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ABSTRACT

Environmental Policy, Public Finance and the Labour Market in a Second-best World*

Environmental and tax policies and the optimal provision of clean and dirty public goods are analysed within the context of a second-best framework of optimal taxation. Households consume both clean and dirty commodities. Degradation of the natural environment occurs due to the consumption of dirty private and public goods, but can be offset when the government engages in abatement activities. The 'double dividend' hypothesis, i.e. raise the dirt tax and reduce the labour tax in order to enhance both environmental quality and employment, fails. Increased environmental concern implies a higher dirt tax, a lower tax on labour, less employment and economic activity and a cleaner environment. If the elasticity of substitution between private consumption commodities and leisure is large, and that between clean and dirty goods is small, public consumption expands while private consumption contracts. Otherwise, public consumption falls.

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NON-TECHNICAL SUMMARY

One of the most pressing problems facing the world today is pollution and the accompanying degradation of the natural environment – the deterioration of the ozone layer, the rapidly disappearing rain forests, falling levels of ground water, the acidification of the natural environment, noise, etc. Households and firms pollute the natural environment too much, mainly because they do not pay a price for the social damage they inflict on the environment and thus on the welfare of other citizens in society. In this sense, it is useful to admit that environmental policy has to deal as best as it can with the problem of missing markets. This is why there is much talk by political parties and commentators on the possibility of introducing markets for pollution permits or, alternatively, of levying Pigovian taxes and subsidies. In other words, governments impose an implicit or an explicit corrective tax on polluting activities in such a way that the private cost is raised to the social cost. When the revenues are handed back to the polluters in a lump-sum fashion, the first-best outcome can be sustained in a decentralized competitive market economy.

Instead, some argue that if property rights are well defined, polluters and the victims of pollution should engage in bilateral negotiations. This may also lead to an efficient outcome – as the recent Nobel prize winner Ronald Coase demonstrated in 1960. Such a 'laissez faire' policy, relying on market forces and private negotiations between citizens and firms, fails in practice, however, because it only applies if transaction costs are neglible and only a few parties are involved. The costs of negotiations in terms of lost time and legal costs may undermine the feasibility of this market solution and thus necessitate government intervention.

What form should government intervention take? In practice, most governments excel in 'wishful thinking' where environmental policy is concerned. Many covenants are signed between governments and polluting firms in which the latter promise on scout's honour that they will do their utmost to clean up their production process and reduce emissions. Unfortunately, many of the covenants do not lead to substantial reductions in pollution. It is usually much better to use market-oriented instruments. The government could set environmental targets for reductions in emissions and introduce a market for pollution permits. The price these permits will fetch on the market will then automatically become equal to the social cost of pollution and an efficient allocation of non-polluting and polluting activities will be found.

If the government has an explicit environmental target which overrides all other economic policy objectives, it is best to introduce a market for pollution permits because this overcomes information problems. After all, a social planner is unable to collect information on the social damages and benefits of pollution from millions of households and thousands of firms. In general, however, governments find that the environmental target involves a complicated calculation in which targets for public goods are set as well. It then becomes unclear whether it is optimal to use marketable pollution permits or a system of Pigovian taxes and subsidies.

Another major and more traditional task of governments, apart from conducting a sound environmental policy, is to raise taxes from a variety of sources in order to finance public spending. This is straightforward if lump-sum taxes are available. In practice, however, governments have to resort to distortionary taxes. The UK government's experience serves as a vivid reminder of the political problems involved in levying a poll tax. In order to minimize the excess burden of distortionary taxation, governments should adopt a mix of tax instruments and, if cross-price effects are not too strong, tax most heavily those commodities that feature relatively low price elasticities. This is relevant when taxes only serve to finance public expenditures. Taxes also have a Pigovian objective, however, in the sense that they can be used to encourage households and firms to economize on polluting activities.

The crucial policy question is, therefore, what governments should do when faced with the dual task of internalizing environmental externalities on the one hand, and raising tax revenues to finance public spending on the other. They must design the appropriate environmental policy in a second-best world in which a sizeable public sector gives rise to serious tax distortions. Many political parties and commentators have argued that the best response in such a situation is to introduce dirt taxes to internalize pollution (or to auction off pollution permits), and to use the revenues to reduce the distortionary tax rates elsewhere in the economy. In particular, may politicians have argued that an increase in the (non-distortionary) dirt tax rate accompanied by a cut in the tax on labour may kill two birds with one stone: (i) an improvement in the quality of the environment; and (ii) a boost in employment and an expansion of the labour tax base due to a lower tax wedge between producer and consumer wages. David Pearce (1991) called this the 'double dividend' hypothesis. One of the main contributions of this paper is to argue that this 'double dividend' hypothesis violates fundamental principles of public finance.

This paper demonstrates that the 'double dividend' hypothesis put forward by many politicians and some economists is a red herring. Although it is optimal to raise the dirt tax and reduce the labour tax in response to increased environmental concern, employment typically falls. Dirt taxes are explicitly designed to encourage a change in the composition of private consumption towards cleaner goods. This imposes a cost in terms of private utility, which tends to worsen pre-existing tax distortions by eroding the base of existing distortionary taxes. Moreover, increased environmental concern may make public consumption more expensive and raise public abatement. The implied higher overall tax level is especially costly if existing taxes impose serious distortions. Indeed, countries with a large public sector often find it costly to implement a tough environmental policy because it may erode the tax base and thus require even higher marginal tax rates, raising the cost of public funds.

To attenuate the adverse effects of environmental policy on labour market distortions and the cost of public funds, it may be necessary to reduce the size of the public sector. Indeed, the paper shows that it is optimal to reduce public consumption in response to increased environmental concern if the elasticity of substitution between private goods and leisure is small and that between clean and dirty goods is large. In this case the environment is improved, in part, through a different composition of economic activity. The costs accompanying this structural change in economic activity raise the cost of public funds, thereby reducing public consumption. If substitution between clean and dirty commodities is more difficult, however, and the productivity of public abatement declines rapidly, a cleaner environment must be achieved primarily through less production and more consumption of leisure. In this case the public sector may expand if the substitution effects due to lower after-tax wages are large. The intuition is that the disincentive effects resulting from high tax levels are no longer undesirable. Indeed, public consumption becomes easier to finance as non-distortionary taxes are available. Hence, in this case, 'green' political parties are compatible with 'red' political parties. More specifically, red and green preferences are likely to be compatible only if clean and dirty goods are complements, dirt and labour taxes induce similar behavioural effects, and the environment is enhanced through a lower level of economic activity.

What is the impact of more priority for public consumption (i.e. a shift towards redder preferences)? The dirt tax falls, the labour tax rises, abatement falls, the composition of public and private composition becomes dirtier, the level of (conventional) public goods rises and the level of marketable goods and private welfare fall. Economic activity falls if the uncompensated labour supply curve slopes upwards. Public consumption and the quality of the natural environment move together if substitution between dirty and clean goods is difficult compared to substitution between leisure and private commodities.

An important direction on the agenda for future research is to abandon the world of representative agents and extend the analysis of this paper to address equity issues. Efficiency considerations may lead governments to tax necessities more heavily than luxuries. This clearly violates the equity objective. Increased environmental concern may imply heavier taxes on dirty necessities, less progressive taxes on labour, and less public consumption. An equitable income distribution may be viewed as a public good – cf., the work of Lester Thurow (1971). It is thus of interest to investigate the trade-off between the various public goods (public consumption of clean and dirty products, the quality of the natural environment and an equitable income distribution). This line of analysis should benefit from abandoning the assumption of homothetic preferences. For example, allowing for subsistence levels of consumption and necessities sheds more light on the issue of basic needs and equity in relation to environmental policy.

Other directions for further research are to extend the framework of analysis to allow open economies and issues of international policy coordination, to allow for polluting factors of production, to allow for pollution as a by-product of production, and to introduce elements of rent seeking and political economy in the analysis.

1 Introduction

One of the most pressing problems facing the world today is pollution and the accompanying degradation of the natural environment. Households and firms pollute too much, because they do not face a price for the damage they inflict on the environment. In this sense environmental policy can be viewed as a problem of missing markets. The government may therefore introduce markets for pollution permits or, alternatively, resort to Pigovian tax and subsidy schemes.[1] In other words, governments impose an implicit or an explicit corrective tax on polluting activities in such a way that the private cost is raised to the social cost. When the revenues are handed back to the polluters in a lump-sum fashion, the first-best outcome can be sustained in a decentralised competivive market economy.[2]

However, another major task of governments is to raise taxes from a variety of sources in order to finance public spending. This is straightforward if lump-sum taxes are available. However, in practice governments have to resort to distortionary taxes. In order to minimise the excess burden of taxation, governments should adopt a mix of tax instruments and, if cross-price effects are not too strong, tax most heavily those commodities that feature relatively low price elasticities (Ramsey, 1927).

The crucial policy question is what governments should do when they face the dual task of on the one hand internalising environmental externalities and on the other hand raising tax revenues to finance public spending. The relevant problem is thus one of designing the appropriate environmental policy in a second-best world in which a sizeable public sector gives rise to serious tax distortions. One might argue that the best response in such a setting is to introduce dirt taxes to internalise pollution (or to auction off pollution permits) and to use the revenues to reduce the distortionary tax rates elsewhere in the economy. In particular, an increase in the (non-distortionary) dirt tax rate accompanied by a cut in the tax on labour may kill two birds with one stone: (i) an improvement in the quality of the environment; (ii) a boost to employment and an expansion of the labour tax base due to a lower tax wedge between producer and consumer wages. Pearce (1991) has called this the "double dividend" hypothesis.

Following Ulph (1991), Bovenberg and de Mooij (1992) and Pezzey (1992), this paper analyses the interaction of environmental and tax policy in a second-best world in which lump-sum taxes are not available. The main innovation of this paper is to analyse the optimal setting of dirt and labour taxes from a Pigou-Ramsey perspective, building on the pioneering

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[1] If the government has an explicit environmental target which overrides all other objectives of economic policy, it is best to introduce a market for pollution permits as this overcomes information problems. In general, however, it is not clear whether it is optimal to use pollution permits or Pigovian taxes and subsidies.

[2] Instead, some argue that, if property rights are well defined, polluters and the victims of pollution should engage in bilateral negotiations. This may lead to an efficient oucome as well (Coase, 1960). However, such a "laissez faire" policy fails in practice because it only applies if transaction costs are neglible and only a few parties are involved.

work of Sandmo (1975), and to simultaneously consider the optimal level and composition of public consumption (cf., Atkinson and Stern, 1974) and the optimal level of abatement activities. In order to obtain more specific results, preferences are restricted in such a way that the private decision on the composition of the private consumption basket in clean and dirty products is weakly separable from the decision to work. Moreover, the consumption of marketable goods, i.e. the basket of clean and dirty private goods and leisure, is weakly separable from the consumption of social goods, i.e. the basket of public consumption goods and the quality of the natural environment. Ulph (1991) and Pezzey (1992) only investigate the optimal setting of the labour and the dirt tax rates under the assumption of Cobb-Douglas preferences and an exogeneous revenue constraint. This paper, in contrast, analyses the optimal tax structure and the optimal level and composition of public consumption and abatement activities in the more general case of homothetic preferences.

