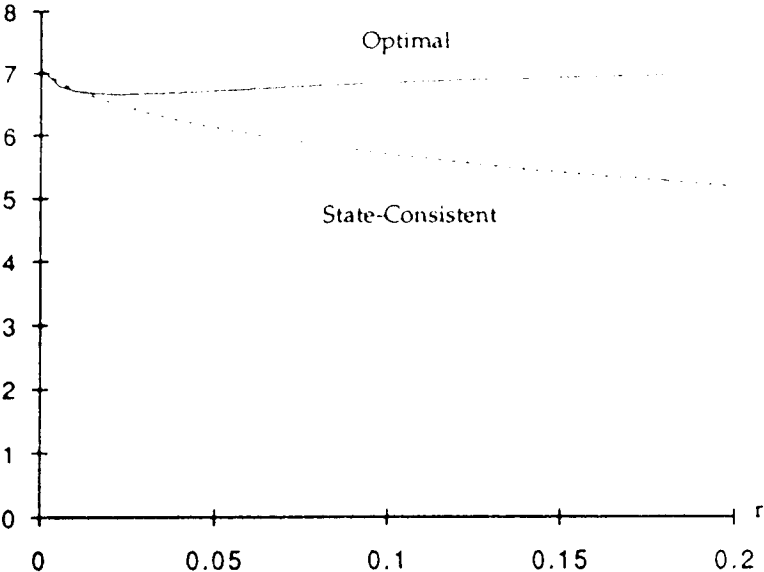


Figure 5: δ and the Optimal and State-Consistent Trigger Points