Section 2 investigates environmental and tax policy in a first-best world where lump-sum taxes and subsidies are available. In this setting it also analyses the optimal provision of clean and dirty public consumption goods and the optimal level of public abatement activities. Assuming that only distortionary taxes are feasible, section 3 discusses the Ramsey tax rules and the optimal provision of public goods if environmental externalities are absent. Section 4 integrates the results of sections 2 and 3. Within the framework of optimal distortionary taxation, it simultaneously considers the correction for environmental externalities, the provision of clean and dirty public consumption goods and the level of public abatement activities. Section 5 restricts preferences in such a way that the demand for clean and dirty private commodities is weakly separable from leisure and that the demand for social goods is weakly separable from the demand for marketable goods. Section 6 loglinearises both the model of private behaviour and the conditions characterising optimal public policy in order to more closely examine the "double dividend" hypothesis. Section 7 explores how an increase in environmental concern affects the optimal level of abatement activities, the optimal provision of clean and dirty public consumption and the optimal tax mix. We find that an increased concern for the environment induces a higher dirt tax (or a higher price for pollution permits), a lower tax on labour, more consumption of leisure, less production and a cleaner environment. If the elasticity of substitution between private goods and leisure is less than unity, "green" and "blue" political parties are compatible in the sense that the basket of marketable goods expands and the basket of public goods contracts if the concern for the environment increases. However, a rise in public consumption may accompany increased environmental concern, if the elasticity of substitution between private goods and leisure is large and between clean and dirty goods is small. In that case "red" and "green" political parties are compatible. In this case, most of the improvement in environmental quality is due to a lower level rather than a different composition of economic activity. With a unitary elasticity of substitution between private goods and leisure, production and unemployment are unaffected and environmental quality is enhanced only through a cleaner composition of economic activity. Section 8 explores the impact of more priority for public consumption. It confirms that public consumption and the quality of the environment move together if substitution between dirty and clean goods is difficult compared to substitution between leisure and private commodities. Section 9 concludes with a summary of results and suggestions for further research.

2 Environmental policy in a first-best world

2.1 The command economy

The representative consumer derives utility (U) from the consumption of marketable goods, namely clean private goods (C), dirty private goods (D) and leisure (V). Furthermore, social goods, *viz.* clean public goods (X), dirty public goods (Y) and the quality of the environment (E), raise utility. The utility function for the representative consumer is thus denoted by $U=\mathrm{u}(C,D,V,X,Y,E)$. The quality of the environment detoriates when private agents or the public sector consume more dirty goods. However, environmental quality improves when the government engages in abatement activities (A), i.e. $E=\mathrm{e}(\mathrm{N}D+Y,A)$ with $\mathrm{e}_{\mathrm{N}D}=\mathrm{e}_{Y}<0$ and $\mathrm{e}_A>0$, where N stands for the number of private agents in the economy. Private agents ignore environmental externalities when they decide on the consumption of dirty products.

Producers face a linear constant-returns-to-scale technology in which output is proportional to employment (L) and can be used for consumption of (clean and dirty) private and public goods and for public spending on abatement activities.[3] The material balance condition for the economy is thus given by:

$$\beta_L \mathrm{N}L = \beta_L \mathrm{N}(1-V) = \beta_C \mathrm{N}C + \beta_D \mathrm{N}D + \beta_X X + \beta_Y Y + \beta_A A, \quad \beta_i>0. \qquad (2.1)$$

Each worker-consumer has one unit of time available which can be used for either work (L) or leisure (V). We assume, without loss of generality, that $\beta_i=1$, $i=C,D,X,Y,A$. The production price associated with these goods acts thus as the numeraire. The production price of labour, the product wage, is denoted by $\beta_L\equiv\beta$.

It is of some interest to consider as a benchmark the first-best outcome, which is attainable in a command economy. In such an economy, the social planner maximises utility of the representative household subject to the material balance condition *(2.1)*. This yields the following optimality conditions:

[3] Relaxing the assumption of fixed producer prices and linear technology does not affect the results as long as producer prices result from competitive behaviour and producers face constant returns to scale (or pure profits are taxed away under decreasing returns to scale) (cf., Auerbach, 1985).

$$u_C = \beta^{-1}\, u_V = Nu_X = u_D + Nu_E e_{ND} = N(u_Y + u_E e_Y) = Nu_E e_A. \tag{2.2}$$

The marginal rate of substitution between leisure and clean products must equal the relevant marginal rate of transformation, i.e. the product wage. The marginal utility of clean private products must be set to the sum of the marginal utilities of clean public goods for each of the private agents (cf., Samuelson, 1954). The marginal utility of clean products must equal that of dirty products minus a term correcting for the environmental externality (note $e_{ND}=e_Y<0$). This correction term drives up the consumption of clean products at the expense of the consumption of dirty products in order to reduce pollution and ensure an optimal quality of the environment. This condition must hold for both private and public goods. Finally, the sum of marginal utilities of a cleaner natural environment resulting from public abatement activities must equal the marginal utility of clean private products.

2.2 Pollution taxes in a market economy: A Pigovian view

The first-best outcome discussed in section 2.1 is only sustainable in a decentralised market economy if lump-sum taxes and subsidies are available. To see this, interpret the material balance condition *(2.1)* as the equilibrium condition for the goods market and $V+L=1$ as the condition for equilibrium in the labour market. If firms maximise profits under perfect competition, the producer prices of clean and dirty private products, clean and dirty public goods and abatement are $\beta_C=\beta_D=\beta_X=\beta_Y=\beta_A=1$ and the producer wage is β_L. The budget constraint for a typical consumer is:

$$P_C\, C + P_D\, D = (1-t_L)\, \beta\, L + T \tag{2.3}$$

where P_C and P_D denote the consumption prices of, respectively, clean and dirty products, t_L stands for the labour tax rate and T represents lump-sum transfers provided by the government to each household. The consumption wage is denoted by $P_L \equiv (1-t_L)\beta$ and corresponds to the opportunity cost of leisure. The government can also levy specific taxes on clean and dirty products, so that $P_i = 1+t_i$, $i=C,D$ where t_C denotes the clean tax and t_D denotes the dirt tax. The optimality conditions for the representative consumer in a competitive market economy are:

$$(u_C/P_C) = (u_D/P_D) = (u_V/[(1-t_L)\beta]) = \lambda \tag{2.4}$$

where λ denotes marginal private utility of income. The government can thus achieve the first-best outcome characterised by *(2.2)*, if it levies no taxes on consumption of clean goods and on labour income, i.e. $t_C=t_L=0$, sets the dirt tax equal to

$$t_D = - N\, u_E\, e_{ND}\, (u_D + N\, u_E\, e_{ND})^{-1} = - N\, u_E\, e_{ND}\, u_C^{-1} > 0 \qquad (2.5)$$

and determines the quantities of the two public goods and of abatement activities from *(2.2)*. This Pigovian dirt tax is denoted by t_D^*. This can also be interpreted as the price of auctioned pollution permits. The government budget constraint then determines the amount of lump-sum transfers (taxes) that must be handed back to (levied on) the public if revenues exceed (fall short of) public spending, i.e. $NT=t_D ND-X-Y-A$. Equivalent to the dirt tax *(2.5)* is a subsidy on clean goods combined with a tax on labour of equal magnitude, namely $t_L=-t_C=-N(u_E e_{ND}/u_D)>0$.

3 Distortions and public goods: Ramsey tax schemes

In practice, the government must resort to distortionary taxes as lump-sum taxes and subsidies are not available (T=0). This section derives the Ramsey tax rules, which yield the least distortionary way of financing a given level of public spending if environmental externalities are absent (u_E=0). It also examines the conditions for the optimal level of public spending. The remaining sections then turn to the general policy problem in which taxes face the dual task of generating revenues to finance public spending and internalising environmental externalities.

3.1 Tax structure

Since a labour tax is equivalent to a uniform tax on clean and dirty private products and a dirt tax is the natural candidate for inducing private agents to pollute less, we assume (without loss of generality) that C is the untaxed good. There is thus no explicit tax on clean private commodities (P_C=1). This assumption accords most closely with the political debate about the interactions between optimal environmental and tax policies and, in particular, about the "double dividend" hypothesis. When a given amount of public spending, $X+Y+A$, has to be financed by distortionary taxation, the optimal tax structure must satisfy (cf., Atkinson and Stiglitz, 1980, Chapter 12; Auerbach, 1985):

$$\lambda\, D - \mu \left[D + t_D \left(\frac{\partial D}{\partial P_D} \right) + t_L\, \beta \left(\frac{\partial L}{\partial P_D} \right) \right] = 0 \qquad (3.1)$$

$$\lambda\, L - \mu \left[L - t_D \left(\frac{\partial D}{\partial P_L} \right) - t_L\, \beta \left(\frac{\partial L}{\partial P_L} \right) \right] = 0 \qquad (3.2)$$

where μ denotes the marginal disutility of raising a unit of government revenues and thus measures the scarcity of public funds. Using the Slutsky decompositions $(\partial I/\partial P_D)=S_{iD}-(\partial I/\partial T)D$ and $(\partial I/\partial P_L)=S_{iL}+(\partial I/\partial T)L$, i=$D$,$L$, where S represents the (symmetric and negative definite) Slutsky matrix, one obtains:

$$\begin{pmatrix} t_D \\ \beta t_L \end{pmatrix} = -\left(\frac{\mu-\lambda'}{\mu}\right)\begin{pmatrix} S_{DD} & S_{LD} \\ S_{DL} & S_{LL} \end{pmatrix}^{-1}\begin{pmatrix} D \\ -L \end{pmatrix}, \quad \lambda' = \lambda + \mu\left[t_D\frac{\partial D}{\partial T} + \beta t_L\frac{\partial L}{\partial T}\right]. \tag{3.3}$$

The marginal *social* utility of private income (λ') may exceed the marginal *private* utility of income (λ) as it takes acount of the increased tax revenues resulting from additional private expenditures. Negative definiteness of the Slutsky matrix in *(3.3)* implies that μ exceeds λ' for positive government revenues (cf., Auerbach, 1985, p.112). Defining $\epsilon_{ik} \equiv P_k S_{ik}/I$ as the compensated elasticity of demand for commodity i (I) with respect to the price of commodity k and using Slutsky symmetry ($\epsilon_{LD} = -\alpha_D \epsilon_{DL}$ where α_D denotes the private budget share of dirty goods), one can rewrite equation *(3.3)* in terms of elasticities:

$$\begin{pmatrix} \theta_D \\ \theta_L \end{pmatrix} \equiv \begin{pmatrix} \dfrac{t_D}{1+t_D} \\ \dfrac{t_L}{1-t_L} \end{pmatrix} = \left(\frac{\mu-\lambda'}{\mu}\right)\begin{pmatrix} \epsilon_{DD} & -\epsilon_{DL} \\ -\epsilon_{LD} & \epsilon_{LL} \end{pmatrix}^{-1}\begin{pmatrix} -1 \\ 1 \end{pmatrix}. \tag{3.3'}$$

Equation *(3.3')* can be solved to give:

$$\theta_D \equiv \left(\frac{t_D}{1+t_D}\right) = \left(\frac{\epsilon_{LL}-\epsilon_{DL}}{\epsilon_{LD}-\epsilon_{DD}}\right)\theta_L \tag{3.4}$$

and

$$\theta_L \equiv \left(\frac{t_L}{1-t_L}\right) = \left(\frac{\epsilon_{LD}-\epsilon_{DD}}{-\epsilon_{DD}\epsilon_{LL}+\epsilon_{LD}\epsilon_{DL}}\right)\left(\frac{\mu-\lambda'}{\mu}\right). \tag{3.5}$$

The marginal disutility of financing public spending (μ) exceeds the marginal social utility of private income (λ') if public revenues are positive. Moreover, negative definiteness of the Slutsky matrix implies that the denominator in the first term in brackets in the expression for θ_L in *(3.5)* is positive. Hence, the tax on labour is positive if and only if ϵ_{LD} exceeds ϵ_{DD}.[4]

Since compensated demand functions depend only on relative prices, $\epsilon_{iL} = -(\epsilon_{iC}+\epsilon_{iD})$, $i=D,L$ must hold. Hence, using Slutsky symmetry (i.e. $\epsilon_{LD} = -\alpha_D\epsilon_{DL}$ and $\epsilon_{LC} = -(1-\alpha_D)\epsilon_{CL}$), we

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[4] If $\epsilon_{LD}=\epsilon_{DL}=0$ and $\epsilon_{LL}=-\epsilon_{DD}$, equation (3.4) shows that it is optimal to have a uniform tax on dirty products and labour supply. It is optimal to have a higher (lower) dirt tax than labour tax if the own price elasticity of the compensated supply of labour (ϵ_{LL}) exceeds (falls short of) that for dirty private goods ($-\epsilon_{DD}$). Similarly, it can be shown that, if $\epsilon_{CD}=\epsilon_{DC}=0$ and $\epsilon_{CC}=\epsilon_{DD}$, it is optimal to levy a uniform tax on clean and dirty products (equivalent to a tax on labour). If the own price elasticity of the demand for clean products ($-\epsilon_{CC}$) is lower (higher) than that of dirty products ($-\epsilon_{DD}$), it is optimal to give (levy) an additional subsidy (tax) on dirty products. These special cases illustrate the Ramsey rule which states that it is optimal to tax goods with a low price elasticity more heavily than goods with a high price elasticity.

can write *(3.4)* as:

$$\theta_D = \left(\frac{t_D}{1+t_D}\right) = \left(\frac{e_{CL}-e_{DL}}{e_{CD}-e_{DD}}\right)\theta_L. \tag{3.4'}$$

Government policy involves a uniform tax on both clean and dirty products, i.e. a tax on labour, and in addition a dirt tax. The sign of the dirt tax depends on the cross-elasticities with leisure. In particular, the dirt tax is positive if clean goods are better substitutes for leisure than dirty goods are ($\epsilon_{CL}>\epsilon_{DL}$). In that case, dirty goods are the relative complement to leisure. Accordingly, it is optimal for the government to levy a uniform tax on clean and dirty products, i.e. tax labour, and to levy an additional tax on the product that is most complementary to leisure.[5]

3.2 Choice between public and private goods

In the absence of lump-sum taxes and subsidies, the marginal rate of transformation between private and public goods no longer corresponds to the (sum of) the marginal rate(s) of substitution between private and public goods as in Samuelson (1954). This may be seen from the following condition for the optimal provision of public goods (cf., Atkinson and Stern, 1974):

$$N\left(\frac{u_X}{u_C}\right) = \eta\left[1 - t_D N\left(\frac{\partial D}{\partial X}\right) - t_L \beta N\left(\frac{\partial L}{\partial X}\right)\right], \quad \eta = \left(\frac{\mu}{\lambda}\right) \tag{3.6}$$

where η denotes the marginal cost of public funds (in terms of the numeraire). There are two reasons why the marginal rate of transformation between private and public goods differs from the corresponding sum of the marginal rates of substitution.

The first reason is that, if public goods are complementary to taxed commodities ($\partial D/\partial X>0$ or $\partial L/\partial X>0$), raising public spending alleviates the excess burden of distortionary taxation by boosting the consumption of taxed commodities. For example, the construction of public highways between suburbs and cities may induce some agents to work more and, therefore, pay more tax on their labour income. Moreover, they may buy more heavily taxed commodities such as petrol and cars. Public libraries, in contrast, may encourage private agents to enjoy more leisure, thereby eroding the base of the labour tax. The social cost of funds devoted to libraries is thus higher than that of funds allocated to highways.

The second reason for the divergence between the marginal rates of transformation and

[5] An alternative is to think of government policy as employing a clean tax and a dirt tax with labour as the untaxed good. The implicit price of clean products then amounts to $(1-t_L)^{-1}$ and the associated tax rate $t_L(1-t_L)^{-1}$. The implicit price of dirty products equals $(1+t_D)(1-t_L)^{-1}$ and the implicit dirt tax rate is $(t_D+t_L)(1-t_L)^{-1}$.

substitution is that, if μ exceeds λ, an increase in government revenues exacerbates the deadweight loss of distortionary taxation and thus raises the marginal cost of public funds. However, if taxed commodities are inferior, μ may be less than λ. In that case, the negative income effect associated with a higher tax level alleviates the excess burden because it raises the consumption of taxed goods. Hence, additional revenues reduce rather than raise the marginal cost of public funds.

4 Pollution and labour taxation in a second-best world

This section again considers a second-best world in which lump-sum transfers are excluded (T=0). In contrast to the previous section, however, environmental externalities are present in consumption. Accordingly, labour and dirt taxes are employed not only to finance public spending but also to internalise environmental externalities. These externalities affect the Ramsey tax rules and the optimal provision of public goods.

4.1 Tax structure

In the presence of environmental externalities, equations *(3.1)-(3.2)* become

$$\lambda D - \mu\left[D + t_D\frac{\partial D}{\partial P_D} + \beta t_L\frac{\partial L}{\partial P_D}\right] - N u_E' e_{ND}\frac{\partial D}{\partial P_D} = \lambda D - \mu\left[D + (t_D - t_{DP})\frac{\partial D}{\partial P_D} + \beta t_L\frac{\partial L}{\partial P_D}\right] = 0 \qquad (4.1)$$

$$\lambda\ L - \mu\left[L - (t_D - t_{DP})\left(\frac{\partial D}{\partial P_L}\right) - \beta\ t_L\left(\frac{\partial L}{\partial P_L}\right)\right] = 0 \qquad (4.2)$$

where u_E' denotes the marginal social utility of the environment:

$$u_E' = \left(\frac{u_E + \mu\left[t_D\frac{\partial D}{\partial E} + \beta t_L\frac{\partial L}{\partial E}\right]}{1 - N e_{ND}\frac{\partial D}{\partial E}}\right). \qquad (4.3)$$

u_E' accounts not only for the direct impact of the environment on utility (u_E>0), but also for the indirect effects of an improved environment on the tax base. Furthermore, if a better environmental quality raises the demand for dirty goods (i.e. $\partial D/\partial E$>0), the net social utility of the environment is reduced (as e_{ND}<0).

The Pigovian component of the optimal dirt tax, t_{DP}, can (using λ=u_C and η=μ/λ) be written as:

$$t_{DP} = \left(\frac{-Ne_{ND}u_E'}{u_C}\right)\left(\frac{1}{\eta}\right) = \left(\frac{-Ne_{ND}}{1-Ne_{ND}\frac{\partial D}{\partial E}}\right)\left(\frac{u_E}{\mu}+t_D\frac{\partial D}{\partial E}+\beta t_L\frac{\partial L}{\partial E}\right). \tag{4.4}$$

The Pigovian dirt tax component corrects for the environmental damage due to the consumption of dirty private products. The Pigovian tax rises with the marginal social utility of the environment (u_E'). If the marginal cost of social funds is unity (i.e. $\eta=\mu/\lambda=1$) and environmental quality (E) is weakly separable from the other arguments in u(.) (so that $u_E'=u_E$), the Pigovian tax (scaled by the environmental damage per unit of dirty private consumption, ($-e_{ND}$)) amounts to the sum of the marginal rates of substitution between the public good of the environment and the clean commodity:

$$\left(\frac{t_{DP}}{-e_{ND}}\right) = \left(\frac{Nu_E}{u_C}\right). \tag{4.4'}$$

If the marginal cost of public funds rises above unity (i.e. $\eta>1$), the environmental tax declines. The reason is that the optimal environmental tax equates the social costs of environmental damage due to dirty private consumption to the social benefit of additional tax revenue due to that consumption. A high marginal cost of public funds indicates that tax revenue is scarce. This implies that dirty consumption has to yield less tax revenue to offset environmental damage. Indeed, the optimal Pigovian tax measures the environmental damage in terms of public revenue (rather than private income). Accordingly, if public revenue becomes more valuable compared to private income (i.e. η rises), the optimal environmental tax decreases. Intuitively, high marginal costs of public funds indicate that public goods are expensive. In these circumstances, the government can afford less tax differentiation aimed at environmental protection as the revenue-raising objective of the tax system becomes relatively more important. In this way, high cost of public funds reduces the demand not only for ordinary public consumption but also for the public good of the environment.

In contrast to previous authors (e.g., Sandmo, 1975; Atkinson and Stiglitz, 1980; Auerbach, 1985), our definition of the Pigovian tax *(4.4)* incorporates a second factor that may cause the Pigovian tax to deviate from the sum of the marginal rates of substitution. In particular, the environmental quality may directly impact the consumption of taxed commodities. For example, if labour supply is taxed and an improved environment induces people to enjoy more leisure and work less (i.e. $\partial L/\partial E<0$), the social value of environmental protection is reduced and the optimal environmental tax falls. In principle it is possible, albeit unlikely, that the Pigovian component of the dirt tax is negative, namely if tax rates are high and if a better environmental quality substantially reduces the demands for taxed goods.

Comparing the conditions for the optimal tax structure without environmental externalities (i.e *(3.1)* and *(3.2)*) with those with externalities in consumption (i.e. *(4.1)* and *(4.2)*), one notices that the terms with the dirt tax t_D in *(3.1)* and *(3.2)* are replaced by terms with the dirt tax net of the Pigovian tax term (t_D-t_{DP}) in *(4.1)* and *(4.2)*. Clearly, the Ramsey tax term (t_D-t_{DP}) is separable from the Pigovian tax term (t_{DP}). Intuitively, the Ramsey tax term measures the social contribution (in terms of government revenues) of additional demand for dirty goods as the difference between a positive and a negative contribution. On the one hand, consumption of dirty private goods boosts the tax base and thus facilitates the financing of ordinary public goods. On the other hand, it damages the environment, thereby reducing the supply of the "collective good" of the environment. It follows that the optimal dirt tax is the sum of the Ramsey and Pigou terms (cf., Sandmo, 1975):

$$\theta_D = \left(\frac{\epsilon_{CL}-\epsilon_{DL}}{\epsilon_{CD}-\epsilon_{DD}}\right)\theta_L + \theta_{DP}, \qquad \theta_{DP} = \left(\frac{t_{DP}}{1+t_D}\right) \tag{4.5}$$

which replaces *(3.4')*. Hence, even if the compensated elasticities of the demands for clean and dirty goods with respect to the price of leisure are identical (i.e. ϵ_{CL}=ϵ_{DL}), a zero dirt tax is not optimal. Although the Ramsey term (i.e. the first term on the right-hand side of *(4.5)*) is zero, a separate Pigovian dirt tax corrects for the environmental externality, namely t_{DP}. This encourages households to consume clean rather than dirty products and thus ensures an optimum quality of the environment.

The formula for the optimal labour tax in *(3.5)* is not affected by environmental externalities. This corresponds to the principle of targeting (e.g., Dixit, 1985), which states that a distortion is best addressed by the instrument that acts directly on the relevant margin. Hence, dirt rather than labour taxes should deal with environmental externalities.

It is instructive to write the condition for the optimal dirt tax as the weighted sum of the Pigou and Ramsey terms (using *(3.5)*, *(4.4)* and *(4.5)*):

$$\theta_D = (1-\eta'^{-1})\left(\frac{(\epsilon_{CL}-\epsilon_{DL})(1-\alpha_D)}{-\epsilon_{DD}\epsilon_{LL}+\epsilon_{DL}\epsilon_{LD}}\right) + \eta^{-1}\left(\frac{-N e_{ND} u_E'}{(1+t_D)u_C}\right). \tag{4.6}$$

The weight of the Ramsey term (ϵ_{CL}-ϵ_{DL}) is positive if government revenue from non-dirt taxes is positive (as the Slutsky matrix is negative definite and $\eta'\equiv\mu/\lambda'>1$). It rises with the scarcity of public funds (reflected in a high value for η'). The weight of the Pigovian term, in contrast, falls with the marginal cost of public funds (η).[6] Hence, if public funds become scarcer, the Ramsey term becomes more important relative to the Pigovian term. Intuitively, the tax structure focusses more on the revenue-raising rather than the environmental objective

[6] Typically, the weight given to the Pigovian term is less than one, but it may be greater than one if taxed goods are inferior.

if government revenue becomes more "expensive".

In a command economy the marginal cost of public funds is unity ($\eta=1$). In a market economy, in contrast, the marginal cost of public funds incorporates the costs of distortionary taxation. This may be seen upon solving equation *(4.2)* for the marginal cost of public funds:

$$\eta = \left(\frac{1}{1-(\theta_D-\theta_{DP})\alpha_D\epsilon'_{DL}-\theta_L\epsilon'_{LL}}\right). \tag{4.7}$$

The marginal cost of public funds exceeds unity if the financing of additional public spending erodes the base of existing 'Ramsey' taxes. This occurs if there is a positive labour tax and the Marshallian labour supply curve slopes upwards ($\epsilon_{LL}'>0$) or if the dirt tax exceeds the Pigovian component ($\theta_D>\theta_{DP}$) and private demand for dirty goods falls as the tax on labour increases (i.e. $\epsilon_{DL}'>0$).

4.2 Public consumption: Level and composition

The condition for the optimal provision of clean public goods *(3.6)* becomes:

$$N\left(\frac{u_X}{u_C}\right) = \eta\left[1-(t_D-t_{DP})N\left(\frac{\partial D}{\partial X}\right)-t_L N\beta\left(\frac{\partial L}{\partial X}\right)\right]. \tag{4.8}$$

Comparing *(3.6)* and *(4.8)*, one notices that the dirt tax in *(3.6)* is again replaced by the Ramsey component of the dirt tax in *(4.8)*. Intuitively, only the revenues from the Ramsey term provide a net social benefit while the revenues from the Pigovian term merely compensate for environmental damage. The optimality condition for dirty public goods is:

$$N\left(\frac{u_Y+u_E'e_Y}{u_C}\right) = \eta\left[1-(t_D-t_{DP})N\left(\frac{\partial D}{\partial Y}\right)-t_L N\beta\left(\frac{\partial L}{\partial Y}\right)\right]. \tag{4.9}$$

Clearly, if public consumption pollutes the environment (i.e $e_Y<0$), the direct marginal utility of public goods must be higher to compensate for the environmental damage caused by these public goods.

4.3 Public abatement

Finally, the government engages in abatement until the sum of the marginal rates of substitution between these activities and private consumption of clean goods equals the social cost of abatement activities:

$$N\left(\frac{u_B\, e_A}{u_C}\right) = \eta\left[1 - N\left((t_D - t_{DP})\frac{\partial D}{\partial E} + \beta\, t_L \frac{\partial L}{\partial E}\right) e_A\right]. \tag{4.10}$$

Note that, if public goods and the quality of the natural environment are weakly separable from the other arguments in u(.), the marginal rate of substitution between the environment and public consumption of clean commodities can with the aid of *(4.4)*, *(4.9)* and *(4.10)* be written as:

$$\frac{u_B}{u_X} = \left(\frac{u_Y - u_X}{u_X}\right)\left(\frac{1}{-e_Y}\right) = \left(\frac{t_{DP}}{-e_{ND}}\right) = \frac{1}{e_A}. \tag{4.11}$$

The relative price of the environment in terms of public goods equals the dirt tax scaled by the damage done by dirty (private) goods to environmental quality. Clearly, the marginal utility of dirty public goods must exceed that of clean public goods (i.e. $u_Y > u_X$) so that (other things being equal) the government consumes less dirty goods.

5 Restrictions on preferences

5.1 The utility tree

To obtain more specific results, we put restrictions on preferences. In particular, preferences are described by the tree given in Figure 1. Together with leisure, the basket of clean and dirty private goods (Q) determines the utility of marketable goods (M). Total utility of the representative consumer (U) is a function of the "private" utility index M, the basket of clean and dirty public goods (G) and environmental quality (E). The budget shares are given at the bottom of the branches of the utility tree given in Figure 1. The utility function of the representative consumer can thus be written in weakly separable form:

$$U = u(C,D,V,X,Y,E) = U(M(Q(C,D),V),G(X,Y),E). \tag{5.1}$$

It is assumed that the utility function U(.) and the sub-utility functions M(.), G(.) and Q(.) are homothetic.

The chosen structure of preferences implies strong restrictions on demand. In particular, the compensated elasticities ϵ_{CL} and ϵ_{DL} are identical. Accordingly, the Ramsey tax term in *(4.5)* is zero and the dirt tax reduces to the Pigovian correction term ($t_D = t_{DP}$). Moreover, neither private demand for dirty goods nor labour supply depends directly on public consumption or the quality of the environment. Hence, the social cost of public goods (i.e. the right-hand side of expressions *(3.6)* and *(4.8)-(4.10)*) reduces to the marginal cost of public funds (η). Furthermore, marginal social utility of the environment (u_E') given in expression *(4.3)* boils down to the direct effect of environmental quality on utility (u_E).

We assume that the function describing the quality of the environment takes the following form:

$$E = e(ND+Y,A) \equiv E_o - \delta\,(N\,D + Y) + e^A(A),\;\; \delta, e^{A\prime}\geq 0,\; e^{A\prime\prime}<0 \qquad (5.2)$$

where E_o denotes the quality of the environment when it is not spoilt by pollution or improved through abatement and δ stands for the emission ratio. There are decreasing returns to abatement activities.

5.2 The optimality conditions revisited

For the chosen structure of preferences, *(4.3)-(4.5)* imply the following tax structure:

$$t_D = t_{DP} = \left(\frac{NU_E\delta}{u_C}\right)\left(\frac{1}{\eta}\right). \qquad (5.3)$$

The optimal structure of public consumption follows from *(4.8)*, *(4.9)* and *(5.3)*:

$$\frac{G_X}{G_Y} = \frac{1}{1+t_D}. \qquad (5.4)$$

The marginal rate of substitution between clean and dirty public goods equals the ratio of the price of clean public goods (normalised to unity) to the social cost of dirty public goods. This latter cost consists of the production price (unity) plus the dirt tax, which represents the environmental costs.

From equations *(4.8)*, *(4.9)*, *(5.3)* and $u_C=\lambda=U_M M_Q/P_Q$, where P_Q denotes the "ideal" price index for the private consumption basket Q (i.e. $P_Q=[C+(1+t_D)D]/Q$), one obtains the following expression for the trade-off between the optimal levels of public and private consumption:

$$\frac{NU_G}{U_M M_Q} = \eta\left(\frac{P_G}{P_Q}\right) \qquad (5.5)$$

where P_G is the "ideal" price index for the basket of public goods (i.e. $P_G=[X+(1+t_D)Y]/G=1/G_X$). Clearly, the sum of the marginal rates of substitution between public and private goods must equal the marginal cost of public funds (η) times the relative price of public goods in terms of private goods (P_G/P_Q).

Two elements cause the sum of the marginal rates of substitution to differ from the marginal rate of transformation in production. First, the relative price of public and private commodities (P_G/P_Q) depends not only on the relative costs in production but also on the relative social costs in terms of environmental damage. Second, the marginal cost of public funds (η) drives a wedge between the sum of the rates of substitution and the rate of transformation. It incorporates the costs of distortionary taxation and, in general, differs from unity. For the chosen structure of preferences, $\theta_D=\theta_{DP}$ so that the dirt tax does not affect the marginal cost of funds directly and *(4.7)* reduces to:

$$\eta = \left(\frac{1}{1-\left(\frac{t_L}{1-t_L}\right)\epsilon_{LL}'}\right). \qquad (5.6)$$

In a decentralised market economy, distortionary taxes raise the marginal cost of public funds above unity if the financing of additional government spending erodes the base of existing distortionary taxes. This is the case if both the labour tax (t_L) and the uncompensated wage elasticity of labour supply (ϵ_{LL}') are positive.

Combining *(5.3)* and *(5.5)*, one obtains an expression for the trade-off between public consumption and environmental quality:

$$\frac{U_G}{U_E} = \left(\frac{p_G}{t_D/\delta}\right). \qquad (5.7)$$

The marginal rate of substitution between public consumption and environmental quality must equal the ratio of the price of public goods to the price for environmental quality (i.e. the dirt tax scaled by the dirt-emission rate for consumption). Note that, as both public consumption and the environment are "collective" goods, the marginal cost of public funds (η) does not enter *(5.7)*.

Combining *(4.10)* and *(5.3)*, one finds that the marginal effect of abatement on the quality of the environment equals the price of abatement in terms of the implicit price of the environment:

$$e^A(A)' = (t_D/\delta)^{-1}. \qquad (5.8)$$

The optimum level of abatement increases with the implicit price of the environment (t_D/δ) and thus with the level of the dirt tax. This illustrates the principle that one should employ a mix of environmental policy instruments.

6 Loglinearisation

We loglinearise the model around an initial equilibrium (in which lump-sum transfers are zero) in order to explore how optimal policy should respond to changes in various parameters. In particular, in sections 7 and 8 we analyse the comparative statics of private behaviour and optimal government policy with respect to changes in environmental concern

(Γ_E) and changes in concern about public consumption (Γ_G), respectively. Levels of variables are denoted by capitals and the corresponding logarithmic deviations from the initial equilibrium values are denoted by small letters. However, arithmetic deviations from initial equilibrium values for t_i, normalised by P_i, are denoted by t_i', i=D,L.

6.1 Private behaviour

Table 2(a) contains the compensated elasticities associated with the Hicksian demand functions. Note that clean and dirty goods are Hicksian substitutes for a given sub-utility level Q. However, for a given utility level M (or U), clean and dirty goods may be Hicksian complements (if σ_M is large relative to σ_Q) as an increase in the price of dirty products induces substitution away from clean (and dirty) products towards leisure. Both clean and dirty products are Hicksian substitutes for leisure. The ideal price indices are given by:

$$p_Q = (1-\alpha_D)\, p_C + \alpha_D\, p_D \tag{6.1}$$

$$p_M = (1-\alpha_V)\, p_Q + \alpha_V\, p_L = (1-\alpha_V)\, [(1-\alpha_D)\, p_C + \alpha_D\, p_D] + \alpha_V\, p_L. \tag{6.2}$$

Table 2(b) reveals that utility of marketable goods (m) rises if lump-sum transfers or the real consumption wage increase. Substitution of m from Table 2(a) yields the Marshallian demand functions and the uncompensated demand elasticities reported in Table 2(b). For example, the uncompensated elasticity of the Marshallian demand for leisure with respect to the consumption wage (P_L) is given by $\epsilon_{VL}'\equiv(1-\alpha_V)(1-\sigma_M)$ and exceeds the compensated elasticity $\epsilon_{VL}<0$. An increase in the consumption wage has two effects on labour supply: (i) substitution away from leisure towards clean and dirty commodities, thereby raising labour supply (measured by $\epsilon_{LL}=\sigma_M\alpha_V>0$); (ii) a boost in real income, which raises consumption of leisure, thereby reducing labour supply (measured by the term $-\alpha_V$). The substitution effect dominates the income effect if the elasticity of substitution between leisure and Q, i.e. σ_M, exceeds unity. Similarly, the uncompensated elasticity of the demand for clean products with respect to the price of dirty products (ϵ_{CD}') depends crucially on the relative magnitudes of the substitution elasticities σ_Q and σ_M. Clean and dirty private goods are likely to be gross substitutes (complements) when the elasticity of substitution between clean and dirty products is large (small) and the substitution elasticity between private goods and leisure is small (large).

6.2 Equilibrium and the government budget constraint

The loglinearised version of the material balance condition for the economy is given by (cf., equation *(2.1)*):

$$\omega_C\, c + \omega_D\, d + \omega_X\, x + \omega_Y\, y + \omega_A\, a = l \tag{6.3}$$

where $\omega_i \equiv I/\beta L$, i=$C,D$ and $\omega_j \equiv J/\beta NL$, j=$X,Y,A$. Substituting the uncompensated demand functions from Table 2(b), one finds (see Appendix):

$$\omega_X\, x + \omega_Y\, y + \omega_A\, a = \Delta\,(t_L' + \alpha_D\, t_D') - (1-t_L)\theta_D\alpha_D(1-\alpha_D)\sigma_Q\, t_D' \tag{6.4}$$

where $\omega_G' \equiv \omega_X+\omega_Y+\omega_A = 1-\omega_C-\omega_D = t_L+(1-t_L)\theta_D\alpha_D$ denotes the national income share of public spending and $\Delta \equiv 1-\omega_G'(1+\epsilon_{LL}')$. In view of Walras' law, *(6.4)* can also be viewed as the reduced-form government budget constraint. Public spending must be financed from labour and dirt tax revenues. Equation *(6.4)* allows for the erosion of tax bases as tax rates rise. To illustrate, an increase in the labour tax rate increases revenues less than proportionally if the uncompensated wage elasticity of labour supply (ϵ_{LL}') exceeds zero. In that case, an increase in the labour tax induces people to work less, thereby eroding the base of the labour and dirt taxes. An increase in the dirt tax rate has an additional adverse effect on the tax base. It encourages consumers to shift from dirty to clean products, thereby eroding the base of the dirt tax.

6.3 Private utility

Table 2(b) shows that private utility, $m=\alpha_V v+(1-\alpha_V)q$, may be written as a decreasing function of both the tax on labour and the dirt tax:

$$m = -\,(1-\alpha_V)\,(t_L' + \alpha_D\, t_D'). \tag{6.5}$$

Using *(6.4)*, $g=\alpha_Y y+(1-\alpha_Y)x$ (where $\alpha_Y \equiv G_Y Y/G$ denotes the (shadow) budget share of dirty goods in public consumption), and

$$x - y = \sigma_G\, t_D' \tag{6.6}$$

(i.e. the loglinearised version of *(5.4)*), we can decompose the effect on private utility as follows (see Appendix):

$$m = -\,(1-\alpha_V)\Delta^{-1}\,[(\omega_X+\omega_Y)\, g + \omega_A\, a + \chi\, t_D'] \tag{6.7}$$

where

$$\chi = (\omega_C + \omega_D)\left(\frac{\theta_D \alpha_D}{1-\theta_D \alpha_D}\right)(1-\alpha_D)\sigma_Q + (\omega_X + \omega_Y)\left(\frac{\theta_D \alpha_Y}{1-\theta_D \alpha_Y}\right)(1-\alpha_Y)\sigma_G. \tag{6.8}$$

Private utility suffers if public consumption (g>0) or public abatement increases (a>0). The term χ captures the costs in terms of private utility of a shift towards a cleaner composition of public and private consumption. In particular, a change in the composition of public consumption towards cleaner goods harms private welfare. Finally, the excess burden of the dirt tax in changing the composition of private consumption towards clean goods reduces utility from marketable goods.

6.4 Environmental quality

Environmental quality depends on a number of effects (see Appendix):

$$e = \left(\frac{1}{\alpha_E(1-\theta_D \alpha_D)}\right)\left[-\theta_D \alpha_D l + \omega_A a + \chi t_D' + \left(\frac{\theta_D(\alpha_D - \alpha_Y)(\omega_X + \omega_Y)}{1-\theta_D \alpha_Y}\right) g\right] \tag{6.9}$$

where from Table 2(b)

$$l = \left(\frac{\alpha_V}{1-\alpha_V}\right)(\sigma_M - 1)m \tag{6.10}$$

and $\alpha_E \equiv (t_D/\delta)E/(N\beta L)$ represents the ratio of the value of the environment to national income. The first term in the square brackets in *(6.9)* shows that a shift to clean leisure improves environmental quality. This shift corresponds to a lower *level* of economic activity (the "volume" effect). The other three terms stand for changes in the *composition* of economic activity. In particular, the second term denotes an increase in public abatement. The third term reflects a shift from dirty to clean commodities in private and public consumption (see *(6.8)*). The last term stands for a different distribution of activity over the public and private sectors. If the (shadow) budget share of clean commodities in public goods is greater than in private goods ($\alpha_Y < \alpha_D$), public consumption is cleaner than private consumption and thus a higher level of public consumption benefits the environment.

7 Shift towards greener preferences

To obtain explicit results about the way in public policy should respond to a change in political preferences, marketable goods, public consumption and the environment are assumed to be perfect substitutes in social utility. Hence, social utility can be written as $U=\Gamma_G G+\Gamma_E E+\Gamma_M M$.[7] Imperfect substitution would mitigate the allocational effects of a change in political preferences on M, E and G. Furthermore, we assume that in the initial equilbrium public and private consumption are equally dirty (i.e. $\alpha_Y=\alpha_D$). Hence, the fourth term in *(6.9)* drops out.

7.1 A closer look at the "double dividend" hypothesis

Environmental policy

The trade-off between public consumption and environmental quality is given by expression *(5.7)*. Substituting $p_G=\alpha_Y t_D'$ into the loglinearised version of *(5.7)* yields:

$$\frac{t_D'}{\theta_D} = \left(\frac{\gamma_E-\gamma_G}{1-\theta_D\alpha_Y}\right). \qquad (7.1)$$

If consumption pollutes the environment (i.e. $\alpha_Y>0$), the relative increase in the dirt tax (t_D'/θ_D) exceeds the relative increase in environmental concern ($\gamma_E-\gamma_G$). Intuitively, more environmental concern requires an increase in the price of the environment (i.e. the dirt tax) relative to the price of public consumption. If public consumption damages the environment, a higher dirt tax raises not only the price of the environment but also the shadow price of public goods, thereby offsetting some of the effect of the higher dirt tax on the relative price of the environment to public consumption. Hence, the dirt tax has to rise more to accomplish the required increase in the relative price of the environment.

Composition of private and public spending

The solution for the dirt tax determines the optimal mix of dirty and clean public goods from *(6.6)*. It also determines the composition of public and private consumption from Table 2 and optimal abatement from *(5.8)*:

$$c - d = \sigma_Q\, t_D' \qquad (7.2)$$

$$a = \left(\frac{t_D'}{\sigma_A\theta_D}\right), \quad \sigma_A \equiv \left(\frac{-e^{A''}A}{e^{A'}}\right) > 0 \qquad (7.3)$$

where σ_A denotes the elasticity of productivity of public abatement. The increase in environmental concern ($\gamma_E-\gamma_G$) pushes up the shadow price of the environment (t_D), which changes

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[7] This function may also be interpreted as a complex interest function in which the weights Γ_i, i=G,E,M represent the bargaining strength of various social pressure groups. The solution to optimising this interest function also corresponds to an outcome of an electoral competition model (Drissen and van Winden, 1992).

the composition of public and private consumption toward clean goods and raises public abatement. The possibilities for substitution, as reflected by the elasticities σ_G and σ_Q, determine the magnitude of the change in the composition of the consumption baskets. The rise in public abatement is inversely related to the concavity of the abatement function $e^A(.)$.

Cost of public funds: tax level and tax composition effects

Expression *(5.5)* determines the trade-off between public and private consumption. It can be written as

$$\left(\frac{N\Gamma_G}{\Gamma_M}\right)\left(\frac{P_Q}{P_G}\right) = M_Q\eta. \qquad (7.4)$$

The dirt tax does not affect the price ratio P_Q/P_G if private and public consumption are equally dirty ($\alpha_D=\alpha_Y$). Accordingly, the left-hand side of *(7.4)* is fixed as public and private consumption (G and M) are perfect substitutes. The right-hand side corresponds to the marginal cost of public funds in terms of private utility (M). The loglinearised version of *(7.4)* is (see Appendix):

$$\gamma_G = d\log(M_Q\eta) = -\left(\frac{\alpha_V}{1-t_L(1-\alpha_V+\sigma_M\alpha_V)]}\right)\left[(\sigma_M-1)\alpha_D t_D' + \left(\frac{\sigma_M\kappa}{1-\alpha_V}\right)m\right] = 0. \qquad (7.5)$$

where

$$\kappa = 1 - t_L[1-(1-\alpha_V)(\sigma_M-1)]. \qquad (7.6)$$

The first term in the square brackets at the right-hand side of *(7.5)* stands for the effect on the cost of public funds of substituting a non-distortionary (dirt) tax for a distortionary (labour) tax, while keeping private welfare (m) constant. This change in the tax structure towards non-distortionary taxation cuts the cost of public funds if the elasticity of substitution between private consumption commodities and leisure exceeds unity ($\sigma_M>1$) and the uncompensated wage elasticity of labour supply is thus positive ($\epsilon_{LL}'=\alpha_V(\sigma_M-1)>0$). Indeed, expression *(5.6)* reveals that lower distortionary labour taxation reduces the cost of public funds if and only if the uncompensated wage elasticity of labour supply is positive. Intuitively, public goods become easier to finance and the marginal cost of public funds declines if dirt taxes aimed at internalising externalities rather than distortionary taxes are employed. This will be called the "tax *composition*" effect because it involves a change in the composition of taxation without affecting the level of private utility m and thus the overall tax level.

The second term in the square brackets at the right-hand side of *(7.5)* is the "tax *level*" effect. It involves a fall in private welfare on account of an increase in the overall burden of taxation. In particular, tax rates rise for three reasons (compare expressions *(6.5)* and *(6.7)*): (i) to compensate for the erosion of the base of the dirt tax; (ii) to pay for a cleaner composition of public consumption; and (iii) to finance public abatement activities. Hence, the "tax level" effects corresponds to the burden of making economic activity cleaner by undertaking public abatement and changing the composition of public and private consumption baskets towards clean commodities. It can be interpreted as the costs associated with an expanding social sector. The social sector includes not only Samuelson-style public consumption goods but also the quality of the environment. In "funding" this sector, the government levies explicit taxes for public abatement and clean public consumption. Furthermore, the private sector pays implicit taxes by incurring costs for the change in the composition of its own consumption basket (i.e. the "excess" burden of dirt taxes). The higher tax level reduces the marginal benefits from working, thereby exacerbating the distortionary effects of labour taxes. Indeed, the expansion of the social sector raises the cost of public funds (in terms of utility) as long as substitution between leisure and private goods is possible (i.e. $\sigma_M>0$).

The sign of the "tax composition effect" is ambiguous and depends on the relative magnitude of income versus substitution effects. In particular, moving toward non-distortionary dirt taxation reduces the cost of public funds only if the substitution effect of a higher after-tax wage is sufficiently large relative to the income effect (i.e. $\sigma_M>1$). The sign of the "tax level" effect, in contrast, is unambiguous; given that κ is assumed to be positive[8], a rise in the tax level always raises the cost of public funds because only substitution effects determine the "tax level" effect. Intuitively, a "tax level" effect imposes a first-order loss in private utility, thereby raising marginal private utility from private consumption goods (M_Q). Hence, the income effects of a higher tax level on the costs of public funds in terms of private utility ηM_Q (i.e. the right-hand side of *(7.4)*) cancel out. On the one hand, adverse income effects associated with a higher tax level reduce the cost of public funds in terms of private consumption (η) by reducing the demand for leisure and thus raising taxed labour supply. On the other hand, these income effects increase the cost of public funds in terms of private utility by reducing private utility (m) thus raising the marginal private utility from private goods (M_Q).

Tax structure: Labour taxation

Upon elimination of m in *(7.5)* and using *(6.5)*, one obtains an inverse relationship between the tax on labour and the dirt tax (see Appendix). Hence, a rise in environmental concern, which is accompanied by a higher dirt tax, reduces the tax on labour. Conventional wisdom about environmental concern causing a shift away from labour taxation is thus correct.

[8] This will be the case if $t_L<\frac{1}{2}$ or $\alpha_V=1$ or $\sigma_M>1$.

Level of economic activity and employment

Table 3 contains the detailed effects of an increase in environmental concern on the optimal levels of various instruments of government policy and on other important variables. Although the (explicit) tax on labour falls, employment does not rise (see Table 3). In fact, if the uncompensated wage elasticity of labour supply differs from zero (i.e. $\sigma_M \neq 1$), labour supply falls. Intuitively, a reallocation from labour to leisure benefits the environment; the consumption of leisure does not pollute while employment indirectly harms the environment because part of labour income is spent on dirty goods. The fall in employment and output is especially large if the (initial) national income share of the public sector is substantial. Hence, the popular version of the "double dividend" argument - increased environmental concern raises employment - fails.

Proposition 1: *A higher political preference for a cleaner environment raises the dirt tax and reduces the labour tax. The higher dirt tax yields a cleaner composition of private and public consumption baskets and a higher level of public abatement. Despite a lower labour tax, employment falls unless the substitution elasticity between private consumption of commodities and leisure is one.*

7.2 Public consumption: Are red and green policies compatible?

Public consumption is affected through two main channels. The first term in square brackets in the expression for public consumption in Table 3 stands for the "tax composition" effect. This effect makes green and red preferences compatible in the sense that public consumption and environmental quality move together. In particular, substituting dirt for labour taxes reduces the marginal cost of public funds if existing distortionary taxes imply a marginal cost of public funds (η) greater than unity, i.e. if the substitution effect of lower labour taxation dominates the income effect ($\sigma_M > 1$) (see expression *(5.6)*).

The second term in square brackets in the expression for public consumption in Table 3 represents the "tax level" effect. It implies a negative link between public consumption and environmental concern and reflects the private welfare costs of environmental protection on account of a cleaner composition of economic activity. These costs exacerbate the deadweight loss associated with distortionary (labour) taxation, thereby raising the cost of public funds.

The overall effect of greener preferences on public consumption depends on the balance between the "tax composition" and the "tax level" effects. In particular, environmental quality and public consumption move together, and red and green preferences are thus compatible, only if the "tax level" effect is small compared to the "tax composition" effect. The "tax level" effect is small if dirt taxes constitute a small part of the price of dirty goods (i.e. θ_D small) and, at the same time, the substitution elasticity between dirty and clean goods (σ_Q) is small (see expressions *(7.5)*, *(6.7)* and *(6.8)*). In that case, changes in the composition of the private consumption basket are not only small (because σ_Q is small) but also cheap (because of the narrow tax base). The "tax composition" effect gains in importance if substitution between leisure and private consumption goods is strong (i.e. if σ_M is large) and hence labour taxes are especially distortionary (see expression *(7.5)*). Accordingly, the "tax composition" effect is large relative to the "tax level" effect if substitution between leisure and private commodities is easy compared to substitution between clean and dirty private commodities. Green and red preferences are thus compatible if dirty and clean private goods are complements. In that case, the tax on labour and the dirt tax yield similar effects. In particular, a labour tax is a good instrument to improve the environment as it reduces private consumption of dirty commodities by raising the consumption of clean leisure. Furthermore, by reducing the real after-tax wage, a dirt tax acts like a labour tax because it leaves the composition of private consumption largely unaffected if the substitution elasticity between clean and dirty goods is small. Hence, a dirt tax is a rather efficient instrument to fund public spending (due to the small excess burden from changing the composition of private demand).

Red and green political preferences can be compatible only if the substitution elasticity between leisure and private commodities (σ_M) is large. Table 3 reveals that in that case labour supply declines substantially. The intuition is that a fall in employment (i.e. more "inactivity") is socially undesirable only if it is due to a distortionary tax on labour. Social welfare is enhanced, however, if employment declines on account of a dirt tax reflecting the social cost of environmental degradation. Indeed, increased environmental concern should make work less attractive, because part of labour income is spent on dirty goods. This is in fact consistent with green parties advocating a reduction in the length of the working week. Since the disincentive effects of high tax levels on labour supply are no longer socially undesirable, environmental concern facilitates the financing of public consumption.[9]

Accordingly, red and green are compatible if most of the improvement in environmental quality is accomplished through a lower level of economic activity and employment. Hence, a tension emerges between the desire to increase the participation (of women) in the labour force while at the same time maintaining public consumption and improving environmental quality. The objectives of enhancing environmental quality and raising labour participation do not conflict only if the substitution elasticity between leisure and private commodities is close to unity ($\sigma_M \approx 1$) while substitution elasticities between clean and dirty goods in private and public consumption (σ_Q and σ_G) are large and the effectiveness of abatement actitivities in cleaning up the environment does not decline rapidly. In that case, the tax *level* effect dominates the tax *composition* effect and the improvement in environmental quality is accom-

[9] A fall in employment is undesirable if it is due to a higher (distortionary) labour tax on account of the "tax level" effect.

plished primarily through a cleaner *composition* of economic activity rather than a lower *level* of that activity.[10] This implies that activity and employment are not discouraged. Hence, the disincentive effects of high tax levels are undesirable and public consumption therefore becomes *more* rather than *less* expensive.

Proposition 2: *Public consumption rises with a higher political preference for a cleaner environment (a "social" double dividend) if and only if σ_M is large (exceeds one) and, at the same time, σ_G and σ_Q are small and σ_A large. Accordingly, clean and dirty commodities need to be complements for red and green preferences to be compatible. Furthermore, the "tax composition" effect should dominate the "tax level" effect in determining the marginal cost of public funds. In the case of a social double dividend, most of the improvement in environmental quality is achieved through a lower level of activity rather than a cleaner composition of that activity.*

7.3 Economic costs of an explicit green target

Environmentalists and government plans often demand explicit improvements in the quality of the environment, but rarely explain how these green targets should be achieved in practice. This subsection explores the question what the best instruments are to achieve a particular absolute green target for the quality of the environment, say E=E*. It also examines the welfare costs of such a target. Social welfare may be split up into three components: (i) the quality of the environment E; (ii) the utility of the basket of public consumption goods G; (iii) the utility of the basket of marketable goods M ("private welfare"). A green target implies an improvement in (i) and, if the economy was at an optimum before the green target was set, must reduce either public consumption or private welfare or both.

Cleaner economic activity: Choice of instruments

If the income and substitution effects of wages on labour supply offset each other (i.e. σ_M=1), the environment is improved by changing the composition rather than the level of economic activity. The higher implicit tax level associated with cleaner economic activity (and hence a cleaner environment, which has the features of a social good) raises the marginal cost of public funds. Hence, the environment crowds out conventional public consumption and thus implies a shift towards "public poverty". Private utility, labour supply and the level of economic activity are not affected in this special case.

Three environmental policy instruments can be employed to render economic activity more friendly to the environment, namely public abatement, cleaner public consumption and a

[10] Note that the tax *level* effect involves a different (and cleaner) *composition* of economic activity while the tax *composition* effect reduces the *level* of activity.

dirt tax aimed at cleaner private consumption. Abatement makes a major contribution to a cleaner environment if the productivity of abatement is not very sensitive to the level of abatement (i.e. σ_A is small). Cleaner public consumption is important if substitution between clean and dirty goods in public consumption is easy (i.e. σ_G is large). The dirt tax plays a large role if economic activity is dirty (i.e. $\theta_D\alpha_Y=\theta_D\alpha_D$ large, see *(7.1)*), substitution between the two public goods is difficult and the productivity of abatement diminishes rapidly with the level of abatement (σ_A large). In that case, most of the improvement in environmental quality must be accomplished by changing the composition of private consumption. This change may be brought about by a dirt tax or, perhaps more naturally, by auctioning off tradeable polluton permits. In the latter case, the price that the government will fetch for a permit equals the dirt tax.

<u>Change in the level of economic activity</u>

Economic activity cannot become cleaner if substitution in private and public consumption is not possible ($\sigma_G=\sigma_Q=0$) and the optimal level of abatement is constant (i.e. $\sigma_A\rightarrow\infty$). In this special case, environmental quality is enhanced only through a reduction in the level of economic activity and employment. If income effects dominate substitution effects ($\sigma_M<1$), the public sector pays for the cleaner environment. The associated lower tax level increases private utility, thereby raising the demand for (clean) leisure. Accordingly, large private incomes due to a small public sector benefit the environment. In this case, there is a "double dividend" in that both environmental quality and private utility increase while public consumption falls. Hence, if $\sigma_M<1$, green and blue preferences are compatible.

However, if substitution between leisure and private commodities is easy and the uncompensated wage elasticity of labour supply is thus positive (σ_M exceeds unity), the environment crowds out private utility and crowds in public consumption. In that case, a higher tax level encourages substitution towards clean leisure. Hence, a large public sector accompanied by a high tax level benefits the environment. In this case, green and red political preferences are compatible as both environmental quality and public consumption increase. This scenario amounts to a more social type of "double dividend".

Proposition 3: *If σ_M is smaller than one, a cleaner environment boosts private welfare and crowds out public consumption. Hence, a double dividend of a cleaner environment and higher private welfare emerges (i.e. blue and green political preferences are compatible). However, if σ_M exceeds one, a cleaner environment crowds out private welfare. If σ_M equals one, environmental quality is enhanced entirely through a cleaner composition rather than a lower level of activity. Public consumption bears all costs associated with the cleaner composition of activity while private welfare is unaffected.*

8 A shift towards redder preferences

Tax structure and employment

Table 4 contains the effects of an increase in the social priority attached to public consumption, i.e. a shift towards redder political preferences (γ_G>0). Equation *(7.1)* shows that the dirt tax falls. This seems counter-intuitive because a rise in public consumption requires higher tax rates. However, labour rather than dirt taxes rise to finance higher public spending. The intuition is that a higher price for public consumption raises the cost of public funds. Hence, revenue raising rather than environmental protection becomes a more important objective of the tax system. The government is thus less able to afford the use of "inefficient" tax instruments from the point of view of raising revenues. Indeed, expression *(4.6)* reveals that the dirt tax falls if the cost of public funds (η) rises. A shift towards more conventional public goods thus requires a higher labour tax rate but a lower dirt tax rate.

Higher public consumption reduces employment if the marginal cost of public funds exceeds unity (i.e. σ_M>1). In that case, the substitution effect of higher taxation reducing after-tax wages (i.e. the disincentive effect) exceeds the income effect of lower private utility.

Proposition 4: *A higher social preference for public consumption raises the labour tax and reduces the dirt tax. Employment declines if and only if σ_M exceeds one. The lower dirt tax yields a dirtier composition of public and private consumption and less public abatement.*

Public consumption, private welfare and environmental quality

Private utility always suffers. Hence, red and blue political preferences are not compatible. The magnitude of the fall in private welfare depends on the substitution elasticity between leisure and private commodities (σ_M). If this elasticity is large, private welfare suffers less. In that case, the cost of public funds increases rapidly as public consumption expands. Hence, public consumption does not rise much.

Comparison of Tables 3 and 4 reveals the following symmetry: the sign and nature of the effect of a shift towards redder preferences on the quality of the environment is the same as the effect of a shift towards greener preferences on public consumption. In particular, environmental quality and public consumption move together and red and green preferences are thus compatible, if dirty and clean goods are complements (i.e. σ_Q small, σ_M large) and if the composition of public spending is insensitive to the (shadow) price for pollution (i.e. σ_G small and σ_A large). If σ_Q and σ_G are small, the lower dirt tax does not induce a large shift toward dirty commodities in private and public consumption. Furthermore, public abatement does not fall much if the productivity of abatement rises rapidly as abatement falls (i.e. σ_A large). Moreover, the higher labour tax acts as an implicit dirt tax and, if σ_M is large, improves the environment by reducing economic activity. Indeed, one reaps a social double dividend of a cleaner environment and more public consumption only if employment falls. In that case, the change in the level and structure of taxes shifts private demand to (clean) leisure rather than dirty commodities.

The environment suffers if the substitution elasticity between leisure and other private goods (σ_M) is small, substitution in private and public consumption baskets is strong (σ_Q,σ_G large), and the productivity of abatement does not rise rapidly as abatement falls (σ_A small). In this case, both the level of economic activity (labour supply) rises and the composition of this activity becomes dirtier. In particular, the adverse private income effect associated with a higher tax level boosts labour supply. Not only does the income effect exceed the substitution effect, but the small substitution effect causes the marginal cost of public funds to rise only slowly as the public sector expands. Accordingly, the tax level, and therefore employment, rise substantially. Higher output raises the sum of private and public consumption. As regards the composition of economic activity, the lower dirt tax induces a lot of substitution towards dirty consumption goods if σ_Q and σ_G are large. Moreover, public abatement falls substantially if σ_A is small.

Public consumption rises most if labour taxes are not very distortionary (i.e σ_M is small) and the lower dirt tax makes a lot of room for public consumption (through less abatement and "cheaper" (i.e. dirtier) public and private consumption). In other words, the lower dirt tax reduces the cost of public funds significantly while the higher labour tax does not raise this cost much. The environment pays for the lower dirt tax as it raises pollution. Private utility suffers on account of the higher labour tax.

Proposition 5: *A higher social preference for public consumption crowds out private welfare (i.e. red and blue prefrences are incompatible). Environmental quality is improved if and only if σ_M is large and, at the same time, σ_Q and σ_G are small and σ_A is large. In this case, economic activity declines but does not become much dirtier while public consumption does not rise much.*

9 Concluding remarks

The "double dividend" hypothesis put forward by many politicians is a red herring. Although it is optimal to raise the dirt tax and reduce the labour tax in response to increased environmental concern, employment typically falls. Dirt taxes are explicitly designed to encourage a change in the composition of private consumption towards cleaner goods. This imposes a cost in terms of private utility, which tends to worsen pre-existing tax distortions by eroding the base of existing distortionary taxes. Moreover, increased environmental concern may make public consumption more expensive and raise public abatement. The implied higher overall tax level is especially costly if existing taxes impose serious distortions. Indeed, countries with a large public sector often find it costly to implement a tough environmental policy because such a policy may erode the tax base and thus require even higher marginal tax rates thereby raising the cost of public funds.

To attenuate the adverse effects of environmental policy on labour market distortions and the cost of public funds, it may be necessary to reduce the size of the public sector. Indeed, we have shown that it is optimal to reduce public consumption in response to increased environmental concern if the elasticity of substitution between private goods and leisure is small and between clean and dirty goods is large. In this case, the environment is improved, in part, through a different composition of economic activity. The costs accompanying this structural change in economic activity raise the cost of public funds, thereby reducing public consumption. However, if substitution between clean and dirty commodities is more difficult and the productivity of public abatement declines rapidly, a cleaner environment must be achieved primarily through less production and more consumption of leisure. In that case, the public sector may expand if the substitution effects due to lower after-tax wages are large. The intuition is that the disincentive effects due to high tax levels are no longer undesirable. Indeed, public consumption becomes easier to finance as non-distortionary taxes are available. Hence, in this case, green political parties are compatible with red political parties. More specifically, red and green preferences are likely to be compatible only if clean and dirty goods are complements, dirt and labour taxes induce similar behavioural effects, and the environment is enhanced through a lower level of economic activity.

An important direction of future research is to abandon the world of representative agents and extend the analysis of this paper to address equity issues. Efficiency considerations may lead governments to tax necessities more heavily than luxuries. Clearly, this violates the objective of equity. Increased environmental concern may imply heavier taxes on dirty necessities, less progressive taxes on labour and less public consumption. Since an equitable income distribution may be viewed as a public good (cf., Thurow, 1971), it is of interest to investigate the trade-off between the various public goods (public consumption of clean and dirty products, the quality of the natural environment and an equitable income distribution). This line of analysis should benefit from abandoning the assumption of homothetic preferences. For example, if one allows for subsistence levels of consumption and allows for necessities, one can shed more light on the issue of basic needs and equity in relation to environmental policy.

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Appendix

Derivation of (6.4)

Substituting expressions for l, c and d from Table 2(b) into *(6.3)* yields:

$$\omega_X x + \omega_Y y + \omega_A a = (\sigma_M - 1)\alpha_V(\omega_C + \omega_D - 1)(t'_L + \alpha_D t'_D) + (t'_L + \alpha_D t'_D)(\omega_C + \omega_D) - \sigma_Q t'_D[\omega_C \alpha_D - \omega_D(1-\alpha_D)]. \tag{A.1}$$

Use of $\omega_C+\omega_D=1-\omega_G'$, $\omega_C=(1-t_L)(1-\alpha_D)$ and $\omega_D=(1-\theta_D)(1-t_L)\alpha_D$ in *(A.1)* yields *(6.4)*.

Derivation of (6.7) and (6.8)

With the aid of *(6.4)*, we obtain

$$t_L' + \alpha_D t_D' = \left(\frac{\omega_X x + \omega_Y y + \omega_A a}{\Delta}\right) + \left(\frac{(1-t_L)\theta_D \alpha_D (1-\alpha_D)\sigma_G t_D'}{\Delta}\right) \tag{A.2}$$

which upon substitution into *(6.5)* yields:

$$m = -(1-\alpha_Y)\Delta^{-1}[\omega_X x + \omega_Y y + \omega_A a + (1-t_L)\theta_D \alpha_D (1-\alpha_D)\sigma_G t_D']. \tag{A.3}$$

Using $g=\alpha_Y y+(1-\alpha_Y)x$ and *(6.6)* gives:

$$\omega_X x + \omega_Y y = (\omega_X + \omega_Y)g + \sigma_G t_D'[\alpha_Y \omega_X - (1-\alpha_Y)\omega_Y]. \tag{A.4}$$

Given that

$$\omega_X = (\omega_X + \omega_Y)\left(\frac{1-\alpha_Y}{1-\theta_D \alpha_Y}\right) \quad \text{and} \quad \omega_Y = (\omega_X + \omega_Y)\left(\frac{\alpha_Y (1-\theta_D)}{1-\theta_D \alpha_Y}\right), \tag{A.5}$$

one can rewrite *(A.4)* as:

$$\omega_X x + \omega_Y y = (\omega_X + \omega_Y)g + (\omega_X + \omega_Y)\left(\frac{1}{1-\theta_D \alpha_Y}\right)[(1-\alpha_Y)\alpha_Y \theta_D]\sigma_G t_D'. \tag{A.6}$$

Substitution of *(A.6)* into *(A.3)* and using $1-t_L=(\omega_C+\omega_D)/(1-\theta_D\alpha_D)$ yields *(6.7)* and *(6.8)*.

Derivation of (6.9)

Loglinearisation of *(5.2)* and multiplication by $\alpha_E \equiv t_D E/\delta N\beta L$ yields (upon substitution of *(5.8)*):

$$\begin{aligned}
\alpha_E e &= -\left(\frac{t_D D}{\beta L}\right)d - \left(\frac{t_D Y}{N\beta L}\right)y + \left(\frac{e^{A'}}{\delta}\right)\left(\frac{t_D A}{N\beta L}\right)a = \\
&- (\theta_D \alpha_D (1-t_L))d - (\omega_X + \omega_Y)\left(\frac{\theta_D \alpha_Y}{1-\theta_D \alpha_Y}\right)y + \omega_A a = \\
&-((1-t_L)\theta_D \alpha_D)\left[d + \left(\frac{\omega_X x + \omega_Y y + \omega_A a}{1-\omega_G'}\right)\right] + \\
&\left[\left(\frac{(1-t_L)\theta_D \alpha_D}{1-\omega_G'}\right)(\omega_X x + \omega_Y y + \omega_A a) - (\omega_X + \omega_Y)\left(\frac{\theta_D \alpha_Y}{1-\theta_D \alpha_Y}\right)y + \omega_A a\right].
\end{aligned} \tag{A.7}$$

Using *(6.4)* (after eliminating $t_L'-\beta'+\alpha_D t_D'$ from *(6.5)*) and the expression for d from Table 2(b), we can rewrite the term in the first set of square brackets on the (last) right-hand side of *(A.7)* as:

Using *(A.6)* and $(1-t_L)(1-\omega_G')^{-1}=(1-\theta_D\alpha_D)^{-1}$, the term in the second set of square brackets on the (last) right-hand side of *(A.7)* may be written as:

$$d+\left(\frac{\omega_X x+\omega_Y y+\omega_A a}{1-\omega_G'}\right)=\left[1-\alpha_V+\sigma_M\alpha_V-\frac{\Delta}{1-\omega_G'}\right]\frac{m}{1-\alpha_V}-(1-\alpha_D)\sigma_Q t_D'\left[1+\frac{(1-t_L)\theta_D\alpha_D}{1-\omega_G'}\right]= \\ \left[\frac{\alpha_V(\sigma_M-1)}{1-\omega_G'}\right]\frac{m}{1-\alpha_V}-\left[\frac{(1-\alpha_D)\sigma_Q t_D'}{1-\theta_D\alpha_D}\right]. \tag{A.8}$$

$$\left[\frac{\theta_D(\alpha_D-\alpha_Y)}{(1-\theta_D\alpha_D)(1-\theta_D\alpha_Y)}\right](\omega_X+\omega_Y)g+\left(\frac{1}{1-\theta_D\alpha_D}\right)\left(\omega_A a+\left(\frac{\omega_X+\omega_Y}{1-\theta_D\alpha_Y}\right)\theta_D\alpha_Y(1-\alpha_Y)\sigma_G t_D\right). \tag{A.9}$$

Finally, substituting *(A.8)* and *(A.9)* into *(A.7)*, using $(1-t_L)=(\omega_C+\omega_D)(1-\theta_D\alpha_D)^{-1}$ and making use of definition *(6.8)* yields *(6.9)*.

Derivation of (7.5)

Using the demand functions of Table 2 and *(6.5)*, we obtain

$$v-q=\sigma_M(t_L'+\alpha_D t_D')=-\left(\frac{\sigma_M}{1-\alpha_V}\right)m \tag{A.10}$$

which yields:

$$d\log\left(\frac{\partial M}{\partial Q}\right) = \left(\frac{\alpha_V}{\sigma_M}\right)(v-q) = -\left(\frac{\alpha_V}{1-\alpha_V}\right)m. \tag{A.11}$$

We use ϵ_{LL}' from Table 2(b) and $\alpha_V=V$ to rewrite *(5.6)* as

$$\eta^{-1} = \left[1-\left(\frac{t_L}{1-t_L}\right)V(\sigma_M-1)\right] \tag{A.12}$$

which upon loglinearisation yields:

$$-d\log(\eta) = -\left[\frac{t_L\alpha_V(\sigma_M-1)v+\alpha_V(\sigma_M-1)t_L'}{1-t_L(1+\alpha_V(\sigma_M-1))}\right]. \tag{A.13}$$

We substitute t_L' from *(6.5)* and $v=-(\sigma_M-1)m$ (see Table 2(b)) and obtain:

$$-d\log(\eta) = \left[\frac{\alpha_V(\sigma_M-1)[t_L(\sigma_M-1)+(1-\alpha_V)^{-1}]m+\alpha_V(\sigma_M-1)\alpha_D t_D'}{1-t_L(1+\alpha_V(\sigma_M-1))}\right]. \tag{A.14}$$

Combining *(A.11)* and *(A.14)* we obtain:

$$d\log(\eta M_Q) = -\left(\frac{\alpha_V}{1-\alpha_V}\right)\left[1+(\sigma_M-1)\left(\frac{1+t_L(\sigma_M-1)(1-\alpha_V)}{1-t_L(1+\alpha_V(\sigma_M-1)}\right)\right]m - \left(\frac{\alpha_V(\sigma_M-1)}{1-t_L(1+\alpha_V(\sigma_M-1))}\right)\alpha_D t_D'. \quad (A.15)$$

Upon substituting $\sigma_M\kappa[1-t_L(1+\alpha_V(\sigma_M-1))]^{-1}$ for the term in square brackets in *(A.15)*, we obtain *(7.5)*.

Derivation of Tables 3 and 4

All entries can be written as functions of $m' \equiv m/(1-\alpha_V) = -(t_L' - \beta' + \alpha_D t_D')$ (using *(6.5)*) and t_D'. t_L' follows immediately from the definition of m'. Equations *(6.6)*, *(7.2)* and *(7.3)* give, respectively, x-y, c-d and a. Eliminating m from the definition of m' and a from *(6.7)* and using the definition of ρ (see Table 1), we obtain:

$$(\omega_X+\omega_Y)g = -\Delta m' - \left[\chi + \left(\frac{\omega_A}{\theta_D\sigma_A}\right)\right]t_D' = -\Delta m' - \rho t_D'. \quad (A.16)$$

Equations *(6.9)* and *(6.10)* with $\alpha_D = \alpha_Y$ (and substituting *(7.3)* to eliminate a and using the definition of ρ) gives

$$e = \left[\frac{-\theta_D\alpha_D(\sigma_M-1)\alpha_V m' + \rho t_D'}{\alpha_E(1-\theta_D\alpha_D)}\right]. \quad (A.17)$$

Equation *(6.10)* gives $l = (\sigma_M - 1)\alpha_V m'$.

To complete Tables 3 and 4, we need expressions for t_D' and m'. Equation *(7.1)* amounts to a reduced-form expression for t_D'. Upon substitution of this expression for t_D' into *(7.5)*, we obtain:

$$m' = -\left(\frac{\sigma_M-1}{\sigma_M\kappa}\right)\left(\frac{\alpha_D\theta_D}{1-\theta_D\alpha_Y}\right)\gamma_E - \left[\frac{\Delta}{\alpha_V\sigma_M\kappa(1-\theta_D\alpha_D)}\right]\gamma_G. \quad (A.18)$$

Proof that t_L falls if $\gamma_E > 0$

Looking at the entry for t_L' in Table 3, we need to establish that:

$$F(\sigma_M) \equiv \sigma_M t_L[1-(1-\alpha_V)(\sigma_M-1)] \le 1. \quad (A.19)$$

Since $\omega_G' = t_L + (1-t_L)\theta_D\alpha_D$ and stability requires $\Delta > 0$, t_L cannot exceed $(1-\alpha_V+\sigma_M\alpha_V)^{-1}$. For $\sigma_M > 1$, it is then possible to show that:

$$F(\sigma_M) < \sigma_M\left[\frac{1+(1-\alpha_V)-(1-\alpha_V)\sigma_M}{(1-\alpha_V)+\sigma_M\alpha_V}\right] < 1. \quad (A.20)$$

For $\sigma_M < 1$, $t_L < 1$ and one can also establish that $F(\sigma_M)' > 0$, $F(1) = 1$ and thus $F(\sigma_M) < 1$. The only case that $t_L' = 0$ is $\sigma_M = t_L = 1$.

Table 1: Notation

N	number of households
U	social welfare
E	quality of the environment
A	level of abatement activities
M	utility of marketable goods
G	utility of the basket of clean and dirty public goods (i.e. public consumption)
X,Y	consumption of clean and dirty public goods
Q	utility of the basket of clean and dirty private goods
C,D	consumption of clean and dirty private goods
V,L	leisure and labour supply
T	(lump-sum) income transfers from government to private agents
P_C,P_D	consumption prices of clean and dirty products in private consumption
t_D	tax (rate) on private consumption of dirty products
t_{DP}	Pigovian component of the dirt tax
P_L,β	consumption wage ($\beta(1-t_L)$) and producer wage
t_L	tax rate on labour income
θ_D	t_D/P_D
θ_L	$t_L/(1-t_L)$
t_i'	deviation of t_i, normalised by P_i, i=D,L
P_Q	ideal price index for the basket of private consumption goods
P_G	ideal (shadow)price index of public consumption
λ	marginal utility of private income
μ,η	marginal disutility and marginal cost (μ/λ) of public funds
η'	μ/λ'
α_D	budget share of dirty products in private consumption ($P_D D/P_L L$)
α_V	budget share of leisure in marketable goods ($M_V V/M=V$)
α_Y	(shadow) budget share of dirty goods in public consumption ($G_Y Y/G$)
α_E	ratio of value of environment to national income ($(t_D/\delta)E/(N\beta L)$)
ω_i	national income share of good i ($I/\beta NL$,i=X,Y,A, $I/\beta L$,i=C,D)
ω_G'	national income share of public spending ($(X+Y+A)/\beta NL$)
ω_E	value of environment as fraction of national income ($t_D E/\delta_D \beta NL$)
σ_M	elasticity of substitution between Q and V
σ_G,σ_Q	elasticities of substitution between X and Y and between C and D
σ_A	elasticity of productivity of public abatement ($-e^{A''}A/e^{A'}$)
ϵ_{ij}'	uncompensated elasticity of demand for good i with respect to price j
ϵ_{ij}	compensated elasticity of demand for good i with respect to price j

Γ_i	weight of I in social welfare $U=\Gamma_G G+\Gamma_E E+\Gamma_M M$
δ	emission ratio for dirty public and private goods
κ	$1-t_L[1-(1-\alpha_V)(\sigma_M-1)]$
Δ	$1-\omega_G'(1+\epsilon_{LL}')=1-\omega_G'[1-\alpha_V(1-\sigma_M)]$
ρ	$\theta_D\alpha_D(1-\alpha_D)[(1-t_L)\sigma_Q+\omega_G'(1-\alpha_Y t_D)^{-1}\sigma_G]+\omega_A(\theta_D\sigma_A)^{-1}$
χ	defined in *(6.8)*

Table 2: **(a) Compensated demand elasticities** (ϵ_{ij})

	p_C	$p_D=t_D'$	$p_L=-t_L'$	m
q	$-\sigma_M\alpha_V(1-\alpha_D)$	$-\sigma_M\alpha_V\alpha_D$	$\sigma_M\alpha_V$	1
v	$\sigma_M(1-\alpha_V)(1-\alpha_D)$	$\sigma_M(1-\alpha_V)\alpha_D$	$-\sigma_M(1-\alpha_V)$	1
l	$-\sigma_M\alpha_V(1-\alpha_D)$	$-\sigma_M\alpha_V\alpha_D$	$\sigma_M\alpha_V$	$-\alpha_V/(1-\alpha_V)$
c	$-\alpha_D(\sigma_Q-\sigma_M\alpha_V)-\sigma_M\alpha_V$	$\alpha_D(\sigma_Q-\sigma_M\alpha_V)$	$\sigma_M\alpha_V$	1
d	$(1-\alpha_D)(\sigma_Q-\sigma_M\alpha_V)$	$\alpha_D(\sigma_Q-\sigma_M\alpha_V)-\sigma_Q$	$\sigma_M\alpha_V$	1

Table 2: **(b) Uncompensated demand elasticities** (ϵ_{ij}')

	p_C	$p_D=t_D'$	$p_L=-t_L'$	T^*
m	$-(1-\alpha_D)(1-\alpha_V)$	$-\alpha_D(1-\alpha_V)$	$1-\alpha_V$	$1-\alpha_V$
q	$-(1-\alpha_D)[(\sigma_M-1)\alpha_V+1]$	$-\alpha_D[(\sigma_M-1)\alpha_V+1]$	$(\sigma_M-1)\alpha_V+1$	$1-\alpha_V$
v	$(1-\alpha_D)(\sigma_M-1)(1-\alpha_V)$	$\alpha_D(\sigma_M-1)(1-\alpha_V)$	$(1-\sigma_M)(1-\alpha_V)$	$1-\alpha_V$
l	$(1-\alpha_D)(1-\sigma_M)\alpha_V$	$\alpha_D(1-\sigma_M)\alpha_V$	$(\sigma_M-1)\alpha_V$	$-\alpha_V$
c	$-\alpha_D\sigma_Q-(1-\alpha_D)[(\sigma_M-1)\alpha_V+1]$	$\alpha_D[\sigma_Q-(\sigma_M-1)\alpha_V-1]$	$(\sigma_M-1)\alpha_V+1$	$1-\alpha_V$
d	$(1-\alpha_D)[\sigma_Q-(\sigma_M-1)\alpha_V-1]$	$-(1-\alpha_D)\sigma_Q$ $-\alpha_D[(\sigma_M-1)\alpha_V+1]$	$(\sigma_M-1)\alpha_V+1$	$1-\alpha_V$

* This column gives semi-elasticities with respect to changes in $T/P_L L$.

Table 3: Shift towards greener preferences

	$\gamma_E > 0$
ι_D'	$(1-\theta_D\alpha_Y)^{-1}\theta_D > 0$
ι_L'	$-(1-\theta_D\alpha_D)^{-1}\theta_D\alpha_D\sigma_M{}^{-1}\kappa^{-1}\{1-\sigma_M\iota_L[1-(1-\alpha_V)(\sigma_M-1)]\} < 0$
a	$(1-\theta_D\alpha_D)^{-1}\sigma_A{}^{-1} > 0$
g	$(1-\theta_D\alpha_D)^{-1}\theta_D[\alpha_D\Delta\kappa^{-1}\sigma_M{}^{-1}(\sigma_M-1)-\rho]\,(\omega_X+\omega_Y)^{-1}$
$x-y$	$(1-\theta_D\alpha_Y)^{-1}\theta_D\sigma_G > 0$
m	$(1-\theta_D\alpha_D)^{-1}\theta_D\alpha_D\kappa^{-1}\sigma_M{}^{-1}(1-\sigma_M)(1-\alpha_V) < 0$ if $\sigma_M > 1$
$c-d$	$(1-\theta_D\alpha_Y)^{-1}\theta_D\sigma_Q > 0$
e	$(1-\theta_D\alpha_D)^{-1}\theta_D[\theta_D\alpha_D{}^2\kappa^{-1}\sigma_M{}^{-1}(\sigma_M-1)^2\alpha_V+\rho]\,[\alpha_E(1-\theta_D\alpha_D]^{-1} > 0$
l	$-(1-\theta_D\alpha_D)^{-1}\theta_D\alpha_D\kappa^{-1}\sigma_M{}^{-1}(\sigma_M-1)^2\alpha_V \leq 0$

Table 4: Shift towards redder preferences

	$\gamma_G > 0$
ι_D'	$-(1-\theta_D\alpha_Y)^{-1}\theta_D < 0$
ι_L'	$(1-\theta_D\alpha_D)^{-1}[\Delta\kappa^{-1}\sigma_M{}^{-1}\alpha_V{}^{-1}+\theta_D\alpha_D] > 0$
a	$-(1-\theta_D\alpha_D)^{-1}\sigma_A{}^{-1} < 0$
g	$(1-\theta_D\alpha_D)^{-1}[\Delta^2\kappa^{-1}\sigma_M{}^{-1}\alpha_V{}^{-1}+\rho\theta_D](\omega_X+\omega_Y)^{-1} > 0$
$x-y$	$-(1-\theta_D\alpha_Y)^{-1}\theta_D\sigma_G < 0$
m	$-(1-\theta_D\alpha_D)^{-1}\Delta\sigma_M{}^{-1}\kappa^{-1}(1-\alpha_V)\alpha_V{}^{-1} < 0$
$c-d$	$-(1-\theta_D\alpha_Y)^{-1}\theta_D\sigma_Q < 0$
e	$(1-\theta_D\alpha_D)^{-1}\theta_D[\alpha_D\Delta\kappa^{-1}\sigma_M{}^{-1}(\sigma_M-1)-\rho]\,[\alpha_E(1-\theta_D\alpha_D]^{-1}$
l	$(1-\theta_D\alpha_D)^{-1}(1-\sigma_M)\Delta\sigma_M{}^{-1}\kappa^{-1} < 0$ if $\sigma_M > 1$

Figure 1: Utility tree